

# Study on Implementation of Hedge Algebra Based Controller in Real Systems

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**Abstract**

In control theory, a fuzzy logic controller has been used to control an object with the lack of information. In recent years, being considered as a new approach to fuzzy logic calculations, Hedge Algebra has remarkably succeeded in the field of control. According to designers' experiences, a Hedge Algebra based Controller (HAC) is capable of dealing with a variety of objects. This paper demonstrates one of the first implementations of HAC applied to a real object in control area. Results of the experiment show high potential of applications of HAC in reality.

**Keywords** - fuzzy logic controller, Hedge algebra theory, Hedge algebra based controller, non-linear, experiment with real object.

## I. INTRODUCTION

Hedge Algebra (HA) is a new approach to the fuzzy logic calculations. HA takes advantages of the reasoning ability of the human to deal with uncertainties and inaccurate information of controlled objects. Therefore, scholars have applied HA to issues of the control and automation field such as approximation problems, control of simple models [4], [5] and [6]. However, using HA in control has not been mentioned in many researches. The study of HAC applications in a real specific object aims at proving the effectiveness of the HA theory and facilitating other applications in reality [2], [7] and [8]. According to the above purposes, the authors have designed and tested HACs that are used for the motion control system.

## II. METHOD TO DESIGN HAC

### A. Introduction of Hedge Algebra

HA is the development basing on the logic perception of linguistics [4]. The input/output relationship in fuzzy logic must define membership functions discontinuously, whereas HA creates an algebraic structure in terms of functions of linguistic input/output variables.

Example: Consider a set of linguistic intervals which is a linguistic domain of TEMPERATURE truth variable including  $T = \text{dom}(\text{TEMPERATURE}) = \{\text{Large, Small, very Large, very Small, more Large, more Small, approximately Large, approximately Small, little Large, little Small, less Large, less Small, very more$

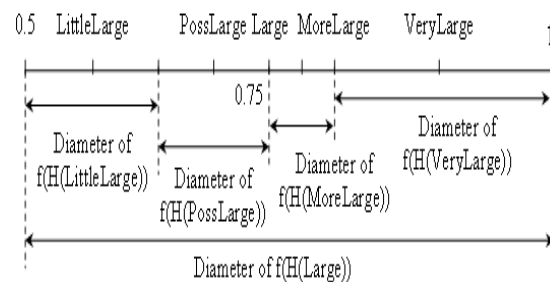
Large, very more Small, very possible Large, very possible Small, ...}.

Then the linguistic domain  $T = \text{dom}(\text{TEMPERATURE})$  can be considered as an algebraic structure  $AT = (T, G, H, \leq)$ , where: "T" is the basedset of AT; "G" is the set of generators (Large, Small); "H" is the set of linguistic hedges (Very, Little, Less...); " $\leq$ " is an semantically ordering relation (Small  $\leq$  Large, more Large  $\leq$  very Large...).

**Definition 1:** A given HA:  $AT = (T, G, H, \leq)$ ,  $f: T \rightarrow [0, 1]$  is the set of semantic quantifying mapping (SQM) of AT if  $\forall h, k \in H^+$  or  $\forall h, k \in H^-$  and  $\forall x, y \in T$ , then:

$$\left| \frac{f(hx) - f(x)}{f(kx) - f(x)} \right| = \left| \frac{f(hy) - f(y)}{f(ky) - f(y)} \right| \tag{1}$$

Considering intervals: Large, very Small, and according to the viewpoint of HA, fuzziness can be defined quite clearly basing on the size of the set  $H(x)$  shown in Fig.1:



**Fig.1. Fuzziness of Linguistic Intervals**

A given semantic quantifying mapping  $f$  of  $X$  and considering  $\forall x \in X$ , fuzziness of  $x$  can be measured by the diameter of  $f(H(x)) \subseteq [0, 1]$ .

### Definition 2: Fuzziness measures

A function  $f_m: T \rightarrow [0, 1]$  is said to be a fuzziness measure if:  $f_m(c^-) = 0 > 0$  and  $f_m(c^+) = 1 - \theta > 0$ , whereas  $c^-$  and  $c^+$  are negative and positive generating elements. Assume set of hedges  $H = H^+ \cup H^-$ ,  $H^- = \{h_1, h_2, \dots, h_p\}$  with  $h_1 > h_2 > \dots > h_p$ ,  $H^+ = \{h_{p+1}, h_{p+2}, \dots, h_{p+q}\}$  with  $h_{p+1} < h_{p+2} < \dots < h_{p+q}$ . Then:  $\forall x, y \in T, \forall h \in H, \frac{f_m(hx)}{f_m(x)} = \frac{f_m(hy)}{f_m(y)}$  this equation does not depend on specific elements and it is called the

fuzziness measure of the hedge h and denoted by  $\mu(h)$ .

**Clause 2.1 Propositions of  $f_m(x)$  and  $\mu(h)$**

$$f_m(hx) = \mu(h)f_m(x), \forall x \in T$$

$$\sum_{i=1}^{p+q} f_m(h_i c) = f_m(c) \text{ With } c \in \{c^-, c^+\}$$

$$\sum_{i=1}^{p+q} f_m(h_i x) = f_m(x) \quad (2)$$

$$\sum_{i=1}^p \mu(h_i) = \alpha$$

$$\sum_{i=p+1}^q \mu(h_i) = \beta \text{ with } \alpha, \beta > 0 \text{ and } \alpha + \beta = 1$$

**Clause 2.2 Construction of SQM Based on the Basics of Fuzziness Measure of Hedges**

Assume fuzziness measures of hedges  $\mu(h)$  and fuzziness measure intervals of base terms  $f_m(c^-)$ ,  $f_m(c^+)$  are given and  $\theta$  is the neutral terms.

A semantic quantifying mapping  $v$  of  $T$  is constructed as follows: with  $x = h_{i_1} \dots h_{i_2} h_{i_1} c$ :

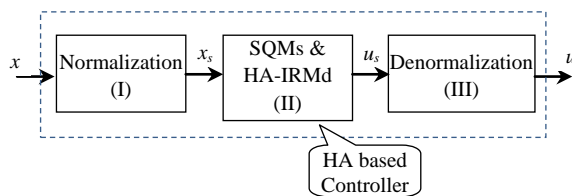
$$v(c^-) = \theta - \alpha f_m(c^-) \quad (3)$$

$$v(c^+) = \theta + \alpha f_m(c^+)$$

$$f_m(x) = f_m(h_{i_1} \dots h_{i_2} h_{i_1} c) = \mu(h_{i_1}) \dots \mu(h_{i_2}) \mu(h_{i_1}) f_m(c)$$

**B. Hedge Algebra-based controller**

The HAC applied in industrial systems comprises 3 blocks as shown in Fig. 2.



**Fig.2. The Diagram of HAC**

Where:  $x$  is the input value,  $x_s$  is the input semantic value, and  $u$  is the control value and  $u_s$  is the control semantic value.

HAC includes the following blocks:

Block I - Normalization (linear transformation from  $x$  to  $x_s$ ): determining the input variable, state variable, control variables (output variables), and the working range of variables. Identifying calculated conditions (choosing the calculated parameters of HA). Calculating the values of semantic quantifying of input variable, state variable, and control variable (apply hedges on the working range of the variables)

Block II - SQMs & HA-IRMd - Semantic quantifying mappings & Hedge Algebra-based Interpolative Reasoning Method (performs semantic interpolation from  $x_s$  to  $u_s$  basing on the semantic quantifying

mapping and rules): changing fuzzy control rules to control rules with semantic quantifying parameters of HA. Solving the approximated problems based on HA to determine the semantic quantifying of control states. Combining the semantic quantifying values of controls and building semantic quantifying curve.

