A Control Method for Nonlinear Systems Using Combined Sliding Mode Control and RBF Neural Network

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Abstract — *In general, the industrial systems are uncertain nonlinear systems with the influence of external disturbance factors. This paper has developed a control method for nonlinear systems combining Sliding Mode Control with Radial Basic Function (RBF) Neural Network to ensure robustness and interference resistance. This paper has developed a control method for nonlinear systems with external measureless disturbance by combining Sliding Mode Control with RBF Neural Network to ensure robustness and interference resistance. The obtained Sliding Mode Control algorithms and weight update rules of the Network have ensured the existence and stability of the Sliding Mode system. The efficiency and feasibility of the proposed method have verified by simulation results in Matlab-Simulink.*

Keywords — *RBF Neural Network, Sliding Mode Control, Lyapunov stability, nonlinear systems, adaptive control.*

I. INTRODUCTION

In practical industries, most of the control objects contain nonlinearities. The nonlinearity exists because of the physical nature of processes or objects, that includes many uncertain factors. That has caused significant difficulties for designing the high-quality control system for the class of uncertain nonlinear objects [1], [2], [3], [4]. Thanks to the possibility of approximation, the neural networks are applicable to design the nonlinear control systems by approximating the uncertain nonlinear functions [2], [3], [6]. For more than a recent decade, the problem of synthesizing nonlinear control systems based on online approximation has attracted the interest of many experts in control engineering [5], [6]. In addition to the difficulties caused by the nonlinearity and uncertainty of control objects, there is a significant difficulty due to the external disturbances. In order to solve the above difficulties simultaneously, the robust adaptive control using sliding mode control has been studied and proposed [6], [7].

In [8], an algorithm for identifying the uncertain nonlinear components using RBF network for the systems containing linear kinematics was developed. A number of algorithms for identifying the uncertain nonlinearity and disturbance affecting to the system based on RBF network with the advantage of convergent speed have been obtained in [9]. The problem of identifying state-dependent disturbance, it means identifying the uncertain nonlinear part in the system containing time-delay and linear stability has been proposed and successfully solved in [10]. However, the [8], [9] and [10] apply to the objects with stable linear kinematics and uncertain nonlinear part. In case of the complicated kinematics including unstable linear kinematics and uncertain nonlinear part has been studied and solved in [14] based on using Neural RBF network and sliding mode and feedback circuits. The Neural RBF and Sliding mode are used in adaptive system to control the class of objects having kinematics with the linear part containing uncertainties and uncertain disturbances [11]. However, it requires a linear mode with the determined dynamic parameters. In recent years, the Neural RBF network with sliding mode has been applied in manipulator control and robotics [12], [13]. In this case, the dynamics of the robot object is described by the Lagrange equation, which is a nonlinear equation with the uncertain factors as matrix parameters while the forms of equations meaning the kinetic structure, is completely defined.

From the above analysis, we can see that the Neural RBF and sliding mode controller have been considered to apply for designing control system for uncertain nonlinear objects. However, depending on the level and character of the uncertainty, there will be different solution and control techniques as well as different algorithms. For the high level of uncertain nonlinear systems (both in structure and parameters) and affected by external uncertain disturbance, a satisfactory solution has not been proposed until now.

This paper presents a design of control system for a class of nonlinear objects containing uncertainties in both structure and parameters under effects of external uncertain disturbances. In this article, the Neural RBF network combined with sliding mode control has been used to simultaneously solve the difficulties caused by the uncertain properties.

II. STATEMENT FORMULATION

Assuming that the dynamics of the control object is described by the following equation:

$$y^{(n)} = f(y,\dot{y},\cdots,y^{(n-1)}) + gu + d, \qquad (1)$$

where u – control signal $u \in \mathbb{R}$, y – the outputs $y \in \mathbb{R}$, d – the external disturbances $d \in \mathbb{R}$, $|d| \le d_0$, $f(y,\dot{y},\cdots,y^{(n-1)})$ - the smooth nonlinear function with uncertain function (structure) and parameters, $g = const > 0$. The raising problem is that the synthesis of the control system for (1) which ensures the output y(t) follows the input signal y_d(t) in the condition of the nonlinear function *f(.)* is completely uncertain and the disturbance *d* is not measurable.

With the signal y_d, the equation (1) is modified to obtain the kinetic equation of the object (1) in form of the error state space equation.

III. CONTROL SYSTEM DESIGN

In case the control objects have high level of uncertainty and subjected to external disturbance, the Neural RBF network and sliding mode control are applicable because they allow us to approximate or identify the smooth nonlinear function with arbitrary precision [2] and the sliding mode control enables the ensuring of invariant system under the effects of disturbances [15], [16].

