Using Extended Kalman Filter to Observe State Parameters of the Twin Rotor MIMO System in Order to Install Model Predictive Control Algorithm Based Phisical Model

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Abstract

Most modern control methods including model predictive control (MPC) need to be measured or observed the state vectors of the object model. However, for real systems are often not fully measured state variables. Meanwhile, we must use the state observations to estimate the state vectors of the object model. One of the effective solutions to estimate the state of the system is to use Kalman filter for linear systems and extended Kalman filter (EKF) for nonlinear systems. This paper, we will mention the construction of extended Kalman filter to estimate the state parameters of the Twin rotor MIMO system (TRMS) and use this estimation to conduct tests on the real system so as to check the solution of the optimal problem by SQP algorithm mentioned in [4] and use variational method to solve the optimal problems, announced in [5] for the TRMS. Specifically the SQP algorithm and variational methods in the MPC given in [1], [2], [3].

Keywords: *Model predictive control, the Twin rotor MIMO system (TRMS), Kalman filter, state parameters, state observation.*

I. INTRODUCTION

The TRMS is the experimental system about the aerodynamics at the lab, have similarly working principles as helicopters, systems with bilinear forms mathematical model, has 6 state variables $(\omega_h \quad S_h \quad \alpha_h \quad \omega_v \quad S_v \quad \alpha_v)$. In which, we only measure two variables are the Yaw angle and the Pitch angle, other state variables are not measured. Therefore, to obtain the state variables in each iteration of the control algorithms need to use a state observer.

II. THE TRMS MODEL

The TRMS was given in figure 1

The TRMS is a bilinear system with two inputs and two outputs. It can be described by the continuous model:

$$\begin{cases} \dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = C(\boldsymbol{x})\boldsymbol{x} \end{cases} \quad (1)$$

State variables, inputs, outputs, respectively are:

$$x = [\omega_h, S_h, \alpha_h, \omega_v, S_v, \alpha_v]^T \quad (2)$$

$$u = [U_h \quad U_v]^T \quad (3)$$

$$y = [\alpha_h \quad \alpha_v]^T \quad (4)$$

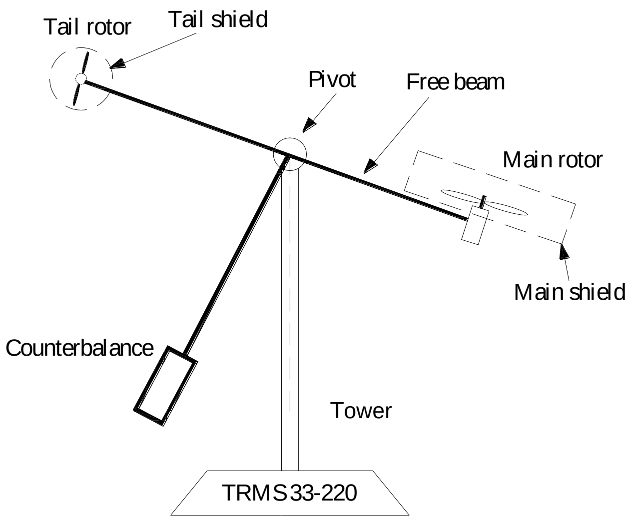

Fig 1. The TRMS

Where:

ω_h: Rotational velocity of the tail rotor (rad/s)

S_h :Angular velocity of the TRMS beam in the horizontal plane without affect of the main rotor (rad/s)

α_h : Yaw angle of the TRMS beam (rad)

ω_v: Rotational velocity of main rotor(rad/s)

S_v: Angular velocity of the TRMS beam in the vertical plane without affect of the tail rotor (rad/s)

α_v: Pitch angle of the TRMS beam (rad)

U_h: Input voltage signal of the tail motor (V)

U_v: Input voltage signal of the main motor (V)

The nonlinear continuous state space equations of the TRMS are expressed in [6], [7], [8] as (7):

Where:

$R_{ah}, L_{ah}, k_{ah}\varphi_h, J_{tr}, B_{tr}, l_t, D, E, F, k_m, R_{av}, L_{av},$
$k_{av}\varphi_v, J_{mr}, B_{mr}, l_m, k_g, g, A, B, C, H, J_v, k_t$

are positive constants, Ω_h and Ω_v is defined by

$$\Omega_h = S_h + \frac{k_m \omega_v \cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \qquad (5)$$

$$\Omega_v = S_v + \frac{k_t \omega_h}{J_v} \qquad (6)$$

$$\frac{d}{dt}\begin{bmatrix} \omega_h \\ S_h \\ \alpha_h \\ \omega_v \\ S_v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -\frac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \frac{B_{tr}}{J_{tr}}\omega_h - \frac{f_1(\omega_h)}{J_{tr}} + \frac{k_{ah}\varphi_h}{J_{tr}R_{ah}} f_6(U_h) \\ \frac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\frac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \frac{B_{mr}}{J_{mr}}\omega_v - \frac{f_4(\omega_v)}{J_{mr}} + \frac{k_{av}\varphi_v}{J_{mr}R_{av}} f_8(U_v) \\ \frac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} + \frac{g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \frac{k_t}{J_v}\omega_h \end{bmatrix} \qquad (7)$$

III. INSTALLING THE KALMAN OBSERVER

Estimating the state parameters of an object is based on the measured parameters at the input and output of them in each time that estimate the other state parameters (not measured) of them.

Assuming a nonlinear system have state - space model:

$$x(k+1) = f(x(k), u(k)) + w(k)$$
$$y(k) = h(x(k)) + v(k) \qquad (8)$$

Extended Kalman Filter algorithm flowchart which estimate the state of the object (8) is shown in Figure 2.

where: $w(k)$ and $v(k)$ are white noise with covariance matrixes $Q(k)$ and $R(k)$. The nonlinear state - space equation can be approximated by the state - space equation depends on the following state:

$$x(k+1) = A(x(k))x(k) + B(x(k))u(k) + w(k)$$
$$y(k) = C(x(k))x(k) + v(k) \qquad (9)$$

The estimation of optimal state variables can be achieved when using the loop equations, is follow

