A Control Method for SMMS Teleoperation System with Affection of Disturbance

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***Abstract* — The article presents an overview of some problems in controlling the Teleoperation SMMS (Single Master – Multi Slave) and establishes a control structure that coordinates the slave robots to perform a common task with regard to the effect of disturbance on the system. Based on the dynamic analysis of the Teleoperation SMMS system, with the specific task of the master robot as well as each slave robot, the combination of the disturbance estimation algorithm and the PD - Virtual Damping control algorithm for each robot ensures that the robot trajectory of the slave robot system is dramatically tracked to the trajectory of the master robot under supervision of the operator. By simulating on a Matlab Simulink for the object of Teleoperation SMMS with 2 DOF planar robot arm indicates the correctness of the proposed control method.**

Keywords — *Teleoperation, SMMS system, PD Controller, Disturbance estimation, Tele-robot.*

I. INTRODUCTION

In general, Teleoperation is a system that allows people employing robots to perform tasks in distant working environments or harsh working environments that humans can not directly perform such as nuclear power plant, ocean exploration, unwanted fires ... [1], [4]. In which, the master robot system receives the direct control signals from the operators, creates the trajectory and sends to the slave robot system. The slave robot system will perform these tasks and send feedback to the master system for the operator monitoring the execution of tasks from the slave system through the master robot. The data communication between the master and slave robot systems is implemented via a communication channel which can be wired or wireless. Depending on the number of robots in slave robot system and master robot system, it can be classified into the following categories: Single Master Single Slave (SMSS); Single Master Multi Slave (SMMS); Multi Master Multi Slave (MMMS) [1], [6].

For Teleoperation SMMS in particular, two practical problems can be mentioned as follows:

Problem 1: The master robot system generates tasks through the operator directly, sends to the slave robot system (including multiple similar slave robots) performing the same independent tasks, for instance: carving the same products on wood or drawing the same paintings on paper. In this problem, we have to pay attention to the unexpected incident or disturbance in one of the robots. It is critical to implement the speed synchronization and trajectory between the slave robots and master robots so that enable the ability of operator detecting the happened problem on specific slave robot in the slave robot system through the master robot.

Problem 2: The master robot system creates the task and the robots in the slave robot system are cooperated to perform the task. For example, the slave robots are coordinated to clamp an object and move the object or the robots work together for attaching or detaching components of the nuclear reactor. In this problem, to control the system, it is necessary to determine the position and function of the slave robots in advance, and it is critical to consider the disturbance and possibility of incidents on each robot in the slave robot system.

In this article, we focus on the development of control algorithms for the object as described in Problem 2, based on the evaluation of disturbance effects on the system and the application of PD-Virtual Damping controllers to control the Robot systems.

In the previous methods, they focused on tracking performance of position and force feedback in both side of Tele-robot system, such as PD control method [4], [5], passive control method [6], impedance control method [7], impedance matched control [8] and almost of recently study research about SMMS system. Moreover, these results should be further improved. In [4], the control law was only applied for position tracking, otherwise, the motions of the master and slave still shows error, and the control of force feedback was not concerned. In addition, on the communication channel, the time delay is constant. A method based on PD method [5], the results are better, the motion has been improved, especially in free motion. In this method, the time delay in the communication lines are variable, however, the force control was not treated. In another research [8], the authors used impedance matched for the teleoperation system, the results are more improved than the method that proposed in [5], however, the time delay is still constant and the feedback force from Slave is not achieved with the force of operator at the Master side. Basing on the analysing of the previous methods and objects, in this paper, we proposed a control law by combining PD control and virtual damping to improve the stability of contact force when slave system contacts with the environment. The time delay in the communication channel is variable. This paper used Lyapunov method to prove the stability of the SMMS system. The simulation results by MATLAB of the proposed control law are shown in this paper. These results indicated the advantages and demonstrate the effectiveness of the method.

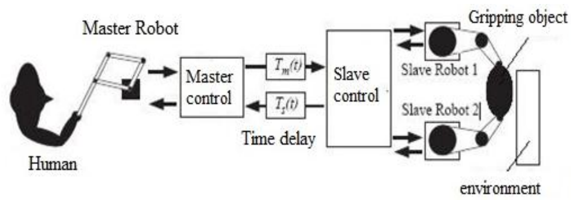

Fig 1: Single Master and Multiple Slaves system (SMMS)

II. PROBLEM FORMULATIONS

A. *Dynamics of Teleoperation system*

In this paper, we consider a pair of robotic systems coupled via communication lines with time-varying delays. Assuming the absence of friction, other disturbances, and gravity term, the equations of a master robot and n slave robot can be described as follow:

$$\begin{cases} M_m\ q_m\ \ddot{q}_m + C_m\ q_m,\dot{q}_m\ \dot{q}_m = \tau_m + J_m^T\ q_m\ F_{op} \\ M_{s_i}\ q_{s_i}\ \ddot{q}_{s_i} + C_{s_i}\ q_{s_i},\dot{q}_{s_i}\ \dot{q}_{s_i} = \tau_{s_i} + J_{s_i}^T\ q_m\ F_{s_i} \end{cases} \tag{1}$$

where the subscript 'm','s' and 'i' denote the master, slave indexes and the order indexes of the slave robots, respectively. $q_m \in R^{m\times1}, q_{s_i} \in R^{n_{s_i}\times1}$ are the joint angle vectors, $\tau_m \in R^{m\times1}, \tau_{si} \in R^{n_{si}\times1}$ are the input torque vectors, $F_{op} \in R^{m\times1}, F_{s_i} \in R^{n_{s_i}\times1}$ are the operational force vector and the grasping force vectors, respectively. $M_m \in R^{m\times m}$, $M_{s_i} \in R^{n_{s_i}\times n_{s_i}}$ are the symmetric and positive definite inertia matrices. $C_m \in R^{m\times m}$ and $C_{s_i} \in R^{n_{s_i}\times n_{s_i}}$ are the centripetal and Coriolis torque vectors. $J_m \in R^{m\times m}$, $J_{s_i} \in R^{n_{s_{s_i}}\times n_{s_i}}$ are Jacobian matrices.

