

# Welding Sequence Optimization Using Artificial Intelligence Techniques, an Overview

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**Abstract**— With heightened emphasis to improve the product quality and process efficiency, the welding industry is challenged to consider innovative approaches like artificial intelligence (AI) techniques. In terms of quality, deformation and residual stress are one of the major concerns. It has been proved that the welding sequence has significant effects on deformation and lesser magnitude for residual stress. On the other hand, robot path planning is a crucial factor to efficiently weld large and complex structures. In this sense, Welding Sequence Optimization (WSO) is suitable for minimizing constraints in the design phase, reworks, quality cost and overall capital expenditure. Traditionally the welding sequence is selected by experience and sometimes a design of experiments is required. However, it is practically infeasible to run a full factorial design to find the optimal one, because, the amount of experiments grows exponentially with the number of welding beads. Virtual tools like finite element analysis (FEA) and robotics simulators allow to run corresponding optimization tasks. In this paper we overview the literature on AI techniques applied to WSO. Additionally, some relevant works that use other methods are taken into account. The reviewed works are categorized by AI technique.

**Keywords**— Welding sequence optimization, welding distortion, welding residual stress, welding process optimization.

## I. INTRODUCTION

In order to succeed in the rapidly evolving global manufacturing landscape, there is a need to increase the competitiveness in the welding industry. Some of the top drivers that still preset are quality, productivity, efficiency, reliability, talent, among others. The key to unlock the future competitiveness are the advanced manufacturing technologies. Nowadays, manufacturers are fully submerged into the digital and physical worlds, where the hardware is combined with software, sensors, and sometimes massive amounts of data is analyzed in a smart way. Therefore smarter products, processes are coming to the market because customers, suppliers and the manufacturing itself are more closely connected [1]. According to this, the Internet-of-Things (IoT), industry 4.0 as well as the development and use of advanced materials will be critical to future competitiveness.

Welding processes are non-linear complex systems with multiple input/output parameters. Owing to this, various optimization methods have been developed. Literature on welding

process optimization (WPO) can be categorized into three main topics: quality, efficiency and simulation. Quality and efficiency are main drivers for competitiveness as we described before. Simulation enables the implementation of AI and ML techniques, because "time to market" and "Do It Right The First Time" are pushing the industry to use virtual tools [2]. This classification and their sub-objective targets are shown in Figure 1.

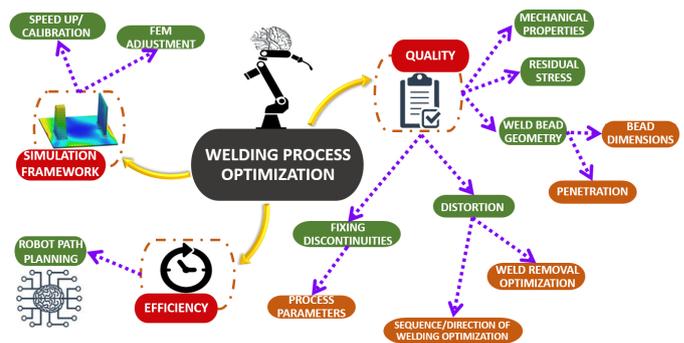


Fig. 1. Welding Optimization

In this paper we present an overview on welding sequence optimization (WSO). There are few works within the perspective of quality where the welding sequence is one of the most promising and widely used technique for minimizing deformation as well as residual stress [3]. So, this overview complements the current available literature. Beyonius *et al.* [4] have done a reference guide where the works were classified basically into weld bead geometry prediction and mechanical properties. Joshi *et al.* [5] describes various statistical and soft computing optimization techniques. Cited works are for diverse applications and welding processes sorted by chronological order.

Deformation and residual stress significantly impact a wide range of industries such as automotive, shipbuilding, aerospace, construction, gas and oil trucking, nuclear, pressure vessels, heavy and earth-moving equipment. [6] [7]. Deformation impacts the assembly process of sheet metal parts, on the other hand, residual stress affects the in-service performance of

welded structures. Hence, there is a need to keep both of them as minimum as possible. Welding deformation and residual stress can be numerically computed through finite element analysis (FEA) without performing expensive experiments. However, under certain circumstances it can be computationally very expensive and time consuming. The conventional approach is to select the best sequence by experience using a simplified design of experiments which often does not offer an optimal sequence [8].