A) *The Predictive Period*

$$\hat{x}(k|k-1) = f(\hat{x}(k-1|k-1), u(k)) \qquad (10)$$

$$M(k) = A(\hat{x}(k|k-1))P(k-1)A^T(\hat{x}(k|k-1)) + Q(k) \qquad (11)$$

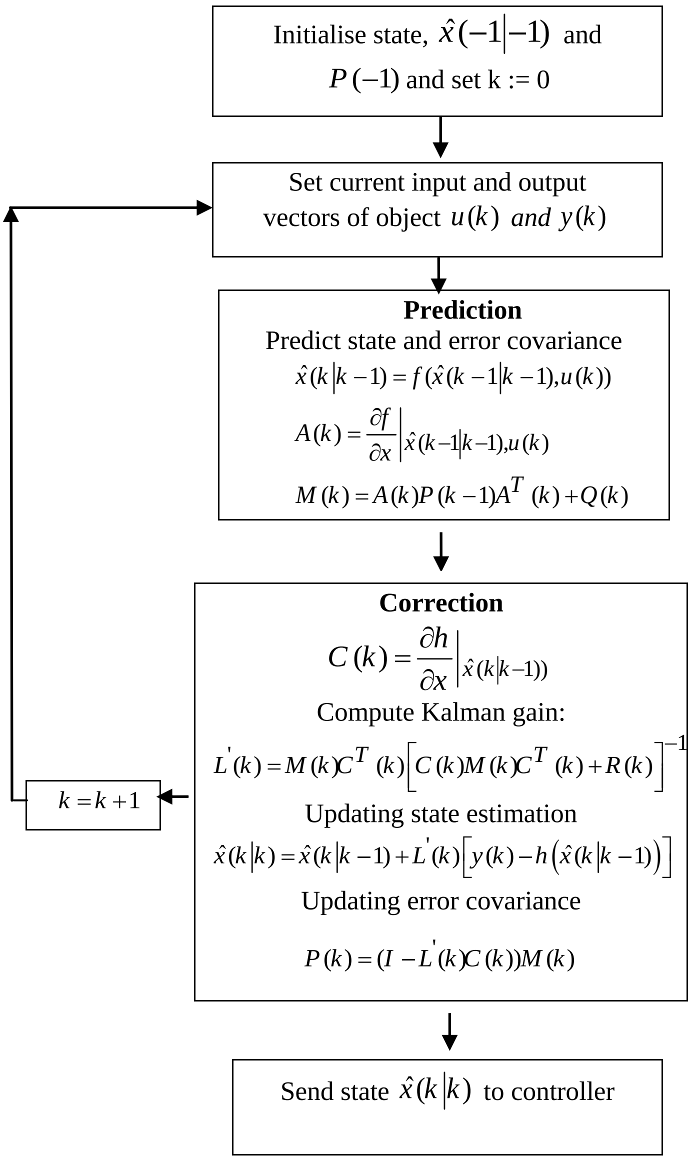

Fig 2. Flowchart of the extended Kalman observer method

where, $\hat{x}(k|k-1)$ is predicted state và $M(k)$ is predicted error covariance.

B. Correction or update

$$L'(k) = M(k)C^T(\hat{x}(k|k-1)).\left[C(\hat{x}(k|k-1))M(k)C^T(\hat{x}(k|k-1)) + R(k)\right]^{-1} \qquad (12)$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + L'(k)\left[y(k) - h(\hat{x}(k|k-1))\right] \qquad (13)$$

$$P(k) = \left(I - L'(k)C(\hat{x}(k|k-1))\right)M(k) \qquad (14)$$

where $L^{'}(k)$ is the Kalman gain, $\hat{x}(k|k)$ is the updated state estimate, và $P(k)$ is the update error covariance.

C. Check the status of observer

In this section, using simulation methods on Matlab-Simulink software to check the accuracy of state estimation algorithm. Simulation test Chart observe the status is shown in Figure 3. In which, the TRMS model block is constructed from the expression (7), the inputs and two outputs of the model (Yaw angle and Pitch angle) are included in the state observer block, the outputs of the status observer block are states that need to estimate of the TRMS. The estimate status is compared with the state of the model. The simulation results are shown from Figure 4 to Figure 9.

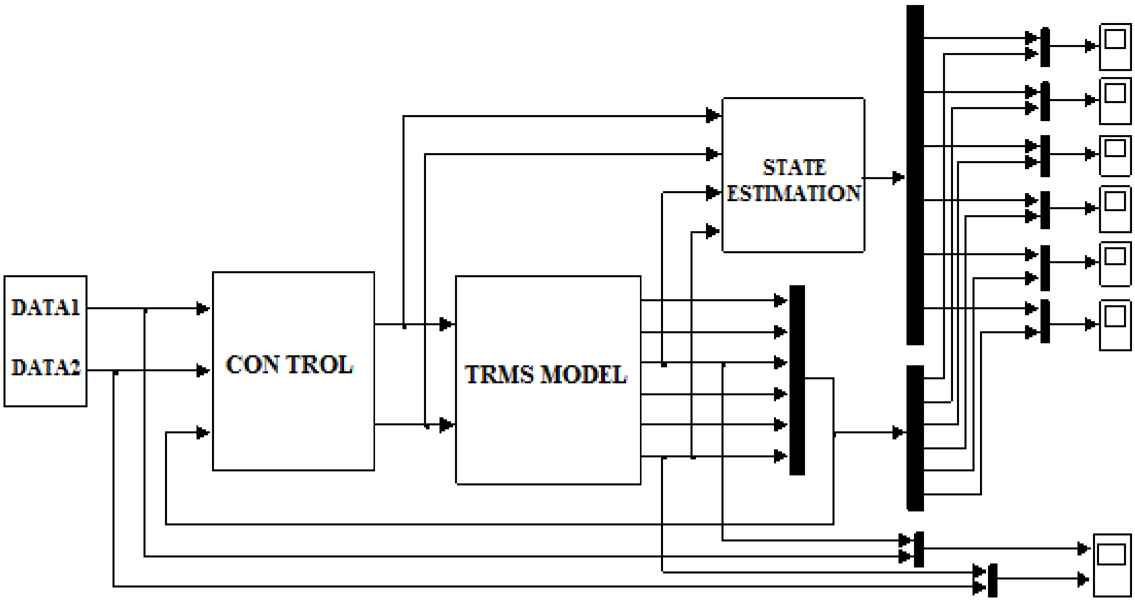

Fig 3. Simulation diagrams check the status of observer

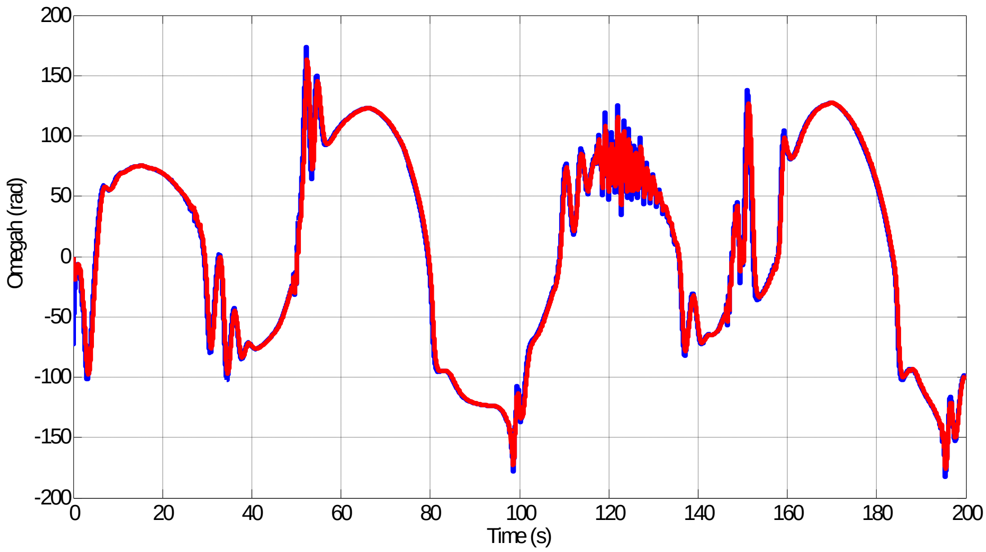

Fig 4. The output response of the status observer with respect to the output response of model of the first state variable (Ω_h)

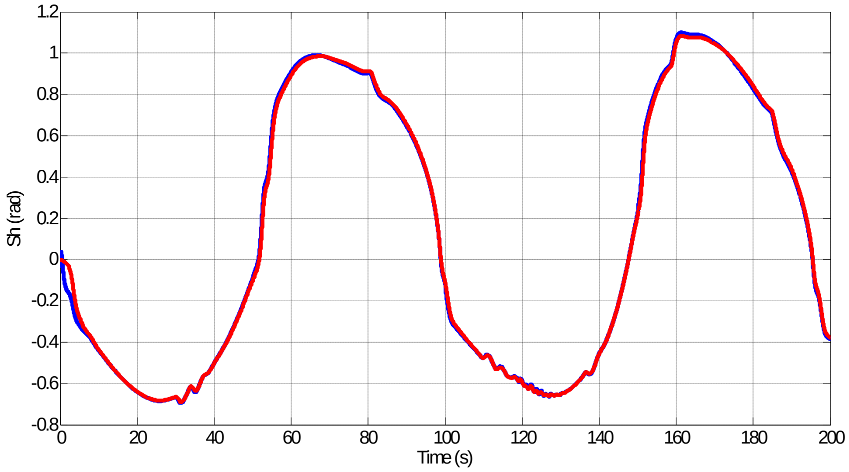

Fig 5. The output response of the status observer with respect to the output response of model of the second state variable (S_h)

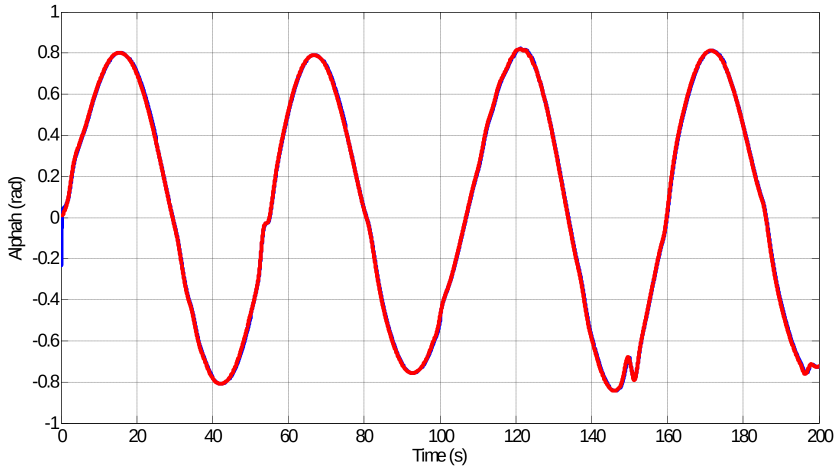

Fig 6. The output response of the status observer with respect to the output response of model of the third state variable (α_h)

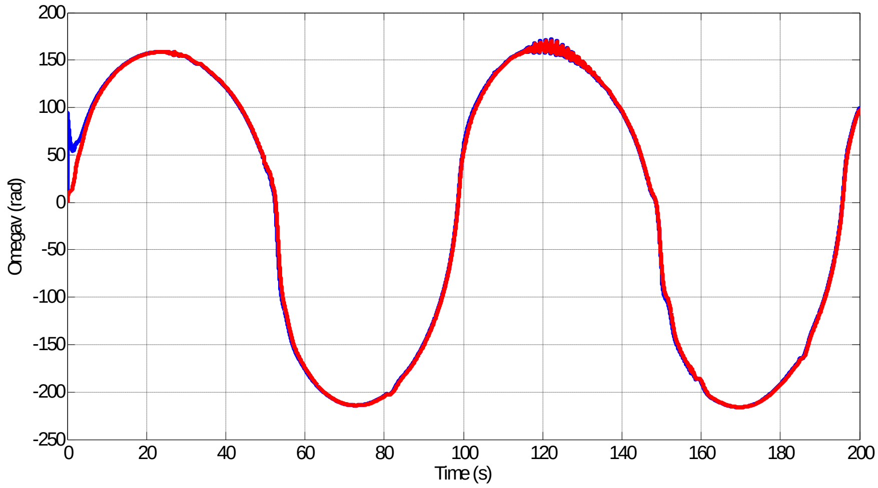

Fig 7. The output response of the status observer with respect to the output response of model of the fourth state variable (Ω_v)

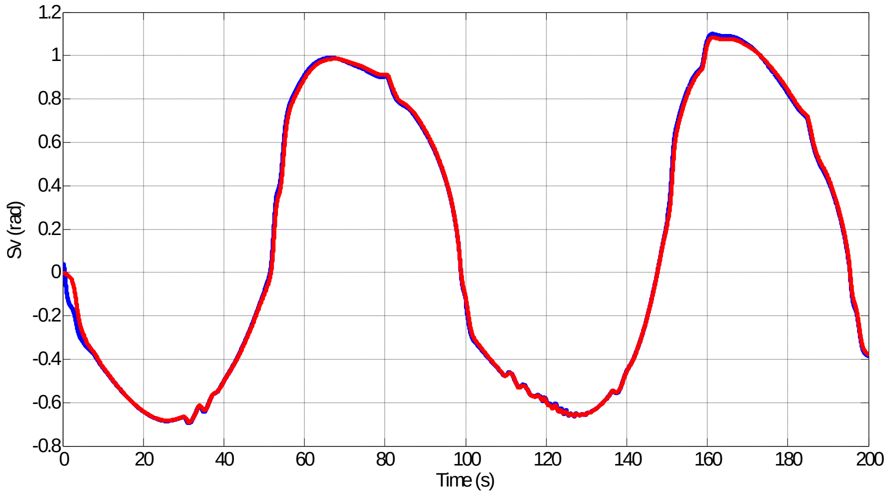

Fig 8. The output response of the status observer with respect to the output response of model of the fifth state variable (S_v)

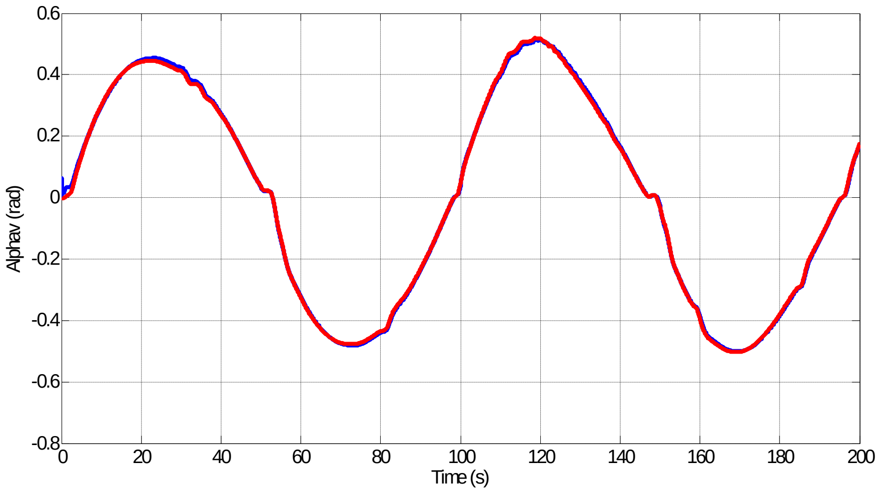

Fig 9. The output response of the status observer with respect to the output response of model of the sixth state variable (α_v)

D. Comment

From the above simulation results, we see that the state variables of the state estimation nearly coincides with the states of the TRMS (largest error is about 0.05% of the initial steps). So we can use this observation to conduct experiments predictive control for the TRMS.

IV. EXPERIMENTING ON PHYSICAL MODEL OF THE TRMS

To check by experiment the optimized predictive control algorithm based nonlinear programming and stable tracking predictive control algorithm according to the output sample signal, the authors have conducted experiments for the TRMS system at the Electricity - Electronics Engineering Lab, Thai Nguyen University of Technology - Thai Nguyen University.

The pairing system with computer through the pairing CARD DSP1103, using real-time tools Workshop of Matlab/Simulink. The output signal which is the Yaw angle and the Pitch angle is measured by Encorder, other states obtained through the state observer above, the experimental results are shown from Figure 10 to Figure 13.

A. Reviews and evaluation of experimental results when using the SQP Algorithm

Experimental results are given in Figure 10 and Figure 11 shows that although the number of times oscillation more when simulation, and an overshoot of the Pitch angle in the 39th seconds is slightly larger when changing the large set signal amplitude (from 2.5 rad to 0 rad) but the output response of the Yaw angle and the Pitch angle keeps tracking on set signal.

The dynamic quality of the system is measured via the standard of deviation squared integral, through the simulation times that value in the range of 0.75 to 0.83.

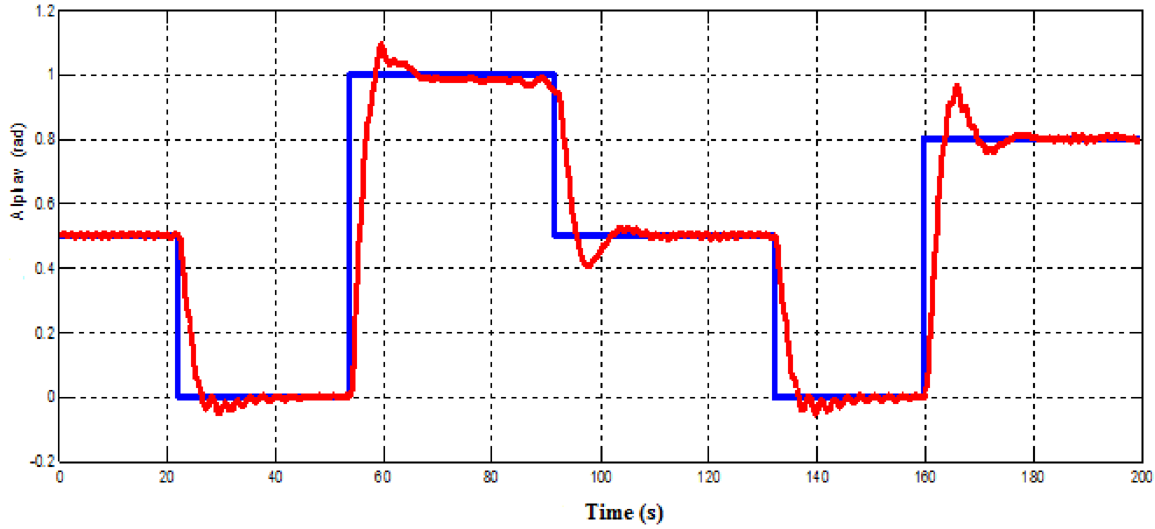

Fig 10. The output response of the Yaw angle when using optimized predictive controller based nonlinear programming

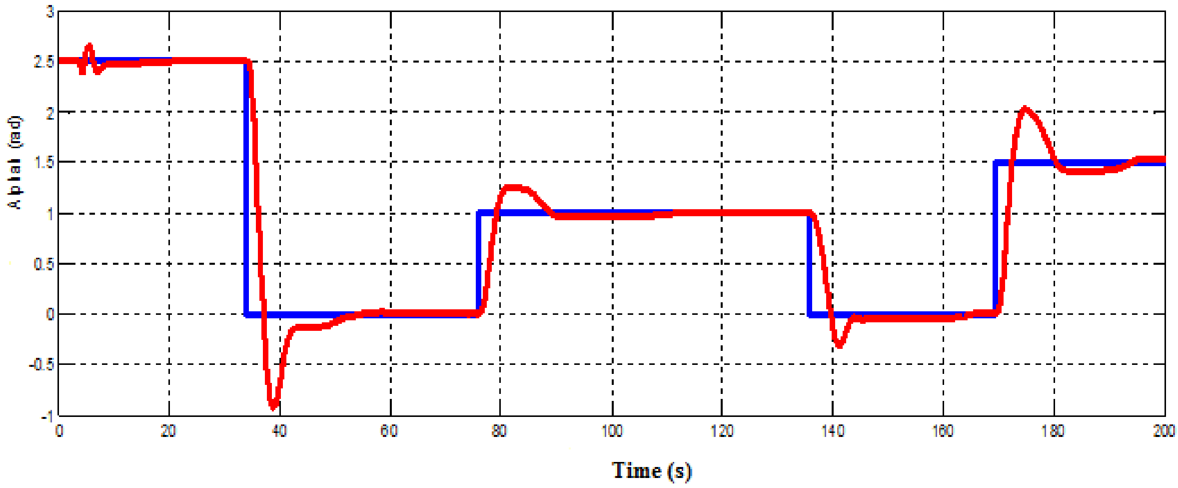

Fig 11. The output response of the Pitch angle when using optimized predictive controller based nonlinear programming

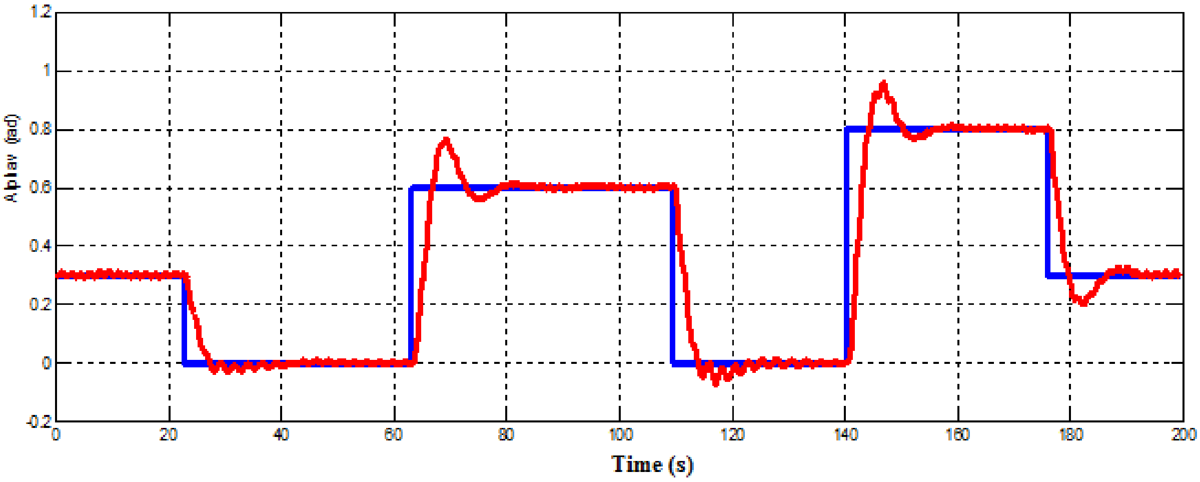

Fig12. The output response of the Yaw angle when using predictive controller stable tracking follow the output sample signal