Let $x_1 = y - y_d$, $\dot{x}_i = x_{i+1}, i = 1,2,\cdots,n-1,$

$$\dot{x}_n = y^{(n)} - y_d^{(n)}.$$

Equation (1) can be written as:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = x_3 \\ &\cdots\cdots\cdots \\ &\dot{x}_n = f(\mathbf{Y}) + gu + d - y_d^{(n)} \end{aligned} \qquad (2)$$

where $\mathbf{Y} = [y \quad \dot{y} \quad y^{(n-1)}]$.

Let $d_{\sum} = d - y_d^{(n)}$. Generally, $y_d^{(n)}$ is the variant parameter in certain limitation $|y_d^{(n)}| \le \rho$. The equation (2) will be:

$$\begin{aligned} &\dot{x}_i = x_{i+1}, i = 1,2,\cdots,n-1; \\ &\dot{x}_n = f(\mathbf{Y}) + gu + d_{\sum} \end{aligned} \qquad (3)$$

To estimate the uncertain nonlinear $f(\mathbf{Y})$ we employ the Neural RBF network:

$$f(\mathrm{Y}) = \hat{f}(\mathbf{Y}) + \varepsilon = \sum_{i=1}^{m} \hat{w}_i \phi_i(\mathbf{Y}) + \varepsilon \qquad (4)$$

where $\hat{w}_i$ is the weight parameter that calibrated during the learning phrase, ε is the approximated discrepancy and $\phi_i(\mathbf{Y})$ are fundamental functions chosen in forms [5].

$$\phi_i(\mathbf{Y}) = \exp\left[-\frac{\|\mathbf{Y}\text{-}\mathrm{C}_i\|^2}{2\sigma_i^2}\right] \Bigg/ \sum_{j=1}^{m} \exp\left[-\frac{\|\mathbf{Y}\text{-}\mathrm{C}_j\|^2}{2\sigma_j^2}\right],$$

where C_i is *n*-dimension vector, representing the center of the i^{th} fundamental function; represents the extension of fundamental function; *m* - the number of fundamental functions $\phi_i(\mathbf{Y})$. Supposing that it is necessary to approximate with the defined accuracy ε^*, therefore, the uncertain nonlinear function is presented through the fundamental function with the weight parameters w_i^*:

$$f(\mathbf{Y}) = \sum_{i=1}^{m} w_i^* \phi_i(\mathbf{Y}) + \varepsilon^*. \qquad (5)$$

Obviously, when we adjust the w_i to satisfy, the Neural network is approximated with the precision ε^*.

The next problem is synthesizing the controller which uses the Neural RBF network to approximate the uncertain nonlinear function $f(\mathbf{Y})$ with the corresponding update weights, ensuring the stability of system and having positive anti-disturbance character. The appropriate solution to this problem is using Sliding mode control because the SMC has outstanding advantages. Specifically, when operating in sliding mode, the system is invariant either with dynamic changing or disturbances [15].

The sliding manifold has been chosen as:

$$\mathbf{S} = \sum_{i=1}^{n} \lambda_i x_i, \;\; \lambda_n = 1 \qquad (6)$$

where $\lambda_i, i = 1,2,\cdots,n-1$ are chosen to satisfy the function (7) is Hurwitz:

$$p^{n-1} + \sum_{i=1}^{n-1} \lambda_i p^{i-1} \qquad (7)$$

where p is Laplace element. That means when the system is operating on the sliding manifold, the system will be stable $x_i \to 0, i = 1,2,\cdots,n$ [15], [16]. The remaining problem is determining the sufficient conditions which ensure that from any point in state space, the system will approach the sliding manifold and be held on the surface. That is the condition for the existence of sliding mode. The condition of existence of sliding mode and the condition for ensuring the roots of characteristic equation (7) stay in the left side of the complex plane result in stability of system with the sliding mode controller [15], [16]. Choose the Lyapunov function as :

$$\mathbf{V} = \frac{1}{2}\mathbf{S}^2 + \frac{\gamma}{2}\sum_{i=1}^{m} \tilde{w}_i^2, \qquad (8)$$

where: $\tilde{w} = w^* - \hat{w}$ (9)

The time derivative of the Lyapunov function (8) has form:

$$\dot{\mathbf{V}} = \mathbf{S}.\dot{\mathbf{S}} + \gamma \sum_{i=1}^{m} \tilde{w}_i \dot{\tilde{w}}_i \qquad (10)$$

Derivative of $\dot{\mathbf{S}}$ in (10) is defined based on (6) and (3):

$$\dot{\mathbf{S}} = \sum_{i=1}^{n} \lambda_i \dot{x}_i = \sum_{i=1}^{n-1} \lambda_i x_{i+1} + f(\mathbf{Y}) + gu + d_{\sum} \qquad (11)$$

