An Efficient Approach for Steiner Tree Problem by Genetic Algorithm

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ABSTRACT : **We** *have applied a genetic algorithm (GA) to solve the Steiner Minimal Tree problem in graphs. In this a simple bit string representation is used, where a 1 or 0 corresponds to whether or not a node is included in the solution tree. The standard genetic operators-selection, crossover and mutation-are applied to both random and seeded initial populations of representations. Various parameters within the algorithm have to be set and we discuss how and why we have selected the values used. A standard set of graph problems used extensively in the comparison of Steiner tree algorithms has been solved using our resulting algorithm. We report our results (which are encouragingly good) and draw conclusions the proposed algorithm has several advantages. One advantage is that, it guarantees convergence to an optimal solution with probability one. And another advantage is that, not only the resultant solutions all are feasible, the solution quality is also much higher than that obtained by the other methods (immensely, in almost every case in our simulations, the algorithm can find the optimal solution of the problem).*

Keywords - *Algorithm, Crossover, Genetic, Graph, Mutation, Steiner Tree.*

I. INTRODUCTION

1.1 Introduction to Steiner Tree Problem

The Steiner Problem in a Graph (SPG) is one of the classic problems of combinatorial optimization. After placement is completed, the routing surface is divided in to smaller routing regions. Then a global router is used to assign each net to a few routing regions to accomplish the required interconnections and finally a detailed router is used. Steiner problem in a graph, SPG is a sub problem of global routing. The SPG arises in a large variety of optimization problems such as network design, multiprocessor scheduling and IC design. Since the SPG is NP-complete, so several of its algorithms have exponential worst time complexities.

In the Steiner tree problem, in order to reduce the length of the spanning tree an extra intermediate vertices and edges may be added to the graph. It results to decrease the total length of connection which is known as Steiner points or Steiner vertices and the resulting connection is a tree, known as the Steiner tree. For a given set of initial vertices there may be several Steiner trees. Generally Rectilinear Steiner Tree (RST) is used.

1.2 Introduction to Genetic Algorithm

The term Genetic Algorithm (GA) are very different from most of the traditional optimization methods and it refers to a class of adaptive search procedures based on principles derived from the dynamics of natural population genetics. Genetic algorithm need design space to be converted in to genetic space. So, genetic algorithms work with a coding of variables. The invention of that technique were developed by Holland33, who also established the theory to explain the subsequent success of the application of GAs to a wide variety of problems, such as classifier systems, network configuration, pattern recognition, and other game problems, general combinatorial optimization. GA is adaptive heuristic search algorithm based on the evolutionary idea of natural selection and genetics.

The simple form of GA is given by the following:

i. Start with a chromosomal representation of a solution to the specified problem;
ii. Select an initial population of solutions taken from the totality of possible solutions to the problem;
iii. The competitive selection of solution representations for reproduction based on some evaluation function. (survival of the fittest);
iv. Idealized genetic operators that recombine the selected representations to create new

structures for possible inclusion in the population;

v. A replacement strategy to maintain a steady population.

II. IMPLEMENTATION OF GENETIC ALGORITHM

2.1 Problem Specification

Suppose an undirected graph G = (V, E) which has a finite set of vertices V, and an edge set, E. Consider we have a set, S $\subseteq$ V, of special vertices. In graph using Steiner minimal tree problem is used to find a sub graph, G' = (V', E'), of G such that

i. V' contains all the vertices in S;

ii. G' is connected; and

iii. $\sum$ {c (e): e ε E'} is minimal.

It is essential that the connected sub graph must be a tree. It may contain other vertices also other than those in S; such vertices, i.e. those in V' - S, referred as Steiner vertices. For example, consider the graph, the vertices which are numbered as 1, 2, 3, 6 and 7 are the special vertices and are shaded in the figure 1.

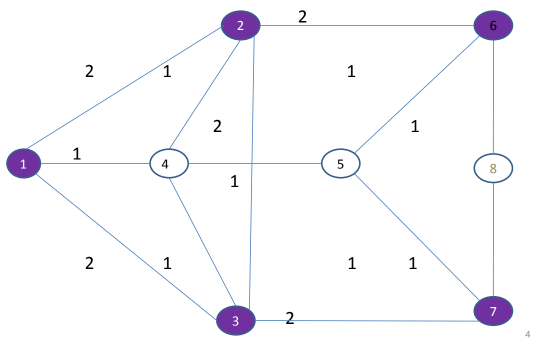

Figure: 1

Now, Steps for applying genetic algorithm in Steiner tree problem

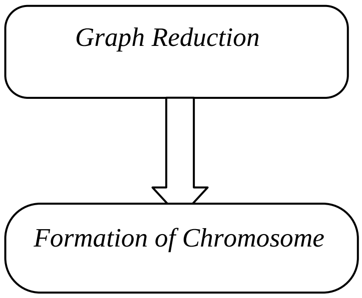

Figure: 2

After the formation of chromosome/initial individual we can apply genetic algorithm.