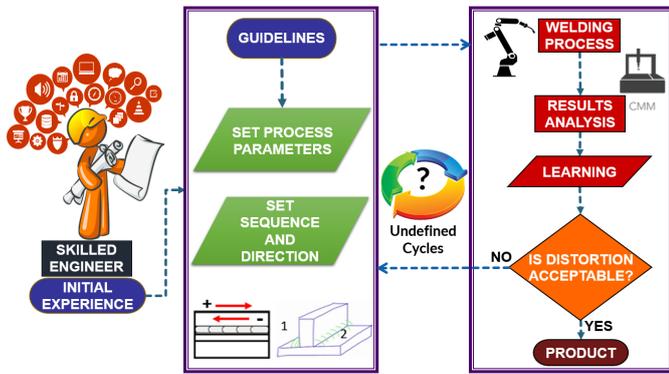


Fig. 2. Conventional approach for selecting a sequence

The optimal welding sequence is only guaranteed using a full factorial design procedure. In this sense, the total number of welding configurations ( $N$ ) are computed by  $N = n^r \times r!$ , where  $n$  and  $r$  are the number of welding directions and beads (seams) respectively. These possible configurations grows exponentially with the number of welding segments. Considering a practical scenario, a complex weldment like an aero-engine assembly might have between 52 and 64 weld segments [9]. Therefore, full factorial design is often practically infeasible even using FEA.

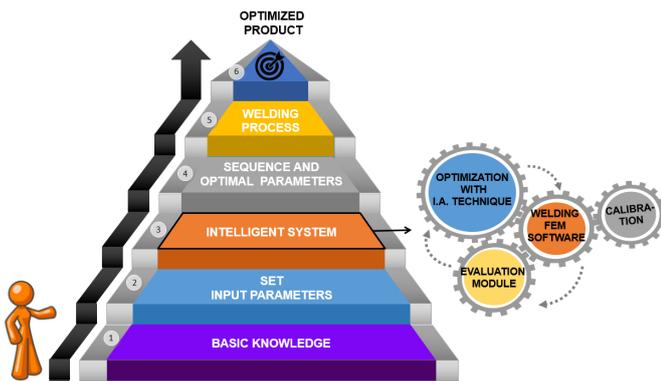


Fig. 3. Modern approach for selecting a sequence

Considering the most important advanced manufacturing technologies which are predictive analytics, smart connected products-(IoT), advanced materials, smart factories-(IoT), digital design - simulation and integration, high performance computing, advanced robotics, additive manufacturing (3D

printing), open-source design/direct customer input and augmented reality, WSO takes special relevance when it comes to additive manufacturing [1]. In particular the Wire and Arc Additive Manufacturing (WAAM) is a promising alternative to traditional manufacturing methods for fabricating large and complex metal components. WSO is totally required in order to plan the path for welding deposition layer by layer introducing as less as possible deformation and residual stress [10].

II. AI TECHNIQUES APPLIED TO WSO

A. Genetic Algorithms

GA emulate natural selection of a set of individuals in order to search the best solution to a problem [11]. The genetic configuration of each individual is a possible solution. The algorithm starts with an initial population and those are submitted to an evolutionary process in such way that the best adapted individuals will continue to reproduce among them and over several generations the best adapted stands out.

Chapple *et al.* [12] have developed a GA approach for welding distortion optimization from two perspectives: (i) weld removal optimization and (ii) a combination of weld removal and welding sequence optimization. They proposed a fitness function in terms of total distortion in a critical region as shown in Equation 1. However, constraints on stress and stiffness were added in weld removal optimization to prevent removing many weld seams. A simplified FEA was used for fitness function evaluation.

$$F = \text{Min}(\text{Max}(D_i)) \text{ if } S_i > T \quad (1)$$

$$i = 1, 2, 3 \dots N \quad i \in R_c$$

Where:  $D_i$  is the total deformation for all nodes  $i$  in the critical region  $R_c$ ,  $S_i$  is the stiffness of the structure and  $T$  is the minimum stiffness defined value. Total deformation is computed by the following equation:

$$D_i = \sqrt{d_{x_i}^2 + d_{y_i}^2 + d_{z_i}^2} \quad (2)$$

Where  $d_{x_i}$ ,  $d_{y_i}$ , and  $d_{z_i}$  are the deformations of node  $i$  along  $x$ ,  $y$ , and  $z$  axis respectively.

Islam *et al.* [13] have implemented GA in order to minimize the distortion in welded structures. They exploited a fitness function in terms of the maximum distortion on the overall structure. They have a conditional that includes a penalty term which is proportional to the number of nodes on the weld seam that have temperature less than melting value Equation 3. The penalty term determines upper and lower bounds for welding process parameters such as current, voltage and speed. They also defined six variables for possible welding direction. A thermo-mechanical FEA was carried out on a specimen as well as an automotive part. Experimental tryouts were done on a specimen using GMAW process.

$$F = \begin{cases} g & \text{IF } Q = 0 \\ g + M_1 & \text{IF } Q > 0 \end{cases} \quad (3)$$

where:

$$g = \text{Min}(\text{Max}(D_i)) \quad (4)$$

$$M_1 = 100Q \quad (5)$$

$D_i$  is the total deformation given by Equation 2,  $Q$  are the number of nodes in the weld seam that are below the melting point;  $M_1$  is a penalty term that is proportional to  $Q$ .

Mohammed *et al.* [7] present an optimization procedure where GA and FEA minimize the welding induced distortion. The fitness function (Equation 6) used in their work is in terms of displacements along Z geometrical axis. This fitness function was developed for the simplified model of an aero-engine part where the distortion on Z axis dominates the other ones.

$$\text{Min } F = \text{Max}(|(d_z)_i|) \quad (6)$$

$$i = 1, 2, 3, \dots, N$$

Where:  $d_z$  is the deformation on z axis and  $N$  the total amount of nodes.

Liao [14] presents an implementation of GA for searching the optimal weld pattern in a spot welding process. The proposed fitness function is computed in two ways, first, in a deterministic mode which means the future states depend from the previous ones. Second, in a stochastic mode where the future states do not depend from the previous ones. FEA was used to compute the fitness function. The fitness function for the deterministic mode is shown in Equation 7:

$$F = \sum_{i=1}^n w_{1i}(D_i)^2 \quad (7)$$

$$i = 1, 2, 3, \dots, N$$

Where  $w_{1i}$  is a weight factor that determines the importance of each node;  $D_i$  is the total deformation on all the nodes  $N$ . The fitness function for the stochastic mode is shown in equation 8:

$$F = \sum_{i=1}^n w_{1i}(U_i)^2 + w_{2i}(V_i) \quad (8)$$

$$i = 1, 2, 3, \dots, N$$

Where  $w_{1i}$  and  $w_{2i}$  are weights,  $U_i$  is the average deformation on every single node and  $V_i$  is the variance of the deformation.

Xie and Hsieh [15] have implemented GA for finding a combined clamping and welding sequence. A multi-objective fitness function is taken into account to minimize cycle time (gun travel path) and assembly deformation as shown in Equation 9. FEA was used to evaluate the fitness function on automotive parts joined by spot welding process.

$$\text{Min } F = w_1 \frac{D_i}{D_{0i}} + w_2 \frac{C}{C_0} \quad (9)$$

$$i = 1, 2, 3, \dots, N$$

Where,  $w_1$  and  $w_2$  are weights that define the importance of each sub-function;  $D_i$  is the total deformation on all nodes for the actual generation.  $D_{0i}$  is the total deformation on all nodes for the initial generation;  $C$  is the cycle time for the actual generation and  $C_0$  is the cycle time for the initial generation. Notice that  $\frac{D_i}{D_{0i}}$  and  $\frac{C}{C_0}$  are considered as normalized functions because the units of deformation and cycle time are different.

Kim *et al.* [16] have implemented GA using a multi-criteria fitness function. This function includes the minimization of gun travel time, avoidance of thermal distortion and smooth robot joint movement. The criteria considered here are Euclidian distance between weld seams, a 30 mm distance considered as heat affected zone and total change of the robot joints respectively. This algorithm is suitable for different arc welding operations such as multi weld lines: singlepass or multipass.

$$\text{Min } F = \text{Min}(w_1g_1 + w_2g_2) \quad (10)$$

Where:  $w_1$  and  $w_2$  are weights. The sub-function that involves gun travel time and distortion criteria  $g_1$  is defined by

$$g_1 = \sum_{a_{ij} \in T} x_{ij} \quad (11)$$

Where:  $T$  is a trajectory.

$$x_{ij} = \begin{cases} c_{ij} & \text{if } a_{ij} \notin h_{ij} \\ c_{ij} + M_1 & \text{if } a_{ij} \in h_{ij} \end{cases} \quad (12)$$

$$c_{ij} = \begin{cases} l_{ij} & \text{if } a_{ij} \in W \\ l_{ij} + M_2 & \text{if } a_{ij} \notin W \end{cases} \quad (13)$$

Where:  $h_{ij}$  is the heat affected zone for each weld seam  $a_{ij}$  in  $W$ ;  $l_{ij}$  is the arc length  $a_{ij}$ ;  $A$  is a set of arcs  $a_{ij}$  from each node  $i \in N$  to each node  $j \in N$ ;  $N$  is a finite set of nodes in the seam  $w$ .  $W$  is a set of arcs that represents a weld seam  $W \subseteq A$ . For the sub-function that involves the smooth robot joint movements  $g_2$  is defined by:

$$g_2 = \sum_{a_{ij} \in T} \sum_{k \in J} \theta_{ijk} \quad (14)$$

Where:  $\theta_{ijk}$  is the angle of change for a joint  $k$  from one node  $i$  to other node  $j$  from the set  $a_{ij}$ .  $J$  is a set of robot joints. The penalty terms  $M_1$  and  $M_2$  are sufficiently large numbers.  $M_1$  ensures that only seams out of the heat affected zone criteria (30 mm) will be selected.  $M_2$  ensures that only valid segments are selected and all of them will be traveled.

Kadivar, M.H. *et al.* [17] have implemented GA for welding sequence optimization to reduce deformation. Their fitness function is in terms of the total deformation and it was scaled using the power law form of scaling to increase the differences among good strings Equation 15. A thermomechanical FEA was used to compute the fitness function. They used 16 digits for making the individuals, where the first eight digits are the order of eight seams and the last eight are the direction, it can be clockwise or anti-clockwise taking values either one or two respectively. A

single-point crossover operator is used. They do not provide details about the selection and mutation operators.

$$F = (1 - |D_i|_{\max})^k \quad (15)$$

### B. Graph Search

Graph search is a problem solving general approach. A graph consists of a set of nodes and a set of directed arcs between nodes. Each situation (state) is represented as a node. Specific states are designated as start and goal. Actions are represented as edges or arcs. The goal is to plan a series of actions that takes us from an initial state to a goal state.

Romero-Hdz, J. *et al.* [18] have implemented a modified lowest cost first search algorithm to reduce welding deformation. The main difference is the fact that in the lowest cost first search, the total cost for reaching a particular node from the source is the sum of the path or arc costs from the source to that particular node. However, the welding deformation is not additive in nature and total deformation cannot be computed for a particular node as the sum of the inner arc or path costs from the source to that particular node. The MLCS algorithm for selecting the welding sequence is demonstrated as follows.

**STEP 1.** Let the number of weld segments be  $N$ . First compute the welding deformation for each element of  $A = \{1+, 1-, \dots, N+, N-\}$  separately. Here,  $i+$  denotes that the welding on segment  $i$  ( $i = 1, 2, \dots, N$ ) will be conducted from right-to-left. Consider a graph  $G$  with root node as a dummy node. Construct a node in  $G$  for each element of  $A$  and join it with the root node. Store the deformation for each element of  $A$  in the respective node in  $G$ . Push the sequence of  $A$  in a priority queue  $Q$  with the sequence having the deformation with increasing order.

**STEP 2.** Pop the first node of  $Q$ , *i.e.*, the node with minimum deformation, say  $i+$ . then construct a new sequence, say  $A1 = \{i+1+, i+1-, \dots, i+(i-1)+, i+(i-1)-, i+(i+1)+, i+(i+1)-, \dots, i+N+, i+N-\}$  (removing  $i+$  and  $i+$  from  $A$  and then add  $i+$  in front of each element of  $A$ ).

**STEP 3.** Perform welding for these new sequences. Add new nodes required for these new sequences and update the graph  $G$ . Store the deformation for each new sequence in the respective node in  $G$ . Delete  $i+$  and  $i-$  from  $Q$  and push these new sequences in the priority queue  $Q$ .

**STEP 4.** Expand the graph  $G$  by performing the step 2 and 3 iteratively until a complete sequence (when the welding operation is performed once on all the segments) is found. Let this sequence be  $S$ . Then  $S$  is considered the pseudo-optimal sequence found.

### C. Artificial Neural Networks

ANNs are a bio-inspired mechanisms that imitate the learning process of the human brain. Multiple models have been developed to solve non-linear and complex problems. ANNs can identify and learn correlated patterns between the input and output data. After training, ANNs can be used to predict a new independent input data.

Fukuda, S. *et al.* [19] have implemented a Hopfield model which is a form of recurrent artificial neural network that uses content-addressable memory with binary threshold nodes. Their investigation considers two approaches, first, a productivity model minimizes the gun travel distance. Second, a quality model minimizes the welding distortion. In the productivity model they regard both ends of a the trajectory as nodes and calculate each distance for these nodes. They have designated direction of welding as constraints. On the other hand, quality model ignores the distances between beads, each seam is represented as node and the amount of shrinkage is determined heuristically using the distance between the weldline and the folded line with respect to neutral axis. In addition, some points of the heuristic knowledge has been considered.

The introduced heuristic knowledge for selecting a welding sequence to avoid deformation and residual stress is as follows.

- 1) Weld from the weldlines with greater restraint and shrinkage.
- 2) Weld alternative weldlines in a member which are symmetrical to the neutral axis.
- 3) Weld the closest weldlines first.
- 4) Weld to avoid abrupt cooling at the ends of crossing weldlines.
- 5) Weld symmetrically structural wise.
- 6) Weld from the members nearest from the center of a structure.
- 7) Weld so as not to produce the unweldable parts after fabrication.

### D. Particle Swarm Optimization

Particle Swarm Optimization is a stochastic computation technique. The population called swarm goes through an evolutionary process where a finite number of individuals or usually called particles are moving around the search space looking for the best solution. Each particle modifies its movement according to its own experience and the behavior of the other ones. In this technique, every single particle tracks all its movements and the global best is determined by selecting the best particle in its neighborhood. A weighted acceleration is modified in each time step.

Wang, Xue-Wu *et al.* [20] have implemented a discrete Particle Swarm Optimization (PSO) technique for welding robot path planing. Initially they stated that fitness function should be multiobjective taking into account the length of the welding path, welding deformation and energy consumption. This last term includes two parts, the welding operation which is up to the process parameters and the energy consumed by the robot. However, they only implemented only single objective function in terms of the shortest path. Crossover, mutation and partition operators were used. In their study they show a comparison between three different approaches: basic PSO, partition PSO (P-PSO) and Partition Mutation PSO (PM-PSO). The study case is a car door which is a complex part with 115 weld joints. Results demonstrate that the hybrid PM-PSO performs better.

### III. OTHER METHODS FOR SELECTING A WELDING SEQUENCE

This section aims to attach some relevant works where the welding sequence assessment is solved from other perspectives, these works can be useful to implement AI and ML techniques as well as getting the domain knowledge.

#### A. Joint Rigidity Method

C. L. Tsai *et al.* [21] have studied the effect of the welding sequence. It reports that Joint Rigidity Method (JRM) is effective in determining the optimum welding sequence for minimum ship panel warping. Basically, the method consist of calculating a rigidity index which is a normalized parameter, this index is defined by a division of the moment applied to the joint and the rotation angle. In this paper several patents are summarized and compared between them. Since this method was applied to flat shipbuilding panels, it can be improved by carry out complex round surfaces validation. It do not consider complex geometries, joint type and number of weld seam for each joint.