Considering that position encoders measure coordinate q_j with $j = m, s_i$, Cartesian coordinate must be related to these coordinates:

$$\dot{x}_j(t) = J_j(q_j)\dot{q}_j,\quad j = m, s_i \tag{2}$$

where $\dot{x}_m, \ddot{x}_m \in R^{m\times1}$ and $\dot{x}_{s_i}, \ddot{x}_{s_i} \in R^{n_{s_i}\times1}$ are the end effector velocities and acceleration vectors, respectively.

Substituting (2) into (1), we can get the dynamics as follows:

$$\begin{cases} \tilde{M}_m\ q_m\ \ddot{x}_m + \tilde{C}_m\ q_m,\dot{q}_m\ \dot{x}_m = J_m^{-T}\tau_m + F_{op} \\ \tilde{M}_{s_i}\ q_{s_i}\ \ddot{x}_{s_i} + \tilde{C}_{s_i}\ q_{s_i},\dot{q}_{s_i}\ \dot{x}_{s_i} = J_{s_i}^{-T}\tau_{s_i} + F_{s_i} \end{cases} \tag{3}$$

where:

$$\tilde{M}_j = J_j^{-T}M_jJ_j^{-1},\ \tilde{C} = J_j^{-T}\left(C_j - M_jJ_j^{-1}\dot{J}_j\right)J_j^{-1}\ k = m,\ s_i$$

x_{s_i} is end-effector of each slave robot in Cartesian coordinate system of multiple slaves. Let us denote the total degree of freedom of the N slave robots by $n = \sum_{i=1}^{N} n_{s_i}$, hence the group dynamics of the N slave robots can be rewritten as

$$\tilde{M}\ q\ \ddot{x} + \tilde{C}_m\ q,\dot{q}\ \dot{x} = \tau' + F \tag{4}$$

where: $x = [x_1^T,...,x_N^T] \in R^{n_{si}}$, $\tau' = [\tau_1^T J_1^{-T},...,\tau_N^T J_N^{-T}] \in R^{n_{si}}$ and $F = [F_1^T,...,F_N^T] \in R^{n_{si}}$ and $\tilde{M} = diag[\tilde{M}_1,...,\tilde{M}_N] \in R^{n_{si}}$, $\tilde{C} = [\tilde{C}_1,...,\tilde{C}_N] \in R^{n_{si}}$ are the inertia matrices and Coriolis matrices, respectively.

In this system, the signals are transferred between both sides of master and slave. Communication delay is assumed as follows:

Assumption 1: Both time-varying delay $T_m(t), T_s(t)$ are continuously differentiable functions and possibly bounded as:

$$0 \le T_j(t) \le T_j^+ < \infty, |\dot{T}_j(t)| < 1,\ j = m, s$$

where $T_j^+ \in R$ is upper bounds of the communication delay.

In the paper, the remote environment is assumed to be a simple spring-damper system with a constant parameter. This system us as a perturbed system described by the equation below in the form of input-to-state stability properties:

$$\begin{cases} \dot{x}_e = \Gamma_{env}(t, x_e, x_L, \dot{x}_L) + g_e(t, x_e, x_L, \dot{x}_L) \\ F_L = \Gamma_{env}\ t, x_e, x_L, \dot{x}_L \end{cases} \tag{5}$$

where $x_e \in R^{n_{si}}$ is a position of the environment, x_L and $\dot{x}_L$ are the position and velocity vectors of the cooperative-slave robots in Lock-system. F_L is the environment force. We assume that $F_{env}(t, x_e, x_L, \dot{x}_L)$ and $\Gamma_{env}\ t, x_e,, x_L, \dot{x}_L$ are piecewise continuous in t and locally Lipschitz in $x_e, x_L, \dot{x}_L$. The input $(x_L, \dot{x}_L)$ is a piecewise continuous and essentially bounded function of t for all $t \geq 0$, $g_e(t, x_e, x_L, \dot{x}_L)$ is the perturbation term. The environment satisfies the following assumptions:

Assumption 2: The cooperative-slave contacts with following spring-damper environment with constant parameter

$$\Gamma_{env}\ t, x_e,, x_L, \dot{x}_L\ \leq | x_e | + a | \dot{x}_L | + b | x_L | \quad (6)$$

holds for all $t \geq 0$, $a, b > 0$ are constant parameters.

