Performance Evaluation of Compressed Sensing Recovery Algorithms for Non-Stationary Signals

G. Gaythri 1, MD. Mahboob Pasha2

1M.Tech, KSRM College of Engineering, Kadapa, Andhra Pradesh.
2Assistant Professor, KSRM College of Engineering, Kadapa, Andhra Pradesh.

1. INTRODUCTION

1.1 Need For Sampling:

Because of the digital revolution several new sensing systems have been increasing which also come with improved resolution and fidelity and also these are arriving as more rigid, flexible, cheaper and more widely used than analog because of the digitization nowadays. The process of conversion from analog to digital is raised from the importance of digital signal in many aspects over analog signal like efficient error controlling by using different codes, digital equipment usage which is cheap, less propagation delay hence high speed, less interference i.e. less affected by noise hence, high security which leads to more accuracy and finally less storage requirement which eases the ways and means of transmission.

1.1.2 Sampling Theorem:

A band limited continuous time signal, with higher frequency fm Hz, can be recovered from its samples unless sampling rate is greater than or equal to twice the highest frequency i.e. according to shannon/Nyquist sampling theorem the sampling should be done at a sampling rate at least 2 times higher than the signal bandwidth to get the signal without any loss. Performing sampling at a rate lesser than Nyquist rate leads to an effect which is called as aliasing.

1.2 Compression:

Compression is a technique which is achieved by acquiring full signal, then computes the coefficients and stores the largest coefficients, which is a wasteful of resources when there is other genius way to obtain compressed version of a signal.

1.3 Compressed Sensing:

Efficient acquisition and reconstruction of signals is possible by using a signal processing technique called compressed sensing. The name compressed sensing [1] is from the possibility of sensing sparse signals from far fewer measurements. This is a new type of sampling theory, whose concept is to recover signals from
incomplete information, which is believed previously highly insufficient to recover the signals. Other terms used instead of compressed sensing are compressive sensing, compressive sampling or sparse recovery, which is a new approach in data acquisition. For simultaneous sensing and compression of finite dimensional vectors a framework is provided by this technique, which depends on linear dimensionality reduction. The basic concept behind CS is to find ways to directly sense the data in a compressed form i.e. at a lower sampling rate rather than first sampling at a high sampling rate and then compressing the sampled data.

\[ m \ll n \]

This technique depends on linear dimensionality reduction process and is a new conceptual framework for signal acquisition which is quite different from that of the formal one used for transmission or storage that involves uniform rate digitization followed by compression. This technique is non-adaptive in the sense that the measurement matrix will be taken random irrespective of the type of signal and the cost to be paid for the reduction in the number of measurements is usage of sophisticated recovery algorithms. It offers basic structure for simultaneous sensing and acquisition. It is a technique which assumes that high-dimensional sparse signals can be recovered from highly incomplete measurements by employing efficient algorithms.

1.4 Applications of compressed sensing:

- Data compression,
- Recovery of missing data,
- Camera design and imaging,
- Computerized Tomography,
- Hyper spectral imaging,
- Geophysical analysis,
- Computational biology,
- Remote sensing,
- Radar imaging,
In Holography (to improve image reconstruction),
A/D conversion.

1.5 Non-Stationary Signals:
A physical quantity which represents some information is called as a signal. There are many types of signals classified based on different parameters. Among them stationary and non-stationary signals are two types. Stationary signal are the one whose frequency components are static with respect to time. Similarly non-stationary signals are the signals whose frequency or spectral components are not constant. Here, the spectral components in the sense mean variance and mean square error etc.

Fig. Sparse representation of a signal using different transforms

2.1 Literature survey
The attractive concept Compressed sensing was introduced by Emmanuel Candes, Justin Romberg, Terence Tao and Donoho in 2004. From the birth of this technique it has been developing drastically and emerging into many fields. Reconstruction by efficient recovery algorithms is an important feature of compressed sensing and several recovery algorithms have been developing based on the requirements and to overcome disadvantages which vary from application to application. These recovery algorithms can be broadly classified mainly into 3 types. They are convex algorithms, combinatorial algorithms and greedy algorithms. Computational complexity of convex algorithms is high even though they require less number of measurements. The combinatorial algorithms are very fast but suffering from a drawback of high number of measurements to be required which will become problematic to obtain sometimes. To name a few HHS pursuit and sub-linear fourier transform are algorithms under this category. Among these greedy algorithms are a good compromise between other two algorithms and in these algorithms signal estimate and support set are approximated through an iteration process. Fast and easy implementation, optimal recovery performance are the attributes of greedy algorithms. A traditional approach in the recovery algorithms is basis pursuit which utilises l1-minimization technique. Later so many algorithms have been developed namely matching pursuit, orthogonal matching pursuit which is improved version of matching pursuit, compressive sampling matching pursuit.

3.1 Orthogonal matching pursuit[OMP]:
Matching pursuit algorithm, which is known as pure greedy algorithm is the base for OMP. Basic matching pursuit is also called as non-orthogonal matching pursuit. It is an iterative process using which approximation is constructed. In matching pursuit, at each iteration only one element is selected and also coefficient is updated which belongs to that column only and also it reselects the same element in order to reduce the minimization error i.e. refinement of approximation. These two are the main drawbacks of matching pursuit and
because of this repeated evaluation computational complexity of matching pursuit is improved.

