Measurement of the Speed of Sound in Air Medium using Lissajous Patterns

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Abstract–Our present work focuses on the beautiful but simple technique for determination of the speed of sound using analysis of the Lissajous patterns obtained on the oscilloscope. The horizontal axis displays the signal input to a loudspeaker while the vertical axis displays the changing signal picked up by a microphone as it is moved to a known distance away from the source. We have taken into consideration the approach of acoustic interferometer in order to predict the Lissajous patterns.

Keywords–*Speed of Sound, Lissajous Patterns, Acoustic Interferometer.*

I. Introduction

A wave is disturbance or oscillation that travels through matter or space, accompanied by a transfer of energy from one point to another, often with no permanent displacement of the particles of the medium; with little or no associated mass transport. Waves are described by a wave equation which sets out how the disturbance proceeds over time.

A wave can be transverse or longitudinal depending on the direction of its oscillation. Transverse waves occur when a disturbance creates oscillations that are perpendicular (at right angles) to the propagation (the direction of energy transfer). Longitudinal waves occur when the oscillations are parallel to the direction of propagation. Mechanical waves propagate through a medium, and the substance of this medium is deformed. The deformation reverses itself owing to restoring forces resulting from its deformation.
For example, sound waves propagate via air molecules colliding with their neighbours. When air molecules collide, they also bounce away from each other (a restoring force). This keeps the molecules from continuing to travel in the direction of the wave
Lissajous figures also called Bowditch curve, pattern produced by the intersection of two sinusoidal curves the axes of which are at right angles to each other [1].

One of the most undertaken approaches to the measurement of speed of the sound in air is the measurement by the acoustic interferometer. The acoustic interferometer is the device which measures the phase difference between two acoustic waves with the same frequencies and different phases [5].

The technique is based on the comparison of the frequency of an unknown source to that of standard oscillator, which can be used to determine the relative phases. The phase difference between two plates of the oscilloscope produces Lissajous pattern on the oscilloscope screen [2, 3, 4].

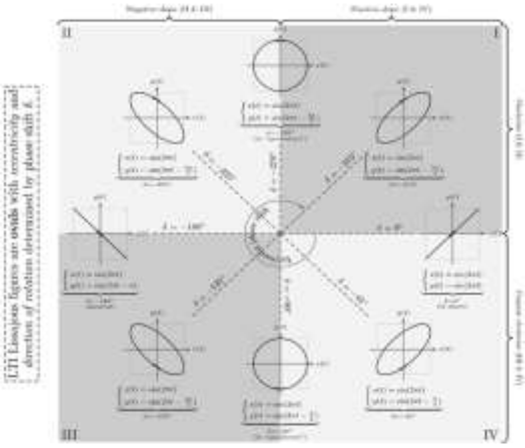

Fig. 1 Lissajous Patterns with eccentricity and directions of rotation.

Let us now assume two harmonic waves moving along two perpendicular directions given by [1]:

$$x = x_0\, cos\omega t$$
$$y = y_0 \cos(\omega t + \phi) \quad (1)$$

Here, x_0 and y_0 are the amplitudes of the two harmonic waves, ω represents the angular velocity of the waves, t represents the time and ϕ represents the phase difference between the two waves. From eq^n (1) we can write:

$$\frac{y}{y_0} - \frac{x}{x_0} cos\phi = -sin\omega t\omega sin\phi \quad (2)$$

Squaring the above eq^n we get,

$$\left(\frac{y}{y_0} - \frac{x}{x_0} cos\phi\right)^2 = (-sin\omega t\omega sin\phi)^2 (3)$$

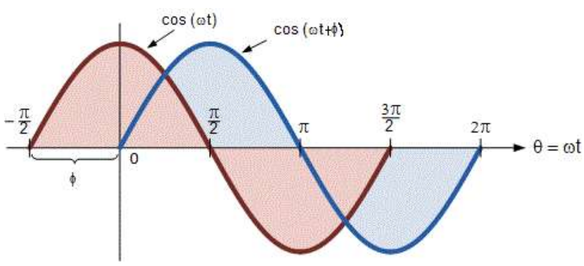

Fig. 1 The Phase difference and intersection of the two harmonic waves.