Assumption 3: Let $x_e = 0$ be a uniformly asymptotically stable equilibrium point of the nominal system (5). There exists a Lyapunov function of the nominal system such that $\alpha_{1e}(| x_e |) \leq V_e(x_e) \leq \alpha_{2e}(| x_e |)$ holds for all x_e and $V_e = 0$ while $x_e = 0$. The time derivative of V_e along trajectories of (5) satisfies:

$$\dot{V}_e(t) \leq -\alpha_{3e} | x_e |^2 + \left| \frac{\partial V_e}{\partial x} \right| g(t, x_e, x_L, \dot{x}_L) \quad (7)$$

where $\alpha_{1e}(|x_e|)$, $\alpha_{2e}(|x_e|)$ are class κ functions and $\alpha_{3e} > 0$. The perturbation $g(t, x_e, x_L, \dot{x}_L)$ satisfies the uniform bound:

$$\left| \frac{\partial V_e}{\partial x} \right| \left| g_e(t, x_e, x_L, \dot{x}_L) \right| < \delta\alpha_{4e} \left| x_e \right| \leq F_L^T s_e(t) \quad (8)$$

For almost all $t \geq 0$, $\alpha_{4e} > 0$ and δ is a perturbation gain. Let us define:

$$s_e(t) = \dot{x}_L(t) + \Lambda_{env} x_L(t) \quad (9)$$

where Λ_{env} is a positive diagonal gain matrix. Note the first bound of the perturbation in (8), we have:

$$\dot{V}_e(t) \leq -\alpha_{3e} \left| x_e \right|^2 + \theta_e \alpha_{3e} \left| x_e \right|^2 - \theta_e \alpha_{3e} \left| x_e \right|^2 + \delta a_{4e} \left| x_e \right|$$

$$= -(1-\theta_e)\alpha_{3e} | x_e |^2 - \left| x_e \right| (\theta_e \alpha_{3e} \left| x_e \right| + \delta a_{4e})$$

$$\leq -(1-\theta_e)\alpha_{3e} | x_e |^2 ; \forall \left| x_e \right| \geq \frac{\delta a_{4e}}{\theta_e \alpha_{3e}} \quad (10)$$

where $\theta_e < 1$ is some positive constants.

Therefore, the upper bound of perturbation satisfied the time derivative of V_e as follow:

$$\dot{V}_e(t) \leq -\alpha_{3e} | x_e |^2 + F_L^T s_e(t) \quad (11)$$

B. *Control objectives*

The SMMS system in this paper is showed in Fig. 1 with a master robot and two slave robots. The cooperative slave robot is similar to dual-arm robot. The object is grasped to transport to a specified place according to the instruction values of a controller from the operator in the task space.

Control object 1: (Autonomous Grasping by Multiple slave robots), in this paper, the achievement of grasping: "a relative position of the end-effectors of the slave robots is shaped in a certain specified form" means that the following condition is accomplished: $x_S = x_S^d$ where x_S is the relative position of the end-effector of the slaves, x_S^d is a desired position.

Control object 2: (Movement of Grasped object) When the grasping is achieved, the center position between the end-effector of the slave robots is same with the center position of the grasped object, then the movement of the grasped object is achieved as: $x_L = x_m$ where $x_L = \alpha x_{L0} - C, x_{L0}$ and x_m are the center position of the end-effectors and the grasped object, respectively. α is the position scale, C is showed a translation value.

Control object 3: (Static Force Reflection) the Tele-robot with static Force Reflection is achieved as $\dot{x}_{m,L} = \ddot{x}_{m,L} = 0$ such that: $F_{op} = -\beta F_L$ where F_L is the contact force of cooperative-slave, $\beta > 0$ is a positive scalar and it expressed a force scaling factor.

III. CONTROL DESIGN

In this section, we propose a control law for the SMSS system to achieve the above control object in control objectives.

A. *Passive-Decomposition*

The First, based on the passive-decomposition [9], the dynamics of multiple slave robots is decomposed into a decoupled system: the shape-system describing "movement of the multiple slaves with grasping object" and the locked-system describing "movement of the multiple slaves according to the instruction from master". Utilizing the passive-decomposition, the velocity of multiple slave robots is rewritten with each system as:

$$\dot{x} = S^{-1} \begin{bmatrix} \dot{x}_S \\ \dot{x}_L \end{bmatrix} \quad (12)$$

where $\dot{x}_S$ and $\dot{x}_L$ are velocities of the shape-system and lock-system, respectively. S is the decomposition matrix. The matrix S is also a positive matrix of a decoupling shape and lock system. In the following formula of $S^{-T}\tilde{M}S^{-1}$, the non-diagonal term are removed as:

$$S^{-T}\tilde{M}S^{-1} = \begin{bmatrix} M_S & 0 \\ 0 & M_L \end{bmatrix} \quad (13)$$

where: M_S, M_L are inertia matrices of the shape-system and lock-system, respectively. In the face that, $\dot{x}_S$ and $\dot{x}_L$ are defined for satisfying (7). In addition, a local compensation of impedance shaping is necessary. The reflection forces from environment relate and the locked-system as follows:

$$\begin{bmatrix} F_S \\ F_L \end{bmatrix} = S^{-T} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \begin{bmatrix} \tau_S \\ \tau_L \end{bmatrix} = S^{-T} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (14)$$

where F_1, F_2 are forces in each end-effector of slave robot that exert on grasping object. From above definitions, we define:

$$\begin{bmatrix} C_S & C_{SL} \\ C_{LS} & C_L \end{bmatrix} = S^{-T} \tilde{M} \frac{d}{dt} S^{-1} + S^{-T} \tilde{C} S^{-1} \quad (15)$$