The control signal u ensures the sliding mode in manifold S=0 has form [15], [16] :

$$u = u_{eq} + u_r, \quad (12)$$

the equivalent control term and u_r is the nonlinear control term in form of Relay.

$$u_{eq} = -\frac{1}{g}\left[\sum_{i=1}^{n-1} C_i x_{i+1} + f(\mathbf{Y})\right] \quad (13)$$

The term u_r is:

$$u_r = -\mathbf{M} sign \mathbf{S}, \quad (14)$$

To obtain u_{eq} in (13), we must use the estimation of the uncertain nonlinear $\hat{f}(\mathbf{Y})$, thus:

$$u_{eq} = -\frac{1}{g}\left[\sum_{i=1}^{n-1} \lambda_i x_{i+1} + \hat{f}(\mathbf{Y})\right] \quad (15)$$

From (11) - (15) we have:

$$\dot{\mathbf{S}} = f(\mathbf{Y}) - \hat{f}(\mathbf{Y}) + u_r + d_{\sum} \quad (16)$$

Putting $f(\mathbf{Y})$ and $\hat{f}(\mathbf{Y})$ from (4), (5) to (6) we obtain:

$$\dot{\mathbf{S}} = \sum_{i=1}^{m} \tilde{w}_i \phi_i(\mathbf{Y}) + \varepsilon^* + d_{\sum} - \mathbf{M} sign \mathbf{S} \quad (17)$$

where: $\tilde{w}_i = w_i^* - \hat{w}_i$

From (10) and (17) we have:

$$\dot{\mathbf{V}} = \mathbf{S}\left[\sum_{i=1}^{m} \tilde{w}_i \phi_i(\mathbf{Y}) + \varepsilon^* + d_{\sum} - \mathbf{M} sign \mathbf{S}\right] + \gamma \sum_{i=1}^{m} \tilde{w}_i \dot{\tilde{w}}_i \quad (18)$$

The expression (18) shows that to take the derivative form of $\dot{\mathbf{V}} < 0$, it is needed to satisfy the following conditions:

$$\mathbf{S}\sum_{i=1}^{m} \tilde{w}_i \phi_i(\mathbf{Y}) + \gamma \sum_{i=1}^{m} \tilde{w}_i \dot{\tilde{w}}_i = 0 \quad (19)$$

$$\mathbf{S}\left[\varepsilon^* + d_{\sum} - \mathbf{M} sign \mathbf{S}\right] < 0 \quad (20)$$

So that:

$$\dot{\tilde{w}}_i = -\frac{1}{\gamma}\mathbf{S}\phi_i(\mathbf{Y}); \quad (21)$$

$$\varepsilon^* + \mathbf{D}_{\sum} < \mathbf{M} \quad (22)$$

where: $\mathbf{D}_{\sum} = d_0 + \rho$

With the conditions (21), (22), the time derivative of the Lyapunov function is always negative. Therefore, (21) and (22) are sufficient conditions to make sure the existence of sliding mode in sliding manifold S=0. When the conditions (21) and (22) are satisfied, the system is always attracted to the sliding manifold.

From (6) and (21) we have :

$$\dot{\tilde{w}}_i = \dot{w}_i^* - \dot{\hat{w}}_i = -\frac{\phi_i(\mathbf{Y})}{\gamma}\sum_{j=1}^{n} \lambda_j x_j; \quad i = 1, 2, \cdots, m \quad (23)$$

Because $w_i^* = const$, $\dot{w}_i^* = 0$, so that (23) will be formed as:

$$\dot{\hat{w}}_i = \frac{\phi_i(\mathbf{Y})}{\gamma}\sum_{j=1}^{n} \lambda_j x_j \quad i = 1, 2, \cdots, m \quad (24)$$

The updated law Neural RBF network to approximate the uncertain nonlinear function $f(\mathbf{Y})$ through expression (24) ensures the convergent evaluation process and robust stable system.

Fig. 1 is the structure of Sliding mode combined with Neural RBF network control system. The adaptive adjustment block AB adjusts the weights $\hat{w}_i$ of the Neural RBF according to the updating law (24).