Block III - Denormalization (linear transformation from  $u_s$  into  $u$ ): basing on the initial conditions of the control problem to solve semantic quantifying curve interpolation and determine the real control value.

**III. EXPERIMENTS AND RESULTS**

**A. Model of the System**



**Fig.3. MEDE5**

MEDE5( Mechatronic Demonstrate Setup-2005) was designed by the Control Engineering group of the Department of Electrical Engineering in The Faculty of Electrical Engineering, Mathematics and Computer Science of The University of Twente as shown in Fig.3 [3]. The mechanical part of the setup is designed mimicking printer technology. It consists of a slider which can move back and forth over a rail. In the model, a DC motor, rail and slider are fixed on a flexible frame which causes vibration during the acceleration of the mass. The model can be applied to real applications such as 2-D, 3-D printers, CNCs and accurate control systems. If control theories are designed well then acceleration and deceleration processes of the slider will be smoother and the vibration of the frame will be reduced. The above system is verified by certain control methods that are integrated with PID, MRAS [3] in order to drive the system to a set position or follow the reference orbit.

In calculation, when neglecting nonlinear terms of the friction force, the system has a 6<sup>th</sup> order linear mathematical model. If the rail connecting the motor with the slider is considered stiff and the mass of rotor of the motor is avoided then the system will have a 4<sup>th</sup> order linear model. In addition, if the frame is stable then the system will have a 2<sup>nd</sup> order model represented by the state equation (4)[3]:

$$\begin{bmatrix} \dot{x}_{loadworld} \\ \dot{v}_{loadworld} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m_l} \end{bmatrix} \begin{bmatrix} x_{loadworld} \\ v_{loadworld} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_l} \end{bmatrix} F \quad (4)$$

$$y = [1 \ 0] \begin{bmatrix} x_{loadworld} \\ v_{loadworld} \end{bmatrix}$$

Where:  $x_{loadworld}$   $v_{loadworld}$  are position and velocity of the slider with respect to the origin.

**B. Design of controllers**

There are two controllers, namely HAC and FLC, designed in this part of the paper. Both of the are designed with 2 inputs: the error input e(t), denoted by E and the integral of the error input e(t), denoted by IE; the output is U.

Labels of E, IE, U are shown as follows:

Very Very Negative (VVN), Very Negative (VN), Negative (N), Little Negative (LN), Neutral (W), Little Positive (LP), Positive (P), Very Positive (VP) and Very Very Positive (VVP)

**Constructing FLC**

FLC control rules are shown in Table 1.

**Table 1.FLC Control Rules**

U	E								
	VVN	VN	N	LN	W	LP	P	VP	VVP
IE	VVN	VVN	VVN	VVN	VVN	VN	N	LN	W
	VN	VVN	VVN	VVN	VN	N	LN	W	LP
	N	VVN	VVN	VN	N	LN	W	LP	P
	LN	VVN	VVN	VN	N	LN	W	LP	P
	W	VVN	VN	N	LN	W	LP	P	VP
	LP	VN	N	LN	W	LP	P	VP	VVP
	P	N	LN	W	LP	P	VP	VVP	VVP
	VP	LN	W	LP	P	VP	VVP	VVP	VVP
	VVP	W	LP	P	VP	VVP	VVP	VVP	VVP

**Constructing HAC**

Choose a parameter set:

$G = \{0, \text{Negative (N)}, W, \text{Positive (P)}, 1\};$

$H = \{\text{Little (L)}\} = \{h_{-1}\}; q = 1;$

$H^+ = \{\text{Very (V)}\} = \{h_1\}; p = 1;$

$v(W) = \theta = 0.5; fm(N) = \theta = 0.5;$

$fm(P) = 1 - \theta = 0.5$

Hedges are selected as shown in Table 2

**Table 2. Selected Parameters of E, IE, U**

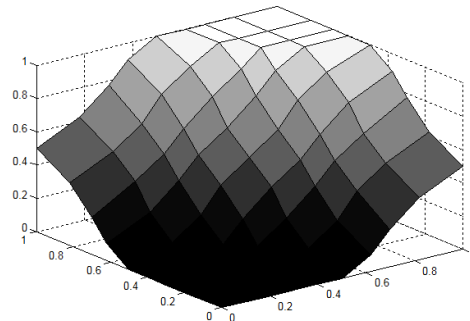
		Input1 (E)		Output (U)	
H		$\mu(h)$		$\mu(h)$	
H-	Little (L)	$\alpha$	0.4	$\alpha$	0.55
H+	Very (V)	$\beta$	0.6	$\beta$	0.45

Calculations of semantic quantifying values for E, IE and U are shown in SAM table (Table 3) which is converted from Table 1.

**Table3. SAM**

U	E								
	0.108	0.18	0.3	0.38	0.5	0.62	0.7	0.82	0.892
IE	0.108	0.0456	0.0456	0.0456	0.0456	0.1012	0.225	0.3762	0.5
	0.18	0.0456	0.0456	0.0456	0.0456	0.1012	0.225	0.3762	0.5
	0.3	0.0456	0.0456	0.0456	0.1012	0.225	0.3762	0.5	0.6238
	0.38	0.0456	0.0456	0.1012	0.225	0.3762	0.5	0.6238	0.775
	0.5	0.0456	0.1012	0.225	0.3762	0.5	0.6238	0.775	0.8988
	0.62	0.1012	0.225	0.3762	0.5	0.6238	0.775	0.8988	0.9544
	0.7	0.225	0.3762	0.5	0.6238	0.775	0.8988	0.9544	0.9544
	0.82	0.3762	0.5	0.6238	0.775	0.8988	0.9544	0.9544	0.9544
	0.892	0.5	0.6238	0.775	0.8988	0.9544	0.9544	0.9544	0.9544

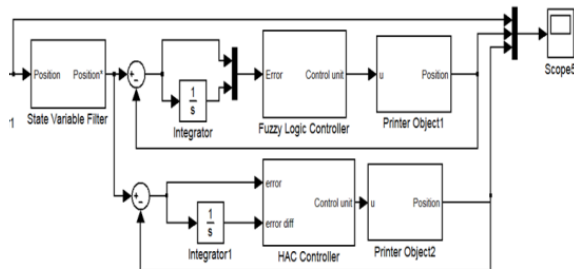
The semantic quantifying curve surface illustrates the in-out relationship as shown in Fig.4.



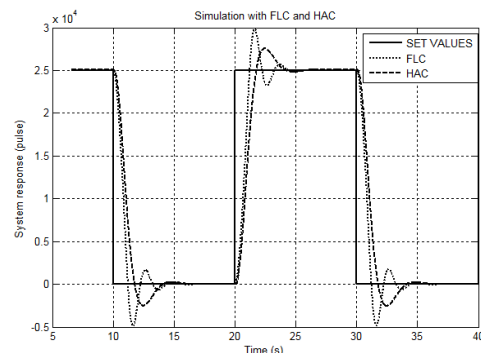
**Fig.4.Semantic Quantifying Curve Surface**

**- The Result of Simulation**

Simulations of FLC and HAC on MATLAB Simulink are shown in Fig. 5 and the results are displayed in Fig. 6.



**Fig. 5.Diagram of FLC and HAC**



**Fig.6. HAC and FLC Simulation Results**

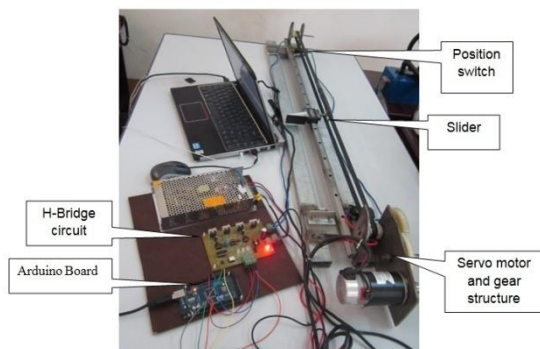
**Remarks of the simulation part:**

PI-FLC and HAC including two inputs and one output have been designed. Simulation results show the stability and accurate tracking of the system. After a certain period of time, the error converges to zero.