The passive-decomposition form with the affection of disturbances is written as:

$$\begin{cases} M_S \; q \; \ddot{x}_S + C_S \; q,\dot{q} \; \dot{x}_S + C_{SL} \; q,\dot{q} \; \dot{x}_L + \tau_S^* = \tau_S + F_S \\ M_L \; q \; \ddot{x}_L + C_L \; q,\dot{q} \; \dot{x}_L + C_{LS} \; q,\dot{q} \; \dot{x}_S + \tau_L^* = \tau_L + F_L \end{cases} \quad (16)$$

Where τ_S^*, τ_L^* are disturbances on the system. Above dynamic equations include friction terms $C_{SL}\dot{x}_L$ and $C_{LS}\dot{x}_S$. However, ignore the remote control by the human, decoupling of the Shape-system and the Locked- system is desired for the slave that maybe autonomous grasping. Therefore, the decoupling control inputs are given as follows:

$$\begin{cases} \tau_S = C_{SL} \; q,\dot{q} \; \dot{x}_L + \tau_S^* - F_S + \tau_S^{'} \\ \tau_L = C_{LS} \; q,\dot{q} \; \dot{x}_S + \tau_L^* - F_L + \tau_L^{'} \end{cases} \quad (17)$$

where $\tau_S^{'}, \tau_L^{'}$ are new control inputs. Substituting (17) into (16), we get:

$$\begin{cases} M_S \; q \; \ddot{x}_S + C_S \; q,\dot{q} \; \dot{x}_S = \tau_S^{'} \\ M_L \; q \; \ddot{x}_L + C_L \; q,\dot{q} \; \dot{x}_L = \tau_L^{'} \end{cases} \quad (18)$$

hence, two above dynamics become a decoupling.

Some properties of this SMMS system are given as follows:

Property 1: $M_i(\mathrm{q})\,(\mathrm{i}=\mathrm{S,L})$ is a positive symmetric matrix and there exist some constant parameters with below relationship as:

$$0 < m_{i1} \le \|M_i\| < m_{i2}, \|C_i\| \le c_i \|\dot{x}_i\| \quad (19)$$

Property 2: $M_i(\mathrm{q}) - 2C_i(\mathrm{q},\dot{\mathrm{q}})$ $(\mathrm{i}=\mathrm{S,L})$ are skew symmetric matrices.

Property 3: $\dot{x}_i, \ddot{x}_i\,(\mathrm{i}=\mathrm{S,L})$ are bounded and $\dot{M}_i, \dot{C}_i$ are also bounded.

Properties 1-3 denote the feature of motion equation of normal robots. The following assumption is formed (1), (18) as follows:

Assumption 4: The operator and the environment can be modeled as passive systems where the velocities $\dot{x}_m, \dot{x}_s$ are system inputs and the bounded force F_{OP}, F_L are the system outputs, respectively. Moreover, these forces are bounded by the function of the velocities, of the master and the Locked-system. The velocities $\dot{x}_m, \dot{x}_s$ also equal to zero for $t < 0$.

B. Disturbance estimation on the Locked- system

The dynamics of the Locked- system were described in (16). We have to estimate $\tau_L^*, C_{LS}\dot{x}_S$, then using the estimated results for control purpose to make the system having the ability of adaptively anti-disturbance. Fig 2 presents the structure diagram of the disturbance estimator on the Locked- system including two main blocks: The model of the Locked-system (MHRS) and the disturbance processing block of the Locked- system (XLNS). The dynamics of the block MHRS is presented as the following equations:

$$\bar{\mathbf{M}}_L(q)\ddot{\bar{q}} + \bar{\mathbf{C}}(q,\dot{q})\dot{\bar{q}} = \tau_L \quad (20)$$

$$\bar{\mathbf{M}}_L(q) = \mathbf{M}_L(q); \; \bar{\mathbf{C}}_L(q,\dot{q}) = \mathbf{C}_L(q,\dot{q}) \quad (21)$$

$\bar{q}, \dot{\bar{q}}, \ddot{\bar{q}}$ are state variables of the Slave robot model MHRS. Let two sides of equation (20) subtract to the corresponding sides of equation (16), regarding to (21), we obtain:

$$\mathbf{M}_L(q)\ddot{\varepsilon}_L + \mathbf{C}_L(q,\dot{q})\dot{\varepsilon} = \tau_e^* \quad (22)$$

with

$$\varepsilon = \bar{q} - q; \quad \dot{\varepsilon} = \dot{\bar{q}} - \dot{q}; \quad \ddot{\varepsilon} = \ddot{\bar{q}} - \ddot{q}; \; \tau_e^* = C_{LS} \; q,\dot{q} \; \dot{x}_s + \tau_L^* - F_L \quad (23)$$

The expressions (22), (23) can be used to determine τ_e^*. The evaluation results from (22), (23) will be the evaluated vector of environmental impact on the Locked- system τ_e^* , with the evaluated error Δ_e depends on the accuracy of the abovementioned sensors: $\tau_e^* - \hat{\tau}_e^* = \Delta_e$ hay $\tau_e^* = \Delta_e + \hat{\tau}_e^*$ (24)

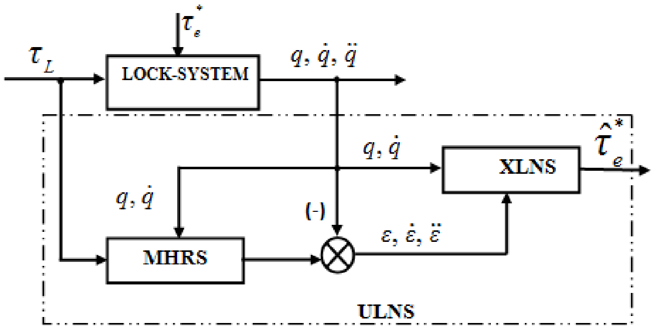

Fig 2: Structure diagram of the environmental impact estimator on the Lock – system