Using the trigonometric identity we get,

$$sin^2\omega t = 1 - cos^2\omega t = 1 - \left({x}/{x_0}\right)^2 \quad (4)$$

$$\therefore \left(\frac{y}{y_0}\right)^2 + \left(\frac{x}{x_0}\right)^2 - \frac{2xy}{x_0 y_0} cos\phi = sin^2\phi \quad (5)$$

Eq^n (5) seems to be similar to that of an ellipse signifying that the interference of the two oscillators perpendicular to each other which have the same frequencies ω, causes points to be moved along the ellipse [1]. The shape and the position of the curve depends upon the amplitudes (i.e. x_0, y_0) and also on the phase difference between them. Fig. (3) represents graphically the formation of the ellipse when the phase difference is 45^0 [5].

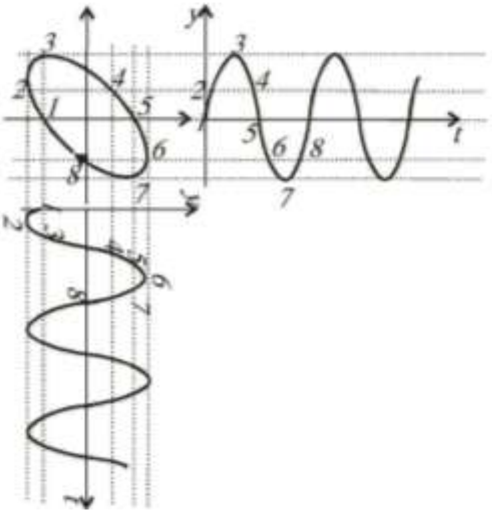

Fig. 3 Graphical representation of formation of the elliptical pattern.

Using similar approach we can easily obtain the Lissajous patterns for 0^0,45^0,90^0,135^0and180^0 as shown in Fig.(4) [5-9].

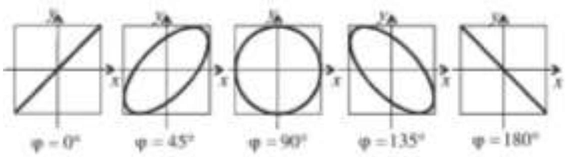

Fig. 4 Graphical representation of formation of the Lissajous patterns for different values of phase difference.

II. Theoretical Approach

The scheme of the acoustic interferometer is shown in Fig. (5). Oscillations spreading from the loudspeaker are intercepted by the microphone thus transforming them to electric signals [1,5].

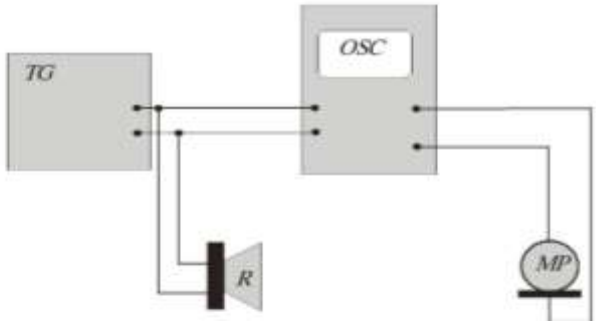

Fig. 5Acoustic Interferometer Scheme where OSC – Oscillator, MP – Microphone, R – Reproducer, TG – Generator.