Park *et al.* [22] have proposed a new model of the JRM. This new model considers the gap between plates, effect of the tacking and the welding sequence by changing FEA modeling parameters. Deformation is computed using an elastic FEA and the equivalent load method. The sequence is determined first by computing the stiffness rate by dividing each stiffness value into the maximum one. The first joint to weld is the one which has the minimum stiffness rate. Once one weld joint is elected, the weld bead is converted into a solid element in the FEA modeling. so, again the unit moment is introduced to the other joints except for the decided one. the process is repeated until the sequence is completed.

#### B. Surrogate Models

Voutchkov *et al.* [23]. presents the use of surrogate models for WSO. This type of models simplified models or representations of complex ones taking some assumptions. A surrogate model usually includes the following steps.

- 1) Define a design of experiments (initial runs).
- 2) Evaluate the DOE accurately.
- 3) Train a surrogate model for getting the relationship between input and outputs.
- 4) Run the optimization using the model.
- 5) Evaluate the optimal output using an accurate model.
- 6) A comparison between surrogate model and accurate one is required.
- 7) If result need to improve, Update the DOE results and re-train the surrogate model.

In their proposal, the total deformation is computed by the sum of the deformations on each seam. Nevertheless they ignored the cooling stages. It is also stated that the first weld from each run will provide the main effect. so after adding a term for improving the accuracy taking into account the

position of the seam, the model is described as in Equation 16.

$$D = \sum_{i=1}^n (M_{wi} + \Delta(w, i)) \tag{16}$$

Where the main effect for welding event  $w$  will be denoted as  $M_w$ . and  $\Delta(w, i)$  is the effect of the position.

#### C. General guidelines

Warmefjord *et al.* [24] report results obtained by exploring strategies for spot welding sequence optimization. Four strategies were explored: general simple guidelines, minimize variation in each step, sensitivity and relative sensitivity. Eight industrial cases were tested, and relative sensitivity is the strategy that offers better results. Welding sequence selected using a strategy provides less deformation than continues welding in all studies done.

## IV. CONCLUSIONS

In this paper we overview the available literature related to WSO. We have found two literature reviews already available related to optimization techniques applied to welding processes [4], [5]. However, the main difference of this paper is that it is focused on a specific topic and it is aligned to provide information for a real industrial need and trends for increase the competitiveness. When it comes to the reviewed works, we found that the most explored technique is GA. Nevertheless, there are some gaps and challenges such as the implementation of multiobjective functions where deformation, residual stress and robot travel time need to be considered Table I. Additionally, the experimental tryouts on a complex parts or real components are limited Table II. Other fact that should be investigated is the error budget when it comes to deformation and residual stress analysis.

TABLE I  
LITERATURE REVIEW ON IMPLEMENTED GA FITNESS FUNCTION.

Author	Main functions				
	Trajectory time	Deformation	Residual stress	Temperature	Others
[15]	Yes	Yes	No	No	No
[16]	Yes	No	No	Yes	Robot joint movements
[14]	No	Yes	No	No	No
[7]	No	Yes	No	No	No
[12]	No	Yes	No	No	Stiffness and stress constraints
[13]	No	Yes	No	Yes	No
[17]	No	Yes	No	No	No

TABLE II  
LITERATURE REVIEW ON GA VALIDATION METHODS.

Author	Validation type				
	Specimen FEA	Specimen tryout	Real part FEA	Real part tryout	Others
[15]	No	No	Yes	No	No
[16]	No	No	No	No	Virtual trajectory Sim
[14]	No	No	Yes	No	No
[7]	Yes	No	No	No	No
[12]	No	No	Yes	No	No
[13]	Yes	Yes	Yes	No	No
[17]	Yes	No	No	No	No

Future work should focus on other AI and ML techniques that have been successfully implemented in similar problems. For example A\*search, reinforcement learning, dynamic programming and some hybrid ones.

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