C. Proposal control law

Concerning the control law of the Shape-system in (18), the control object of this system is: $x_S = x_S^d$, then the position tracking with this control law is proposed as:

$$\tau'_S = M_S(\ddot{x}_S - K_P^S(x_S - x_S^d(t)) - K_D^S(\dot{x}_S - \dot{x}_S^d(t))) + C_S\dot{x}_S - F_S \quad (25)$$

Substituting (25) into (18) we obtain the following closed-loop systems:

$$\ddot{e} + K_D^S\dot{e} + K_P^S e = 0, \quad e = x_S - x_S^d \quad (26)$$

where K_d^S, K_d^S are positive definite diagonal gain matrices.

In the Locked-system, note the control objectives 2 and 3, the cooperative control law is defined as follow:

$$\begin{cases} \tau_m = J_m^T(-K_p(x_m - \hat{x}_L) - K_D(\dot{x}_m - \dot{\hat{x}}_L) + \\ +K_i \int \hat{x}_L dt - F_{op} - K_F(\beta \hat{F}_L - F_{op})) - \mathrm{K}_D \dot{x}_m \\ \tau_L = J_L^T(-K_p(x_L - \hat{x}_m) - K_D(\dot{x}_L - \dot{\hat{x}}_m) + \\ +K_i \int \hat{x}_m dt - F_L + K_F(\beta^{-1}\hat{F}_{op} - F_L)) - \mathrm{K}_D \dot{x}_L \end{cases} \tag{27}$$

where β is force scaling factor. $T_m(\mathrm{t}), T_s(\mathrm{t})$ are assumed to be time-varying delays. Similar to the exerted and contact forces, the following signals

$$\hat{x}_m = x_m(\mathrm{t} - \mathrm{T}_m(\mathrm{t}));\ \hat{x}_L = x_m(\mathrm{t} - \mathrm{T}_s(\mathrm{t})) \tag{28}$$

$$\hat{F}_{op} = F_{op}(\mathrm{t} - \mathrm{T}_m(\mathrm{t}));\ \hat{F}_L = F_L(\mathrm{t} - \mathrm{T}_s(\mathrm{t})) \tag{29}$$

are available for the controller on both sides of Tele-robot. We define:

$$x_e^m = x_m - \hat{x}_L\ ,\ \dot{x}_e^m = \dot{x}_m - \dot{\hat{x}}_L \tag{30}$$

$$x_L^m = x_L - \hat{x}_m\ ,\ \dot{x}_L^m = \dot{x}_L - \dot{\hat{x}}_m \tag{31}$$

Substituting (27), (30), (31) into Lock-system in (18) and (3), we obtain a closed-loop system as follow:

$$\begin{cases} \tilde{M}_m(q_m)\ddot{x}_m + \tilde{C}_m(q_m, \dot{q}_m)\dot{x}_m = -K_p(x_m - \hat{x}_L) + \\ -K_D(\dot{x}_m - \dot{\hat{x}}_L) + K_i \int \hat{x}_L dt - K_F(\beta \hat{F}_L - F_{op}) - \mathrm{K}_D \dot{x}_m \\ M_L(q)\ddot{x}_L + C_L(q, \dot{q})\dot{z}_L = -K_p(x_L - \hat{x}_m) + \\ -K_D(\dot{x}_L - \dot{\hat{x}}_m) + K_i \int \hat{x}_m dt + K_F(\beta^{-1}\hat{F}_{op} - F_L) - \mathrm{K}_D \dot{x}_L \end{cases} \tag{32}$$

where $K_p^j,\ K_d^j, K_F^j \quad j = m, L$ are gains and defined as follows: $K_i^m = \mathrm{k}_m K_i^m,\ K_i^L = \mathrm{k}_L K_i^L,\ i = p, d, F$

Where: K_i^j are positive definite diagonal control gain matrices. $k_m, k_L > 0$ are constant gains.

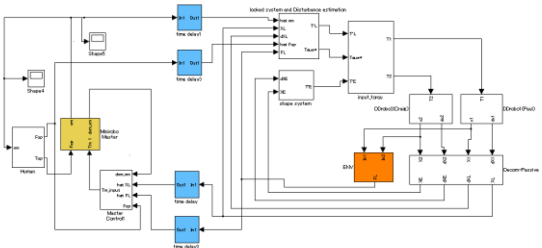

Fig 3: The control structure of the SMMS system

IV. STABILITY ANALYSIS

A. *Stability of Shape-system*

The below theorem concerns the Shape-system.

Theorem 1: Consider the closed-loop Shape-system (26), the desired value of the relative position of spaces between the slave robots is conversed as follows:

$$e = x_S - x_S^d \to 0 \qquad \text{as} \qquad t \to 0$$

(33)

Proof. The equation (26) can be written as follows:

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \phi \begin{bmatrix} e \\ \dot{e} \end{bmatrix},\ \phi = \begin{bmatrix} 0 & I \\ -K_p^S & -K_d^S \end{bmatrix} \tag{34}$$

where K_p^S, K_d^S are positive diagonal matrices, eigenvalues of ϕ becoming negative, therefore following errors of position and velocity are achieved:

$$e = x_S - x_S^d \to 0 \qquad \text{as} \qquad t \to 0$$

(35)

$$\dot{e} = \dot{x}_S - \dot{x}_S^d \to 0 \qquad \text{as} \qquad t \to 0$$

(36)

It means the control objective 1 is achieved and the autonomous grasping of multiple slaves is also achieved.

B. *Stability of Lock-system*

This section deals with the stability of the overall tele-robot system that includes the master and the cooperative slave subsystems of Locked-system.

Lemma 1: Consider the closed-loop master subsystems to be a piecewise continuous in t and locally Lipschitz in the state $x_m = (\mathrm{x}_m^T, \dot{\mathrm{x}}_m^T)^T$.

The input $u_m = (\hat{x}_L^T, \dot{\hat{x}}_L^T, \hat{\mathrm{F}}_L^T)^T$. There exists a continuous differentiable, positive definite, radially unbounded Lyapunov function $V_m : R^r \to R$ of the subsystems that satisfies the inequalities:

$$\alpha_{1m}\left(|x_m|\right) \le V_m \le \alpha_{2m}\left(|x_m|\right) \tag{37}$$

$$\frac{\partial V_m}{\partial t} + \frac{\partial V_m}{\partial x} f\left(t, x_m, u_m\right) \le -\alpha_{3m}\left(|x_m|\right) \tag{38}$$

$\forall |x_m| \ge \rho_m\left(|u_m|\right) > 0 \quad \forall t \ge 0, D = \left\{x_m; |x_m| < r_m\right\}$, $D_u = \left\{u_m; |u_m| < r_{mu}\right\}$,

where $\alpha_{1m}(|x_m|),\ \alpha_{2m}(|x_m|),\ \alpha_{3m}(|x_m|)$ and ρ_m are class κ functions, then the subsystem is locally input-to-state stable.

Proof. First, consider an ISS-Lyapunov function candidate as follows:

$$V_m = \dot{x}_m^T \tilde{M} \dot{x}_m + x_m^T K_p x_m - 2K_F \int_o^t F_{op}(\xi)\dot{x}_m(\xi) d\xi \tag{39}$$

where $\tilde{M}_m,\ K_p,\ K_F$ are positive definite matrices. Following the *Assumed 4*, the environment and the manipulator are passive, then V_m is the positive function. We also easily check that this function satisfies (37) and $V_m = 0$ while $x_m = 0, \dot{x}_m = 0$.

Since $\alpha_{1m}\left(|x_m|\right), \alpha_{2m}\left(|x_m|\right)$ are radially unbounded, hence V_m is said to be radially unbounded.

The derivative of the above function along trajectories of the system (32) with concerning Property 2 as

$$\dot{V}_m = 2\dot{x}_m^T \tilde{M}\ddot{x}_m + 2x_m^T K_p \dot{x}_m - 2K_F F_{op}\dot{x}_m + x_m^T \dot{\tilde{M}} x_m$$

$$= 2\dot{x}_m^T K_p \hat{x}_L - 4\dot{x}_m^T K_D \dot{x}_m + 2\dot{x}_m^T K_D \hat{\dot{x}}_L - 2K_F \beta \dot{x}_m^T \hat{F}_L$$
$$+ 2\dot{x}_m^T K_i \int_0^t \hat{x}_L dt + 2\dot{x}_m^T \theta_m \dot{x}_m - 2\dot{x}_m^T \theta_m \dot{x}_m$$
$$= -2\dot{x}_m^T (2K_D - \theta_m)\dot{x}_m - 2\dot{x}_m^T (\theta_m \dot{x}_m - K_D \hat{\dot{x}}_L +$$
$$- K_p \hat{x}_L - K_F \beta \hat{F}_L - K_i \int_0^t \hat{x}_L dt)$$
$$\forall |\dot{x}_m| \geq (K_p |\hat{x}_L| + K_D |\hat{\dot{x}}_L| + K_F \beta |\hat{F}_L| + K_i \int_0^t |\hat{x}_L| dt)\theta_m^{-1}$$

where θ_m is some positive constants. We can choose θ_m to satisfy the derivative of V_m to be negative as follows: $\theta_m \leq 2K_D$ (40)

Using the fact that, the signal $\dot{x}_m$ is bounded, the feedback force from slave side is also bounded or the input u_m is bounded. Following the *Theorem 5.2* [10], we can choose a class κ function $\gamma_m = \alpha_{1m}^{-1} \circ \alpha_{2m} \circ \rho_m$, positive constant $k_1 m = a_{2m}^{-1}(\mathrm{a}_{1m}(\mathrm{r}_m))$ and $k_2 m = \rho_m^{-1}(\min\{k_{1m}, \rho_m, (\mathrm{r}_m)\})$.

For any initial state $x_m(\mathrm{t}_0)$ and any bounded input $u_m(\mathrm{t})$, we can choose r_m, r_{mu} large enough that satisfies the inequalities given below:

$$|x_m(t_0)| < \alpha_{2m}^{-1}(\alpha_{1m}(r_m)) \quad (41)$$

$$\rho_m\left(\sup_{t\geq 0}|u_m|\right) < \min\{\alpha_{2m}^{-1}(\alpha_{1m}(r_m)), \rho_m(r_{mu})\} \quad (42)$$

Using the Definition 5.2 [10] we have the solution $x_m(\mathrm{t})$ exists and satisfies:

$$x_m(t) \leq \beta(|x_m(t_0), t - t_0|) + \rho_m\left(\sup_{t_0 \leq \tau \leq t}|u_m(\tau)|\right) \quad (43)$$

$$\forall 0 \leq t_0 \leq t$$

where β is a class κL function. Then the solution $x_m(\mathrm{t})$ only depends on $u_m(\tau)$ for $t_0 \leq \tau \leq t$ and the master system is locally input-to-state stable.