2.2 Graph Reduction

Before the GA itself is executed, an attempt to reduce the size of the graph using standard graph reduction technique – Routine graph Reduction performs 4 types of simple reduction. Such as

Compute D (G);

While improvements made Do

reduction (3); Graph Reduction

reduction (2);

reduction (4);

reduction (1);

Due to the required update of D (G) a single reduction requires time O (n^2).In total, routine graph Reductions time complexity is O (n^3).

D (G) is the distance graph of G. Let $e_{v,w}$ denotes the edge between vertices v and w, and let sp(v, w) $\subseteq$ E denote the shortest path between v and w.

The four reductions are:

Reduction 1: - Assume deg (v) = 1 and e_{vw} □ E. If v є W any MStT must include e_{vw}. Hence, v and e_{vw} can be removed from G and w is added to V' if it is not already there. If v є V \ W, no MStT can include e_{vw} i.e. in this case v and e_{vw} can also be deleted.

Reduction 2 :- If v є V, deg(v) = 2 and e_{uv}, e_{vw} є E, then v, e_{uv} and e_{vw} can be deleted from G and replaced by a new edge between u and w of equivalent cost.

Reduction 3:- If $e_{vw} \in E$ and $c(e_{vw}) > c(sp(v,w))$ then no MStT can include e_{vw}, which therefore can be deleted.

Reduction 4:- Assume that $v \in W$ and denote the closest neighbor to v by $u \in V$, and the second-closest neighbor by $w \in V$. Since G is connected, u always exists. If w does not exist, assume $c(e_{vw}) = \infty$. Let z be a vertex in $W \setminus \{v\}$ which is closest to u. If $c(e_{vu}) + c(sp(u, z)) \leq c(e_{vw})$ then any MStT must include e_{vu}. Therefore, G can be contracted along this edge.

2.3 Reduced graph

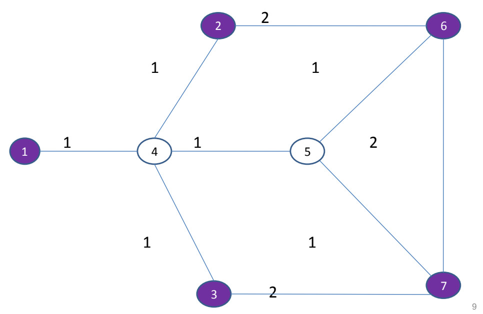

Figure 3: Reduced Graph

Now, each individual solution represented uniquely so that it is appropriate to the application and amenable to the GA. In most of the applications, a bit string representation method is sufficient and preferable. In this method, each bit position i corresponds to a node V_i, and a '1' means that vertex Vi ε V', a '0' means that Vi ∉ V'. For example, referring to Figure, we see that the bit string 1110011 represents the sub graph containing only the special vertices. Different techniques are used to generate the initial population. As the population can be created at random, while in the second set, this randomly created population can be seeded by including a feasible solution equivalent to the MST of the entire graph

2.4 Genetic Algorithm

- produce an initial population of individuals
- evaluate the fitness of all individuals
- while termination condition not met do
- select fitter individuals for reproduction
- recombine between individuals
- mutate individuals
- evaluate the fitness of the modified individuals
- generate a new population End while

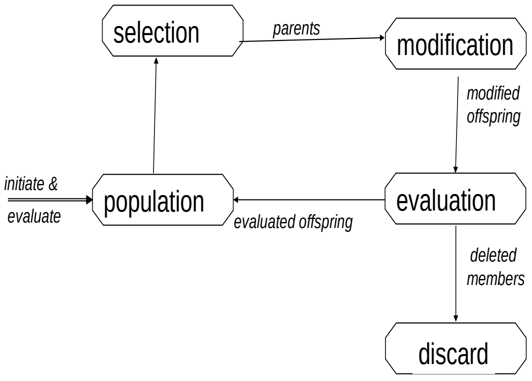

Figure 4: Flow Diagram of Genetic Algorithm

2.5 Selection Scheme

Selection is the process of choosing two parent from the population for crossing. The purpose of selection is to emphasize fitter individuals in the population in hopes that their offspring have higher fitness. The various selection schemes are

Roulette Wheel selection scheme

Tournament selection scheme

Rank Selection

Boltzmann Selection

We are using here Tournament selection scheme.