![Original Signal](image1.png)

![Reconstructed Signal using OMP](image2.png)

**Fig.** Chirp signal of length 1000 samples reconstructed using OMP for number of measurements, p=300 and a sparsity level, k=150.

3.2 Compressive sampling matching pursuit:

One of the latest algorithms developed based on OMP is CoSamp, which is a greedy iterative method. It is just like an extension of OMP. Aim of both this algorithms is similar in the following aspects. First is in the correlation of residual error vector with the measurement matrix columns. Second is finding solution for the least squares problem for elected columns. Both these steps are performed in both the algorithms at each iteration. This algorithm has both element addition and removal process at each iteration as well as keep track of an active set of non-zero elements T. It takes a number of elements to build approximate at each iteration. At each iteration signal is approximated using largest elements. the recovery process is repeated until the missing portion of the signal is recovered, which is reflected by finding the residue and thus measurements are updated. sampling i.e. number of samples required for the signal reconstruction, type of samples and implementation of the sampling schemes in practice and secondly reconstruction i.e. recovery algorithm used for the efficient signal approximation recovery are the two important criterions.

![Original Signal](image3.png)

![Reconstructed Signal using CoSaMP](image4.png)

**RESULTS, ANALYSIS**

We have executed mat lab code for orthogonal matching pursuit and compressive sampling matching pursuit algorithms and have observed number of measurements, mean square error and time parameters. The main performance criterion here is mean square error. All these are executed in a system of AMD Radeon R2 graphics processor with 1.35 GHz, 2GB RAM, 64 bit operating system with windows 8.1 single language bing. The results are obtained for chirp signal of length 1000.
samples, which is a non-stationary signal. The first plot is drawn by varying number of measurements and calculating mean square error for different number of measurements. From fig.1 it is clear that the MSE decreases as the number of measurements increases and also mean square error is less for CoSaMP compared to that of OMP. By observing table 1. it is clear that for every value of p (number of measurements) the MSE of CoSaMP is almost less than half to that of MSE of OMP. A plot is drawn taking sparsity level, k on x-axis and value of MSE on y-axis. For different values of sparsity MSE values are calculated and tabulated in table 1. From the plot of fig.2 it is very much clear that there is very much difference between MSE of CoSaMP to that of MSE of OMP. Of course there is difference in terms of time which may be not a concern when the quality of the recovered signal is considered CoSaMP is the best algorithm which is better than two times as that of OMP.

<table>
<thead>
<tr>
<th>S. NO.</th>
<th>Sparsity level, k</th>
<th>OMP (MSE)</th>
<th>CoSaMP (MSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50</td>
<td>5.9</td>
<td>5.0</td>
</tr>
<tr>
<td>2.</td>
<td>100</td>
<td>4.8</td>
<td>3.9</td>
</tr>
<tr>
<td>3.</td>
<td>150</td>
<td>3.7</td>
<td>2.3</td>
</tr>
<tr>
<td>4.</td>
<td>200</td>
<td>3.4</td>
<td>1.5</td>
</tr>
<tr>
<td>5.</td>
<td>250</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>6.</td>
<td>300</td>
<td>2.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 1: values of MSE for different levels of sparsity for OMP and CoSaMP

Fig. sparsity level, k vs. MSE for OMP and CoSaMP
Table 2: values MSE and time for different number of measurements for OMP vs. CoSaMP

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>P(NO OF MEASUREMENTS)</th>
<th>MSE</th>
<th>TIME(seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>OMP</td>
<td>CoSamp</td>
</tr>
<tr>
<td>1.</td>
<td>50</td>
<td>1.2</td>
<td>0.5</td>
</tr>
<tr>
<td>2.</td>
<td>100</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>3.</td>
<td>150</td>
<td>5.03</td>
<td>1.04</td>
</tr>
<tr>
<td>4.</td>
<td>200</td>
<td>1.05</td>
<td>8.06</td>
</tr>
<tr>
<td>5.</td>
<td>250</td>
<td>9.06</td>
<td>6.06</td>
</tr>
<tr>
<td>6.</td>
<td>300</td>
<td>8.06</td>
<td>5.06</td>
</tr>
</tbody>
</table>

Fig. number of measurements, p vs. MSE for OMP and CoSaMP

Fig. number of measurements, p vs. Time for OMP and CoSaMP

**Conclusion:**

In this project mean square error, time of execution of OMP and CoSaMP algorithms are studied for different number of measurements and sparsity levels from the perspective of one-dimensional signal. And the main concept of this project is based on a parameter called mean square error, which is the main performance criterion. Based on this parameter performance of OMP and CoSaMP algorithms is evaluated. MSE is computed and compared for different number of measurements and sparsity levels. Data acquisition capabilities, processing and performance will be improved very much using compressed sensing. In this project, the performance of CS recovery algorithms OMP and CoSaMP is compared in terms of MSE and time for different levels of sparsity and number of measurements. CoSaMP outperforms the OMP and has less mean square error for the same number of measurements and sparsity levels compared to that of OMP. For sparse signals to achieve lower sampling rate compressed sensing is an effective approach.

References:


