Modelling and Simulation of Armature-Controlled Direct Current Motor using MATLAB

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Abstract

Technology at present needs faster and easier means of controlling equipment with few numbers of components. One of such equipment is the direct current (DC) motor, whose speed is directly proportional to the supply voltage. Using this established relationship between the speed and the supply voltage, this paper investigates the effects of different loads and inputs on the output response of the armature controlled direct current motor. The aim is to establish relationship between the speed of DC motor and the load torque at different voltages as well as investigating the performance of closed-loop systems when different voltages are applied to the armature circuit of the motor when a constant voltage is supplied to the field circuit of the motor. The study was carried out in two stages. The first stage involved mathematical modeling of the system while the second stage centered on the simulation of an armature controlled direct current motor using Simulink environment in MATLAB. The performance analysis of the modeled system shows that usage of feedback enhances the performance of the transient response of the armature controlled DC motor. In addition the result obtained shows that at different input voltages, the speed the DC motor is inversely proportional to the load torque.

Keywords - *Model, DC motor, Back emf, DC motor classifications, Electrical and mechanical variables*

I. INTRODUCTION

The universe we live undergoes continual change. This change is not always apparent if the time scale is long as experienced in some geological processes. However in engineering where situations that are time-dependent are important, ideal description of objects or processes must include proper timing. This kind of description is called or known as model.

A model is simply a representation of the construction and working of some systems of interest [1]. According to the author, a model is similar to but simpler than the system it represents. The process of producing a model is known as modeling. Basically, a good model is a judicious tradeoff between realism and simplicity. One of the important issues in modeling is model validity, which is the process of simulating the model under known input conditions and comparing model output with system output [1].

Basically, simulation of a system is the operation of a model of the system, which can be reconfigured and experimented with. Thus, operation of the model can be studied in order to infer some details on the properties concerning the behavior of the actual system or its subsystem. Generally, according to [1], simulation is a tool to evaluate the performance of a system, either existing or proposed, under different configurations of interest and over long periods of real time.

Hence, for this armature controlled DC motor modeling, the analysis was carried out using both the step and impulse responses. All the data based on internal circuit of the modeled armature controlled DC motor for this study were analyzed both by control system design calculation and by MATLAB software.

In order to enhance both the understanding and proper presentation of the study reported in this paper, the remaining parts of this paper are organized as follows. In Section II, in depth information on DC motor such as its principle of operation, classification and different methods of controlling the speed of DC motors were presented. Section III provides in detail, the methodology used in carrying out the study reported in this paper. The results obtained are presented, interpreted and discussed in Section V while the conclusions made are presented in Section V.

II. DIRECT CURRENT MOTOR

The DC motor is one of the first machines devised to convert electrical power into mechanical power motor [2 – 4]. Its origin, according to [3], can be traced to disc-type machines conceived and tested by Michael Faraday, who formulated the fundamental concepts of electromagnetism.

The DC motor uses electricity and a magnetic field to produce torque, which causes it to turn. It requires two magnets of opposite polarity and an electric coil, which acts as an electromagnet. The repellent and attractive electromagnetic forces of the magnets provide the torque that causes the motor to turn. It also consists of one set of coils, called armature winding, inside a set of permanent magnets, called the stator. Applying a voltage to the coils produces a torque in the armature, resulting in motion.

The principle of operation on a DC motor as shown in Figure 1 requires a DC voltage source. When the two ends of the coil are connected across a DC voltage source, it will cause current, I, to flow through it. Hence, a force is exerted on the coil as a result of the interaction of magnetic field and electric current. The force on the two sides of the coil is such that the coil starts to move in the direction of force.

In a DC motor, several such coils are wound on the rotor, all of which experience force, resulting in rotation. The greater the current in the wire, or the greater the magnetic field, the faster the wire movement because of the greater force created. At the same time, torque is being produced as the conductors are moving in a magnetic field. At different positions, the flux linked with it changes, which causes an electromagnetic force (emf) to be induced as shown in Figure 2. This voltage is in opposition to the voltage that causes current flow through the conductor and is referred to as a counter-voltage or back emf.

The value of current flowing through the armature is dependent upon the difference between the applied voltage and this counter-voltage. The current due to this counter-voltage tends to oppose the very cause for its production according to Lenz's law. It results in the rotor slowing down such that the force created by the magnetic field equals the load force applied on the shaft. Then the system moves at constant velocity. Basically, the operation of DC motor is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a force. This force according to [5], plays an important role to produce a powerful torque. The magnitude of the force is given mathematically in [5] as;

$$F = BIl \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where B is the flux density in Weber per square meter (Wb/m^2), I is the current in ampere (A) and l is the length of the conductor in meter (m).

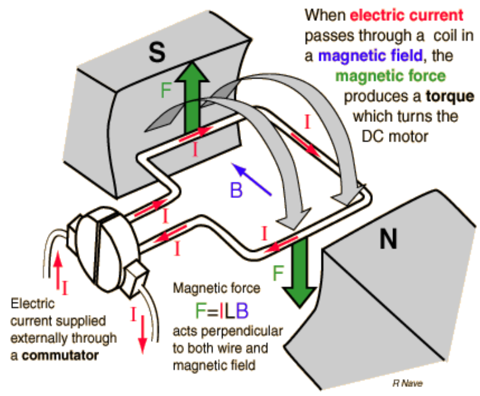

Figure 1: Torque Production In A DC Motor [6]

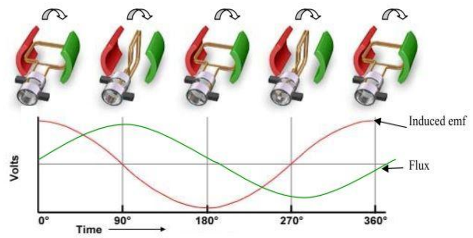

Figure 2: Induced Voltage in Armature Winding Of DC Motor [6]

Basically, there are three types of DC motors. The two commonly criteria usually use in classifying them are their characteristics and the connection of their exciting windings or circuits. Based on these criteria, the three common types are: shunt, series and compound motors.

In shunt DC motor, only one exciting winding, which is connected across the armature terminals and is thus in parallel or in shunt with the armature. The field winding consists of a large number of turns of fine wire on each pole, and usually the windings on all the poles are connected in series in one circuit. The current in the field depends upon the line voltage and upon the resistance of the field winding. The resistance of the field winding is purposely made high so that the field current will be between 1 and 5 per cent of the full-load current of the motor. In this class of DC motor, a rheostat is normally connected in series with the field to control the motor when the speed is above normal rating [7].

The series DC motor, on the other has only one exciting winding, which is connected in series with the armature, so that all current flows through the field as well as the armature. The field winding consists of a few turns of thick wire on each pole, and the windings on all poles are connected in series. The current in the field depends upon the load and is thus large with heavy load and small with light load. The resistance of the field winding is purposely made low so that the loss of voltage and power in that circuit will be small. According to [7], the only thing that limits the speed of a series motor is the amount of load connected to it.

The third class known as the compound or cumulative motor has both a series winding and a shunt winding on each pole, which are wound and connected so that the two windings assist each other in the protection of magnetism. It is a combination of a shunt and a series motor. The compound-wound motor has two excitation windings, both on the main field poles. According to [8], in this type of DC motor the majority of the flux results from the conventional shunt winding with additional excitation from series-connected winding.

Generally, DC motors are much more adaptable speed drives [4] than alternating current (AC) motors which are associated with a constant speed rotating field. Indeed one of the primary reasons for the strong competitive position of DC motors in modern industrial drives is the fine speed control. The speed of a DC motor according to [9] is given as;

$$N = K\frac{(E_a - I_a R_a)}{\phi} \quad \ldots\ldots\ldots\ldots.. \quad (2)$$

where, $(E_a - I_a R_a)$ is the back emf (e_b).

There are three methods of controlling the speed of DC motor: armature voltage speed control, field flux speed control and voltage control. In this study, the armature method was employed. The modeling and simulating procedures involved in carrying out the study are presented in Section III.

III. MODEL DEVELOPMENT

In modeling the armature controlled DC motor for this study, simple electrical circuit of armature controlled DC motor diagram as shown in Figure 3 was employed while the activities involved were broken down into four stages. The four stages involved are presented in the following sub-sections as follows.