It can be seen that HAC performance is stable with small overshoots. These results reveal that the applicability of HAC is similar to that of FLC.

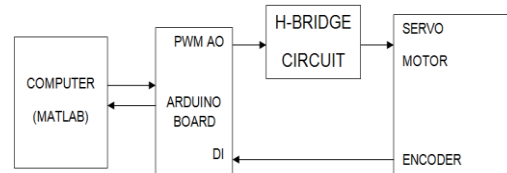
**C. Experiments**

A model of the accurate tracking driven system has been built depending on principles of MEDE5 [3]. System model composition is shown in Fig. 7.



**Fig.7. Model Of The Accurate tracking Driven System** Where:

- Arduino board: Receives the feedback signal from a position sensor (encoder) and communicates with a computer, sends signals to H-bridge power circuit to drive the motor. Arduino is a microprocessor board that is used to program and interact with sensors, motors and other devices. Arduino is chosen to be the brain of so many systems from simple ones to sophisticated ones because of its advantages such as ease to use, simple programming language, low cost and open source for software and hardware. With Arduino, it is easy to estimate control theories because it can integrate with Matlab, Labview and so on.
- Servo motor and gear structure used in the driven system are DB M60-8 with the 24V and 60W power supply, the speed of 3000rpm and the 1000ppr encoder.
- H-Bridge circuit: controls power MOSFETs, allows reverse, protects short circuit, 10A permissible current.
- Position switch LXW5-11G1 is used to limit the movement scope of the slider.
- The controllers designed on MATLAB Simulink have been mentioned in the previous part. They are connected to the tracking system via Arduino circuit as shown in Fig. 8.

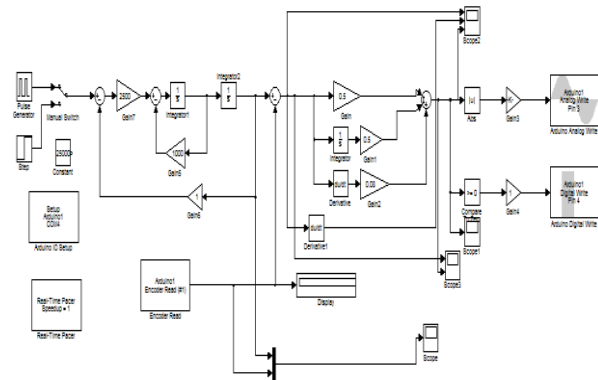


**Fig.8. Control Diagram of the Accurate Tracking Driven System**

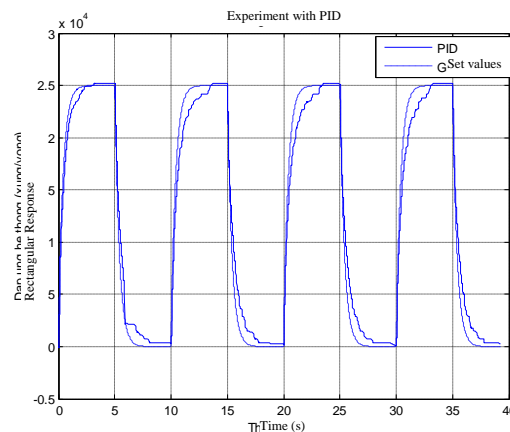
It is evident that the position signal from an encoder is transferred to a controller on computer through Arduino board and compared with the reference input of the system. Accordingly, the controller will calculate the control signal and send to drive the servo motor in order to make the output follow the reference signal.

Before doing experiment with FLC and HAC, authors have implemented practically PID in order to test the controller’s workability as well as to determine required values in physic area. Then, FLC and HAC have been designed.