Lemma 2: State of the closed-loop cooperative-slave subsystem is assumed as $x_S = (\mathrm{x}_L^T, \dot{\mathrm{x}}_L^T, \mathrm{x}_e^T)^T$ and input $u_s = (\hat{x}_m^T, \hat{\dot{x}}_m^T, \hat{\mathrm{F}}_{op}^T)^T$. We suppose the environment dyna- mic satisfy Assumption 2 and 3. There exists a continuous differentiable, positive definite, radially unbounded Lyapunov function V_s of the subsystem that satisfies the below inequalities:

$$\alpha_{1s}(|x_S|) \leq V_S \leq \alpha_{2s}(|x_S|) \quad (44)$$

$$\frac{\partial V_S}{\partial t} + \frac{\partial V_S}{\partial x} f(t, x_S, u_S) \leq -\alpha_{3s}(|x_S|) \quad (45)$$

$$\forall |x_S| \geq \rho_S(|u_S|) > 0$$
$$\forall t \geq 0, D = \{x_m; |x_m| < r_m\},$$
$$D_u = \{u_m; |u_m| < r_{mu}\},$$

where $\alpha_{1m}(|x_m|)$, $\alpha_{2m}(|x_m|)$, $\alpha_{3m}(|x_m|)$ and ρ_m are class κ functions, then the subsystem is locally input-to-state stable.

Proof. First, consider an ISS-Lyapunov function candidate as follows:

$$V_L = \dot{x}_L^T \tilde{M} \dot{x}_L + x_L^T K_p x_L + 2K_F \int_o^t F_L(\xi)\dot{x}_L(\xi)d\xi + V_e \quad (46)$$

Where V_e was introduced in Assumption 3 $\tilde{M}_m$, K_p, K_F are positive definite matrices. Similar to the master subsystem, we also have V_S is the positive function and satisfies the inequality (39). We also can easily check that $V_L = 0$ while $x_S(\mathrm{x}_L = 0, \dot{\mathrm{x}}_L = 0, \mathrm{x}_e = 0)$.

The derivative of V_L along the trajectories of the system (27) with concerning Property 2 as follows:

$$\dot{V}_L = 2\dot{x}_L^T(-K_p(x_L - \hat{x}_m) - K_D(\dot{x}_L - \hat{\dot{x}}_m) + K_i \int \hat{x}_m dt$$
$$- K_F(\beta^{-1}\hat{F}_m - F_L - \tilde{C}\dot{x}_L) - K_D \dot{x}_L) \quad (47)$$
$$+ 2x_L^T K_p \dot{x}_L - 2K_F F_L \dot{x}_L + x_L^T \dot{\tilde{M}} x_L + \dot{V}_e$$

Note the derivative of V_e in (11) and the expressions of F_L and s_e in Assumption 3 and applying Young's quadratic with $|A^T B| \leq (\varepsilon/2)|A|^2 + (1/2\varepsilon)|B|^2$ that holds for all $\varepsilon > 0$, we can have inequality(11)

$$\dot{V}_e(t) \leq -\alpha_{3e}|x_e|^2 + \frac{\lambda}{2}|x_e|^2 + \alpha|x_L|^2 + \frac{1}{\lambda}|x_L|^2$$
$$+ \frac{\Lambda_{env}}{\lambda}|x_L|^2 - b|x_L^T|\Lambda_{env} x_L + b|x_L^T|(b - a\Lambda_{env})\dot{x}_L \quad (48)$$

choose $a = b = 1/\lambda$, $\Lambda_{env} = I$, we have:

$$\dot{V}_s \leq -2\dot{x}_L^T(2K_D - \theta_L)\dot{x}_L - (a_{3e} - \frac{\lambda}{2})x_e^2 - 2\dot{x}_L^T(\theta_L \dot{x}_L$$
$$- K_D \hat{\dot{x}}_m - K_p \hat{x}_m - K_F \beta^{-1}\hat{F}_{op} - K_i \int_0^t \hat{x}_m dt) \quad (49)$$

$$\forall |\dot{x}_L| \geq (K_p |\hat{x}_m| + K_D |\hat{\dot{x}}_m| + K_F \beta^{-1} |\hat{F}_{op}| + K_i \int_0^t |\hat{x}_m| dt)\theta_L^{-1}$$

where θ_L is some positive constant. We can choose the value λ, θ_m to satisfy the derivative of V_S to be negative as follows:

$$\begin{cases} \theta_L \leq K_D \\ \lambda \leq 2a_{3e} \end{cases}$$

Similar to the master subsystem case, we can conclude that the slave and environment systems are also locally input - to-state stable.

So, we can conclude this Tele-robot system is input-to-state stable.

V. THE SIMULATION RESULT

In the simulation, the SMMS system is constructed by a master with two DOFs parallel link type arm and two slaves with two DOFs series link type arms. The model and parameters of a mini master robot and slave robot is taken in [11], [12]. The simulation results obtained by using MATLAB Simulink software.