The frequency of the standard oscillator is driven by the second channel of the oscillator whereas the electrical signals are driven by the first channel of the oscillator. These can be mathematically described as below [1].

$$x_1 = x_{01}\cos(\omega t - \phi_1) = x_{01}\cos\omega\left(t - \frac{d_1}{v}\right) \quad (6)$$

$$x_2 = x_{02}\cos(\omega t - \phi_2) = x_{02}\cos\omega\left(t - \frac{d_2}{v}\right) \quad (7)$$

Here, v is the velocity of the sound in air, d_1 and d_2 are the two distances from the microphone and amplifier while the amplitudes of oscillations at distances d_1 and d_2 is given by x_{01} and x_{02} respectively.

$$\therefore \Delta\phi = \frac{\omega}{v}(d_1 - d_2) \quad (8)$$

Here we assume the path difference between the two oscillations to be equal to the wavelength λ.

$$\therefore d_1 - d_2 = \lambda \quad (9)$$

Thus, we find out the phase difference reduced to 2π. So, we can compute the frequency regarding the approach as,

$$f = \frac{\omega}{2\pi}$$

III. Experimental Setup

Fig. 6 Experimental Setup and Approach

A. Procedure

1. Connect the AFO to the loudspeaker. Switch on the AFO, and set it at about 2 kHz. Place the two microphones in front of the loudspeaker. Connect the microphones to the other two amplifiers to amplify the sound signal as shown in the diagram.

Connect the output of this amplifiers to the X and Y-plates as an input of the CRO [2-4].

2. Connect the microphone amplifier output to the X-plates of the CRO. Turn the time-base control of the CRO off. Adjust the settings of both AFOs & CRO until a clear trace (probably elliptical) is obtained on the CRO's screen. (Care is taken such that a very high volume is not used.)

3. The trace on the CRO's screen should thus turns into a straight line, then another ellipse, then another straight line (but of opposite slope to the first), then an ellipse, then a straight line (of same slope as the first), then the cycle continues.

4. Measure the distance between the microphones. A straight line is seen when the microphone position is such that the wave is directly sent to the CRO's -plates is either in phase or completely out of phase with the wave picked up by the microphone and then sent to the CRO plates [1-4].

5. Adjacent straight lines (of opposite slope) are seen when the microphone is moved through half a wavelength.

IV. Results And Discussion

1. All The distance is measured as the microphone is moved and as it passes through a number of straight lines. Also, the frequency of the loudspeaker AFO is noted Procedure is repeated for more frequencies [1]. Results are tabulated in Table 1.

Phase Difference (ϕ)	Distance of speaker(x)(m)		
	Frequency=1.45 KHz	Frequency=2 KHz	Frequency=2.5 KHz
0	12.2	9	7
$\pi/4$	15.2	11.23	8.8
$\pi/2$	18.2	13.5	10.5
$3\pi/4$	21.3	15.7	12.3
π	24.3	17.9	14.1
2π	36.5	26.7	21.1

Table 1. Experimental Data for the procedure.

2. The theoretically calculated speed is compared with that of the experimentally derived speed and is found to be in good agreement with the available data.Our results are listed below in Table 2.

Frequency (KHz)	Phase Difference $\phi = \omega t$	Time $t = \frac{\phi}{2\pi f}$ (sec)	Distance between speaker and mic x (m)	Mean Velocity $V = \frac{\Delta x}{t}$ (m/s)
1.45	0	0	12.2	350.3
	$\pi/4$	1/8f	15.2	
	$\pi/2$	1/4f	18.0	
	$3\pi/4$	3/8f	21.3	
	π	1/2f	24.3	
	2π	1/f	36.5	
2	0	0	9.0	356.83
	$\pi/4$	1/8f	11.2	
	$\pi/2$	1/4f	13.5	
	$3\pi/4$	3/8f	15.7	
	π	1/2f	17.9	
	2π	1/f	26.7	
2.5	0	0	7.0	354.166
	$\pi/4$	1/8f	8.8	
	$\pi/2$	1/4f	10.5	
	$3\pi/4$	3/8f	12.3	
	π	1/2f	14.1	
	2π	1/f	21.1	

Table 2. Resultant Data from Experimental Procedures.

3. Also, the final velocity of sound measured via our procedure leads out to be 353.21 m/s which is in good agreements with the available data.

4. Furthermore, we also have calculated the percentage error in our approach which turns out to be 0.1162%.

V. CONCLUSION

We can easily conclude that Lissajous patterns can be used to determine the frequency of sound both theoretically as well as experimentally. Also, we can see that our approach provides us with fairly accurate results which are in good agreement with the available data.

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