Principle: - Each chromosome will have equal probability to be selected for crossover.

- Pick up two chromosomes randomly.
- Compare the fitness functions of both chromosomes.
- Select the best chromosome.
- Repeat the process for 16 times.

Selected chromosomes will be paired up sequentially to participate in crossover.

2.6 Fitness Function

Given a population P = {p0, p1. . . pM−1} then computes the fitness of each individual as follows. (Population is referred as a collection of individual)

Let C (p) be the cost of individual p, i.e. the cost of the Steiner tree represented by p, and assume that P is sorted so that C (p0) ≥ C (p1) ≥ . . . ≥ C (pM−1). The fitness F of pi is then computed as

$$F(p_i) = 2i / M-1 \qquad i = 0, 1 . . . M - 1.$$

M is the number of individuals.

2.7 Crossover

Probability of crossover (Pc)?

Indicates how many chromosomes from the parent population will be selected to participate in the crossover operation. If n = 20 & Pc = 0.8. Then 16 chromosomes will be selected. Usually the Pc value is kept high i.e. its range: - 0.7 to 1. Here, we are taking two chromosomes 1110011, 1001111 for performing crossover.

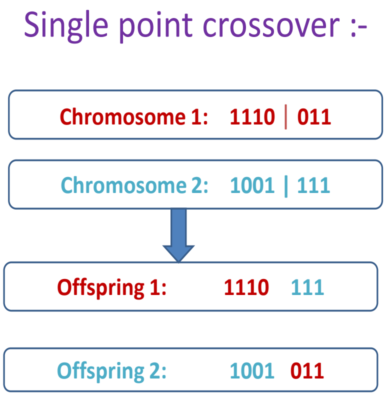

Figure: 5 Single Point Crossover

2.8 Mutation

Probability of Mutation (Pm)? Indicates how many bits of the crossover children will participate in mutation. Usually the Pm value is kept low i.e. its range is: - 0 to 0.2.

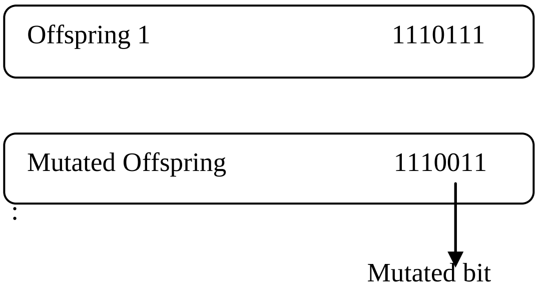

Figure: 6 Mutation

III. RESULT

All new solutions replace solutions in the old population. The whole generation cycle is repeated many times unless and until there is no marked improvement is noticed in the fitness value of the best individual, or some other stopping criterion has been met. Here in the mutated offspring the bit stream was 1110011. Here bit 1 represents the terminal node and bit 0 represents the non terminal node. So the tree for this result is as:

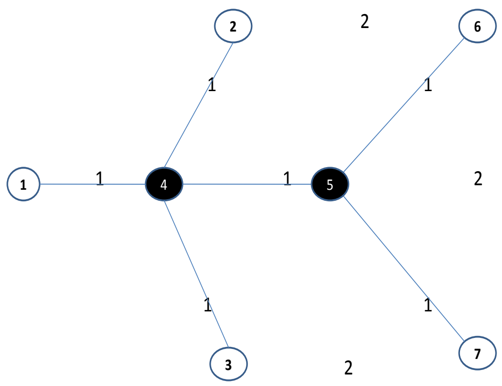

Figure 7: Resultant Graph

IV. CONCLUSIONS

Since GA is intrinsically parallel, so our main conclusion is that Genetic Algorithms provide a noteworthy successful method for finding solutions to the Steiner tree problem in sparse graphs, non-sparse graph and also other NP-hard combinatorial optimization problems. We are currently researching this. On the solution of the Steiner Tree problem in graphs we are applying the GA paradigm and found that the technique is extremely powerful in the solutions to a set of test problems were always found even with relatively large variations in the values of the parameters used.

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