Time delay of communication system assumed to be as follows:

$$T_m = T_s = 0.05\sin\left(t + 0.05\right) \qquad (50)$$

Two kinds of simulation conditions are given as:

- Control the grasping object without any contact with the environment. (free space)
- Control the grasping object in contact with the environment. (contact task)

From the results, in the first case, Fig 4 shows the result of disturbance estimation, that confirms the accuracy of the estimation algorithm. Fig 5 shows the position of the master mini robot and the Locked-system. The master robot from location $(x, y) = (0.2, 0.2)(m)$ at $t = 0(s)$ goes to $(0.17, 0.25)(m)$ (location of object) at $t = 5(s)$, and reaches the location $(0.25, 0.17)(m)$ at $t = 15(s)$. We can see the positions of both side are achieved. Fig 6 shows the time responses of the end-effector position of slave of the shape-system, in this figure, we can conclude that the relative position between two slaves following the target trajectory with grasping object is achieved. It also means the object is grasped from $t = 8(s)$ to $t = 18(s)$ and the object is moved from location $(0.17, 0.25)(m)$ to $(0.25, 0.17)(m)$.

In the free space case, the Locked-system does not contact with the environment, so the force data from Locked-system is zero (Fig 7). In the contact task case, the Locked-system contacts with the environment a $y = 0.23(m)$. In this case, the Locked-system cannot go to the position where master robot requires (Fig 8 - Y position) and the human can feel the force from Locked-system contacts with the environment (Fig 9 - Y force). We can see that the reflecting force from the environment and the scaling force of the human are same values.

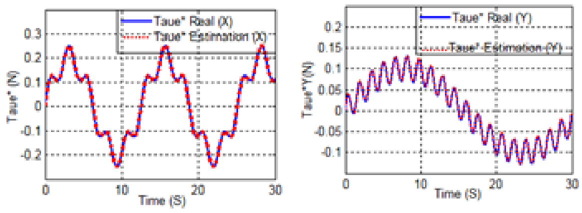

Fig 4: Disturbance estimation affects the Lock-system

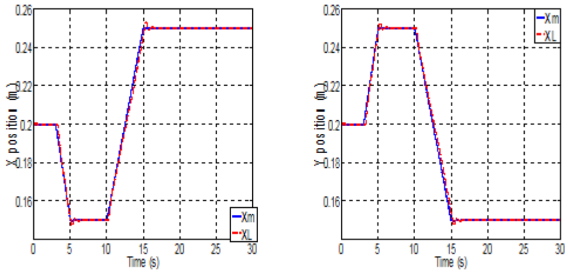

Fig 5: Position of Master and Locked-system in free space

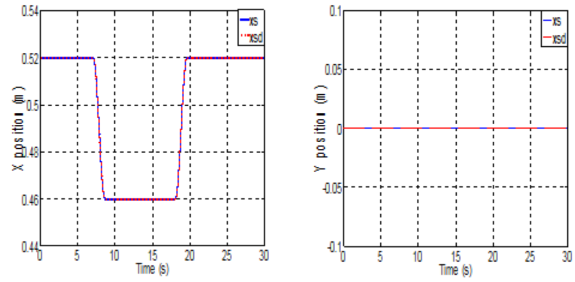

Fig 6: Position of Shape-system in free space

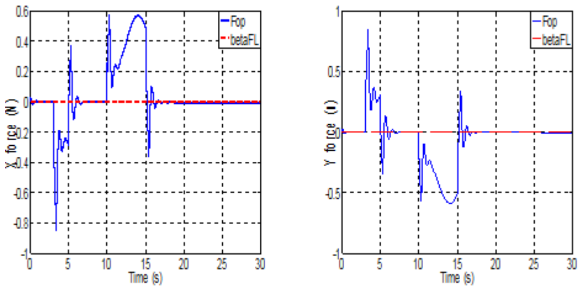

Fig 7: Force data in free space

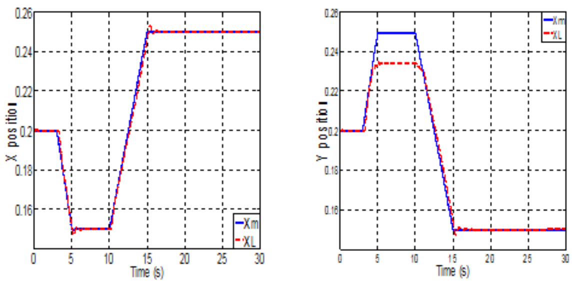

Fig 8: Position of Master and Locked-system in contact task

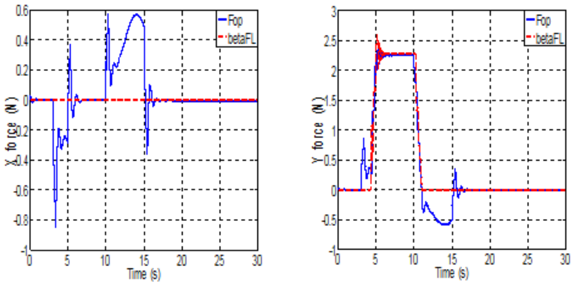

Fig 9: Force data in contact task

VI. CONCLUSION

In this paper, we proposed a new control law for a SMMS Tele-robot system based on ISS small gain theorem. This proposal resolves the dynamics of multiple slaves robot such as the Shape-system dynamics and the Locked-system dynamic of the control law. The simulation results have shown the effectiveness of the given method. In the next study, a number of slave robot need to be developed more than two and setup for experiment.

ACKNOWLEDGMENT

This work was supported and funded by Thai Nguyen University of Technology for Science and Technology Research grant funded No. T2018-B10

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