A. ARMATURE CONTROLLED DC MOTOR CIRCUIT DIAGRAM REPRESENTATION

This stage focus was on development of an armature controlled DC motor as shown in Figure 3 with torque and rotor angle.

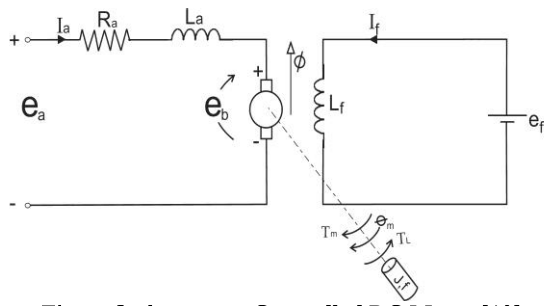

Figure 3: Armature-Controlled DC Motor [10]

In Figure 3, R_a is the resistance of the armature, L_a is the inductance of the armature winding, i_a is the armature current, i_f is the field current, e_a is the applied armature voltage, e_b is the back emf, T_M is the torque developed by the motor, ϕ is the angular displacement of motor-shaft, J is the equivalent moment of inertia of motor shaft and load referred to the motor while f is the equivalent coefficient of motor and load referred to the motor shaft.

B. DERIVATION OF THE SYSTEM EQUATION

The activity in this stage involves the development of both the electrical and mechanical variables in Figure 3. The variables of interest on the electrical side are voltage and current while the variables of interest on the mechanical side are torque and speed. The two basic equations relate these four variables and form the foundation of armature controlled DC motor analysis.

According to [8], the developed torque by motor is defined mathematically as;

$$T = \frac{z_p}{2\pi a} i_a \phi_p = K_t i_a \phi_p \quad \ldots\ldots. \quad (3)$$

where $\frac{z_p}{2\pi a}$ was denoted as K_t.

Similarly, the authors defined the back electromotive force (e_b) mathematically as;

$$e_b = \frac{z_p}{60a} n\phi_p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

However, since;

$$n = \frac{60\omega}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \quad (5)$$

Substitute (5) into (4), to obtain

$$e_b = \frac{z_p}{2\pi a}\omega\phi_p = K_b\omega\phi_p \;\;\ldots\ldots\ldots \qquad (6)$$

where $\frac{z_p}{2\pi a}$ was denoted as K_b.

The factor $\frac{z_p}{2\pi a}$ is frequently referred to as torque constant K_t in (3) and voltage constant K_b in equation (6). Since $K_b = K_t$, a new constant, K', which is defined in [8] as;

$$K' = K_t = K_b = \frac{z_p}{2\pi a} \;\;\ldots\ldots\ldots\ldots \qquad (7)$$

Since the field strength is fixed, the flux per pole ϕ_p is merged with K' to yield another constant, K, which is expressed mathematically as;

$$K = \frac{z_p}{2\pi a}\phi_p \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (8)$$

where K is referred to as the machine constant. Hence, (3) can be re-expressed as;

$$T = Ki_a \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

Likewise, (6) can be re-expressed as;

$$e_b = K\omega \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (10)$$

For clarity purposes so that it will be easy to differentiate between electrical and mechanical variables, the constant K in (9) and (10) are replaced with K_t and K_b to obtain (11) and (12) respectively giving as follows;

$$T = K_t i_a \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (11) \text{ and}$$

$$e_b = K_b\omega \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

Applying Kirchhoff Voltage Law (KVL) in Figure 3, the differential equation of the armature circuit is;

$$L_a\frac{di_a}{dt} + R_a i_a + e_b = e_a \;\;\ldots\ldots\ldots\ldots \qquad (13)$$

Similarly, the armature current produces the torque that is applied to the inertia and friction to obtain the torque equation;

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T = K_t i_a \;\;\ldots\ldots\ldots \qquad (14)$$

C. DERIVATION OF THE SYSTEM EQUATION

Taking the Laplace transform of (12), (13) and (14) respectively, assuming zero initial conditions;

$$E_b(s) = K_b s\theta(s) \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (15)$$

$$(L_a s + R_a I_a)(s) = E(s) - E_b(s) \;\;\ldots\ldots\ldots\ldots \qquad (16)$$

$$(Js^2 + fs)\theta(s) = T(s) = K_t I_a(s) \;\;\ldots\ldots\ldots \qquad (17)$$

Substitute $E_b(s)$ in (15) in (16) to obtain;

$$(L_a s + R_a I_a)(s) = E(s) - K_b s\theta(s) \;\;\ldots\ldots \qquad (18)$$

Make $I_a(s)$ the subject of the formula in (18) to obtain;

$$I_a(s) = \frac{E(s) - K_b s\theta(s)}{L_a s + R_a} \;\;\ldots\ldots \qquad (19)$$

Substitute (19) in (17) to obtain;

$$(Js^2 + fs)\theta(s) = \frac{K_t E(s) - K_t K_b s\theta(s)}{L_a s + R_a} \;\;\ldots\ldots \qquad (20)$$

Re-arrange (20) to obtain;

$$\theta(s)\left[(Js^2 + fs)(L_a s + R_a)\right] + K_t K_b s\theta(s) = K_t E(s)$$

Hence, the system transfer function is expressed mathematically as;

$$\frac{\theta(s)}{E(s)} = \frac{K_t}{\left[(Js^2 + fs)(L_a s + R_a)\right] + K_t K_b s\theta(s)} \;\;\ldots \qquad (21)$$

D. CONVERSION OF TRANSFER FUNCTION TO MODEL BLOCK

From (21), the block diagram for the armature controlled DC motor was developed. The block diagram is shown in Figure 4. The block diagram of the armature controlled DC motor was implemented in the simulink environment in the MATLAB. The model created in simulink toolbox of MATLAB is shown in Figure 5.

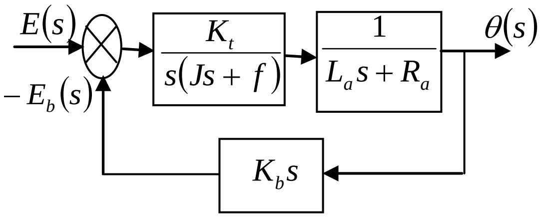

Figure 4: Block Diagram of an Armature Controlled DC motor

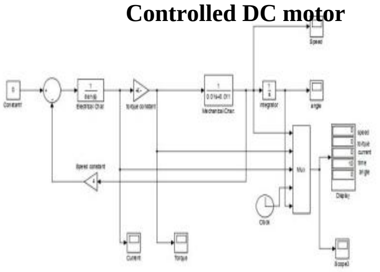

Figure 5: Model Created in SIMULINK Toolbox of MATLAB

IV. SIMULATION RESULTS AND DISCUSSIONS

The simulated model was analyzed based on its transient and steady state responses. The standard test input signals for transient response were step function and impulse function. However, for the steady state response, the graphs of torque-speed characteristics for different armature voltages and armature speed characteristics were considered. The motor specifications are 3hp, 125V and 1500rpm while its parameters: $R_a = 0.6\Omega$, $L_a = 6mH$, $J = 0.093kgm$, $K_t = 0.7274$, $K_b = 0.6$, $B_m = 0.008Nmrad^{-1}s^{-1}$.

For the analyses, all the initial conditions are assumed to be zero, so there is no current flowing through the motor when it first started. The values above were input into the model and simulation took place. The respective results under the steady state response and transient response are presented in the following sub-sections.

A. STEADY STATE RESPONSE

The steady state responses of this armature controlled DC motor model show the performance of the motor under different operating conditions. The steady state responses to be considered are: armature voltage speed characteristics and torque speed characteristics for different armature voltages.

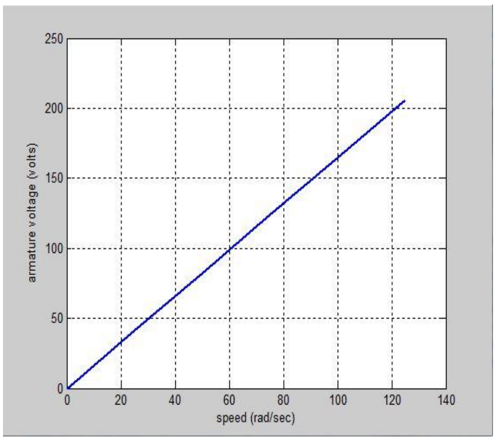

Figure 6: Plot of Armature Voltage Against Motor Speed

The steady state response as shown in Figure 6 shows the variation in the speed as the voltage increases. The result obtained as shown in Figure 6 shows that the voltage-speed characteristics of armature controlled DC motor speed increases with increase in voltage. This implies that the speed of the DC motor is directly proportional to the armature voltage.

In addition, under the steady state response, the variation of the torque speed characteristics for the DC motor at different armature voltages was examined. The obtained result as shown in Figure 7 shows that at different input voltages, the higher the speed the lower the load torque. This implies that the speed of the DC motor at different voltages is inversely proportional to the load torque. The Figure shows that the torque-speed curve is shifted upward by increasing the armature voltage while the slope of the curve remains constant as it is theoretically expected.

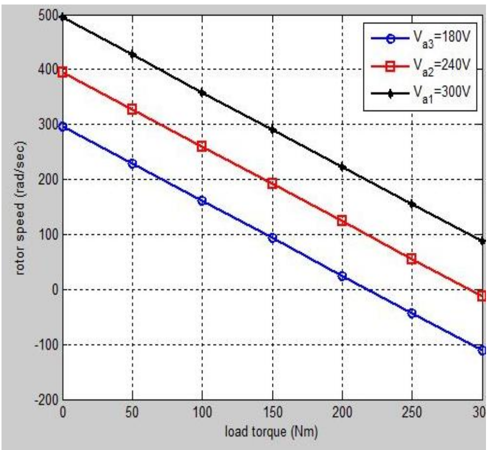

Figure 7: Torque-Speed Characteristics for Three Different Armature Voltages

B. TRANSIENT RESPONSE

Similarly, the transient response of the modeled armature controlled DC motor was examined. The response shows how the use of feedback helps in improving both the performance and stability of the motor.

On performance analysis, the motor performances under different values of K_a with unit input were considered. The responses obtained were shown graphically in Figures 8 - 10. In Figure 8, the amplitude starts at zero at time, t, equals zero. However, as time, t, increases, the value of amplitude tends to 1.649rps. By this, it takes 0.6 time unit for amplitude in Figure 8 to reach the steady state value. It can also be seen from Figure 8 that the amplitude increased steadily without any oscillation. Since the performance objective is to have amplitude track the unit step, it is reasonable to say that the modeled motor performs reasonably well as expected though it a long time for amplitude of Figure 8 to get to its steady state. However, when the value of K_a was increased in Figure 9 and Figure 10, the responses become faster. The responses now reach the steady state in 0.4 time unit and 0.2 time unit respectively. The results show that as the value of K_a is increasing, the response rate is increasing.