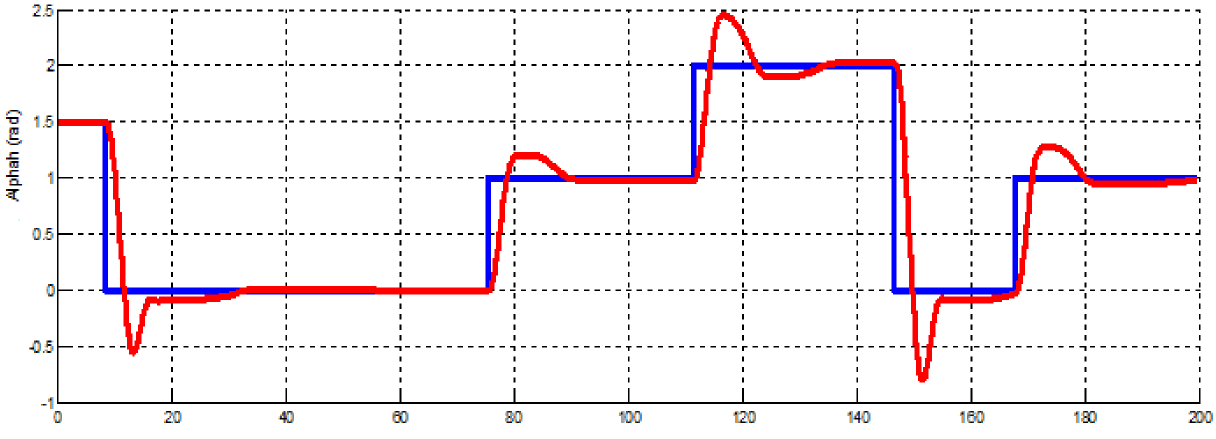

Fig 13. The output response of the Pitch angle when using predictive controller stable tracking follow the output sample signal

B. Reviews and evaluation of experimental results when using the Variational Algorithm

The experimental results obtained when using variational method is given by Figure 12 and Figure 13 shows the danamic characteristics of the system close the set signal. The dynamic quality of the system is measured via the standard of deviation squared integral, through the simulation times that value in the range of 0.83 to 0.86.

V. CONCLUSION

The experimental results have confirmed the correctness of the two algorithms that the authors have proposed in [4] and [5] and the feasibility of them when applying to control for real system. However, this paper still don't test when having the disturbances, so here is the future work in the next study.

REFERENCES

[1] Phuoc. Nguyen.D, (2016) "Optimization in control and Optimal control". Publishing Technology, Ha Noi University of Sciences and Technology.

[2] Nocedal. J and Wright. S.J, (1996) Numerical Optimization. Springer - New York.

[3] Grüne. L and Pannek. J, (2010) "Nonlinear model predictive control," Theory and Algorithms, Springer.

[4] Huong. Nguyen.T.M, Thai. Mai.T, Chinh. Nguyen. H, Dung. Tran.T and Lai. Lai.K, (09-2014), "Model Predictive Control for Twin Rotor MIMO system", The University of Da Nang Journal of science and Technology, 12[85], pp. 39 - 42.

[5] Huong Nguyen T. M., Thai Mai T., Lai Lai K. (2015), "Model Predictive Control to get Desired Output with Infinite Predictive Horizon for Bilinear Continuous Systems", International Journal of Mechanical Engineeringand Robotics Research, Vol. 4, No. 4, pp. 299 - 303.

[6] A. Rahideh, M.H. Shaheed. "Constrained output feedback model predictive control for nonlinear systems", Control Engineering Practive 20, 2012, pp. 431-443.

[7] Huong. Nguyen.T.M, Thai. Mai.T, Chinh. Nguyen. H and Lai. Lai.K, "Researching effects of state parametters in Twin Rotor Mimo System (TRMS)," Journal of science and Technology of Thai Nguyen University, vol. 120, 2014, issue 06, pp. 87 – 92.

[8] A. Rahideh and M.H. Shaheed, "Mathematical dynamic modelling of a twin rotor multiple input–multiple output system,", Proc. of the IMeche, Part I. Journal of Systems and Control Engineering 221, 2007, pp.89-101.