Experimental interface with PID and the results are shown in Fig. 9 and Fig. 10 respectively.



**Fig. 9. PID Controller Diagram**



**Fig. 10. PID Performance**

Experimental interface with FLC is illustrated in Fig. 11 and the results are shown in Fig. 12

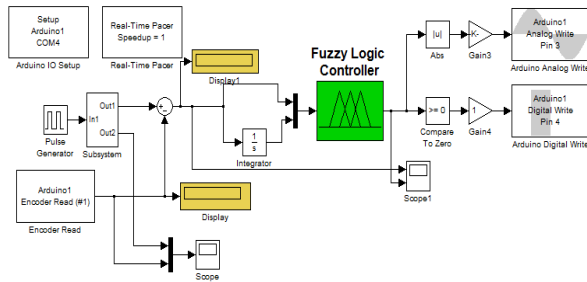


Fig.11. Fuzzy Logic Controller Diagram

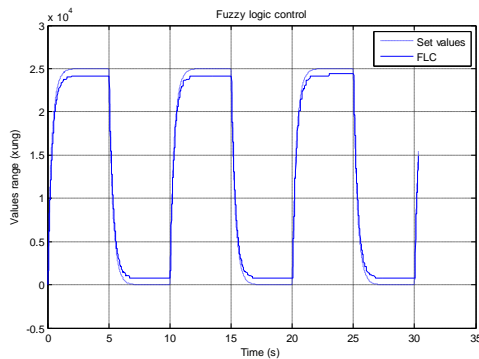


Fig.12. FLC Performance

Experimental interface with HAC and the results are shown in Fig. 13 and Fig. 14 respectively.

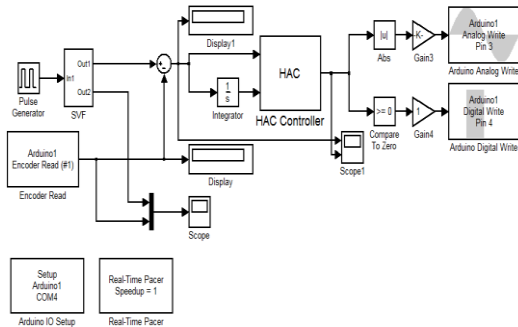


Fig.13.Hedge Algebra Based Controller Diagram

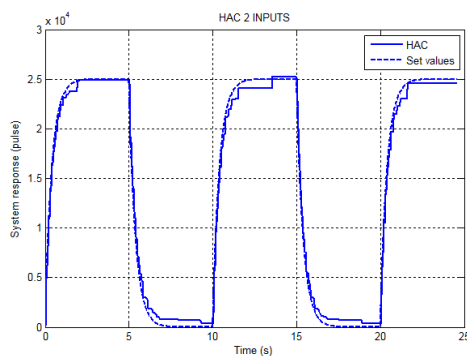


Fig.14.Hac Performance

**Remarks of the experimental part:**

PID, FLC and HAC have good performance. The output tracks the reference input well, responses of the system with input's changes have small overshoots and tiny steady-state errors.

The conducted experiment demonstrates a quick response with the short settling time and the output converges to the reference input with negligible error (there is no steady-state error in simulation).

From the experiment, HAC can be applied to the tracking control system with strict requirements such as quick responses and continuous reverses; therefore, it can be applied to control industrial objects and ensure the quality of the system.

**IV. CONCLUSIONS**

This paper has proposed a new method to design a controller in which HA is used as a soft-computing tool applied in control. The authors have designed and simulated HAC on computer with various objects. In this study, it has become one of the very first experiments in the control field for real objects using HAC. Experimental results show that HAC meets the quality requirements and opens the possibility of applications in reality. HAC has created an algebraic structure with a functional relationship to lead the establishment of a variable set that can be arbitrarily large to express the in-out relationship; therefore, the control quality of HAC can be better than FLC.

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