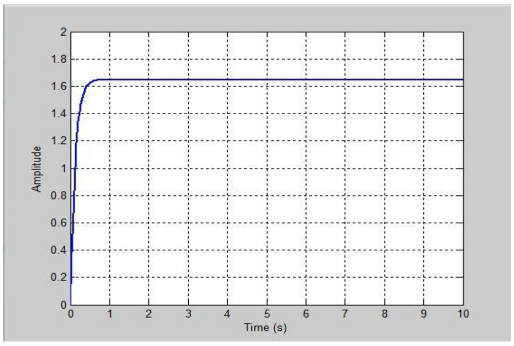

Figure 8: Response of the Motor to a Unit Step input, at $K_a = 0.7274$

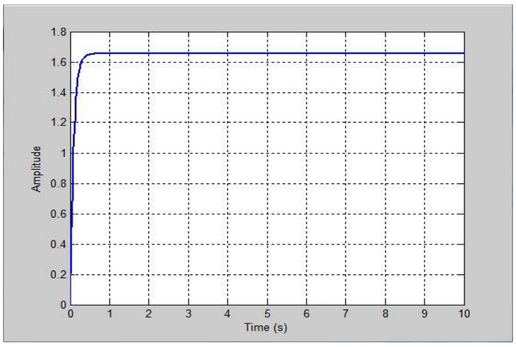

Figure 9: Response of the Motor to a Unit Step Input, at $K_a = 1.0$

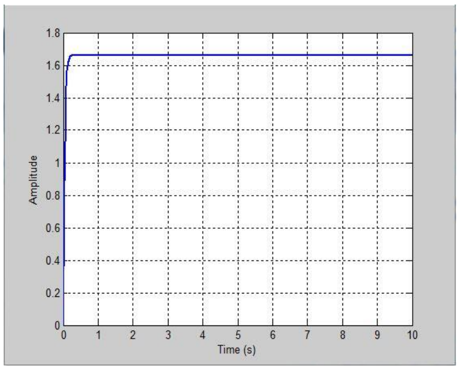

Figure 10: Response of the Motor to a Unit step Input, at $K_a = 2.0$

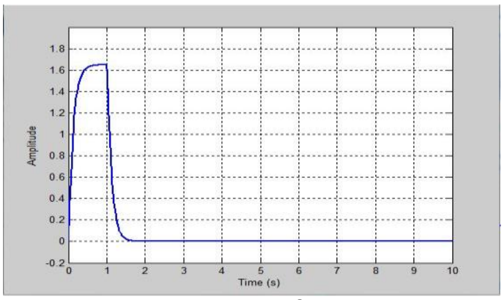

Figure 11: Response of the Motor to an Impulse Input, at $K_a = 0.7274$

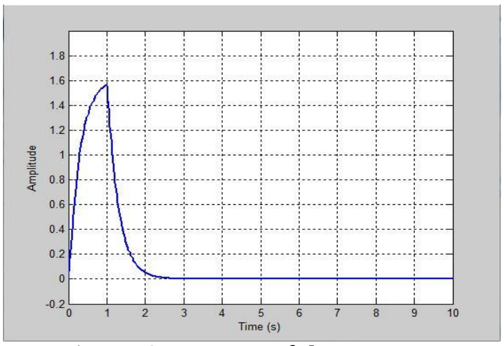

Figure 12: Response of the Motor to an Impulse Input, at $K_a = 0.3$

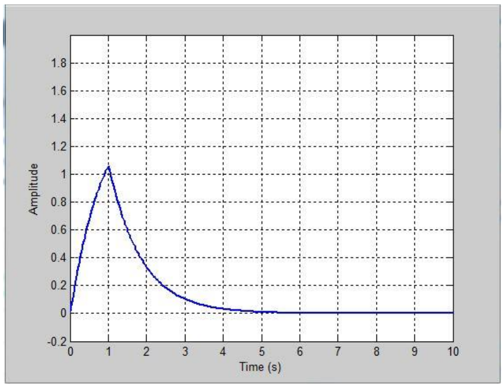

Figure 13: Response of the Motor to an Impulse Input, at $K_a = 0.1$

In order to further evaluate the performance of the modeled motor, its performance was also examined using impulse signal at different values of K_a. The responses obtained were shown in Figures 11 – 13. The graphs show the shapes obtained for the three values of K_a were similar except that it took the responses different time units to return to zero. For instance, while it takes about 0.5 time units in Figure 11 when $K_a = 0.7274$, it takes infinitesimal time unit in Figure 12 when $K_a = 0.3000$ while the response in Figures 13 returns to zero immediately when the value of $K_a = 0.1000$.

On the stability of the motor, the speed of the motor at when subjected to disturbance at different values of K_a were observed. The results obtained presented graphically above show that the speed of the motor is directly proportional to the value of K_a. The results also show that the initial introduction of the disturbance causes a drop in the speed of the motor. The result further shows that disturbance does affect the speed of motor.

V. CONCLUSION

In this paper, the simulation model of an armature controlled DC motor was developed using MATLAB/Simulink. The analysis and performance evaluation of the developed motor show that the simulated motor correctly predicts the effect of armature voltage on the armature voltage-speed characteristics and torque-speed characteristics of the armature controlled dc motor. Furthermore, the results of the study show that the speed of the DC motor is directly proportional to the armature voltage.

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