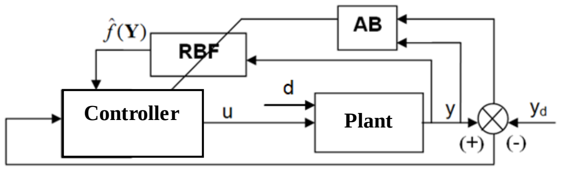

Fig 1: Structure diagram of Sliding mode combined with Neural RBF network control system

IV. THE ILLUSTRATIVE EXAMPLE

The object to be considered is the inverse pendulum with the dynamics is described as following equation [2] :

$$\ddot{\theta} = f(.) + g(.)u + d$$
$$= \frac{\mathbf{g}\sin\theta - ml\dot{\theta}^2\cos\theta\sin\theta/(m_c+m)}{l(4/3 - m\cos^2\theta/(m_c+m))} + \frac{\cos\theta/(m_c+m)}{l(4/3 - m\cos^2\theta/(m_c+m))}u + d$$

where:

θ is the rotation angle of the inverted pendulum

$\dot{\theta}$ is the angular velocity of the pendulum

$\ddot{\theta}$ is the angular acceleration of the pendulum

$\mathbf{g} = 9.8m/s^2$ is the gravitational acceleration

$m_c = 1kg$ is the mass of the pendulum

$m = 0.1kg$ is the mass of bar

$l = 0.5m$ is the half-length of the bar

u is the control signal of the rotation motor

d is the disturbances affects the pendulum system.

The structure of the Neural RBF network is the a single hidden layer network; with the number of input is two, the Neural number is seven and there is one output as shown in Fig. 2.

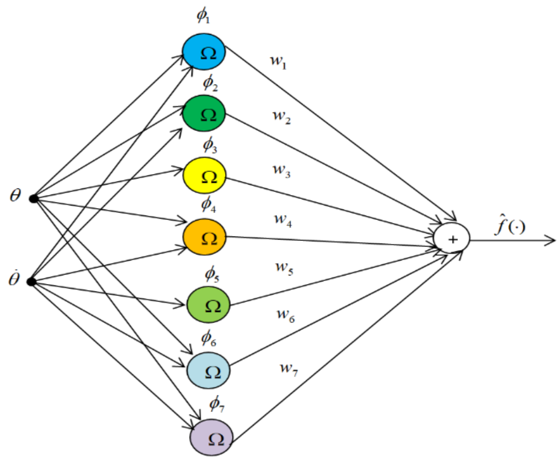

Fig 2: The structure of the Neural RBF network using to estimate the uncertain term *f(.)*

The parameters using in simulation:

$\lambda = 12.5$; $M = 0.1$; $\gamma = 0.035$;

$c_i = [-1.25\ \ -1\ \ 0.5\ \ 0\ \ 0.5\ \ 1\ \ 1.25]$; $\sigma_j = 0.75$

A. *Simulation Results*

Fig. 3 is a schematic diagram of the pendulum control structure applying sliding mode controller based on the uncertain disturbance identification *f(.)* employing Matlab and Simulink software. Based on the evaluation algorithm using Neural RBF network, it is possible to roughly approximate the uncertainty of the object, thus generating the control signal as (12), (14), (15), ensuring the sliding mode in the system, enabling the system both identifying uncertain term and controlling.

To clearly see the effectiveness of the proposed control algorithm, the numerical simulation has been investigated with a variety of inputs and with different uncertain disturbances. The obtained results shown from Fig. 4 to Fig. 7 illustrate that the output can track the reference well under various of uncertain disturbances.

By carrying out different simulations with various motion paths as well as disturbances to the inverted pendulum, we obtained the evaluation results of uncertain term *f(.)* as shown in Fig. 4, Fig. 6 and the motion path of the inverted pendulum as can be seen in Fig. 5 and Fig. 7.

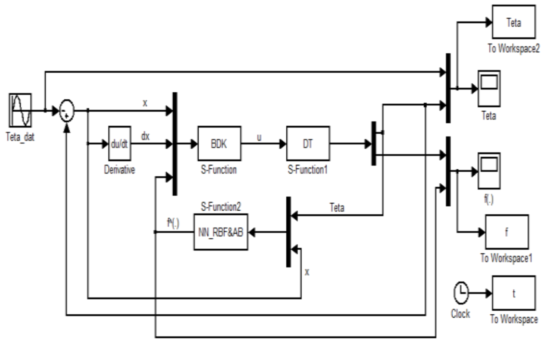

Fig 3: The simulation schematic diagram of the inverted pendulum system using Sliding mode controller based on uncertain disturbance identification *f(.)*

Case 1:

$$\theta = 0.5sin\left(0.3t\right);\ d = 0.02sin\left(1 - 0.25t\right)$$

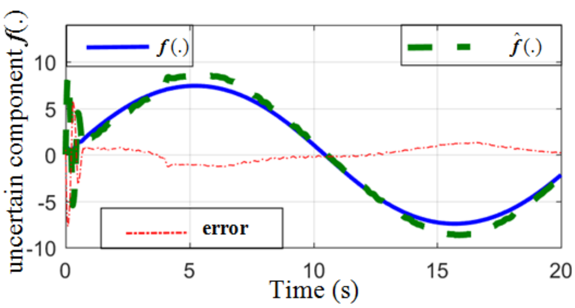

Fig 4: The estimation of uncertain term *f(.)*

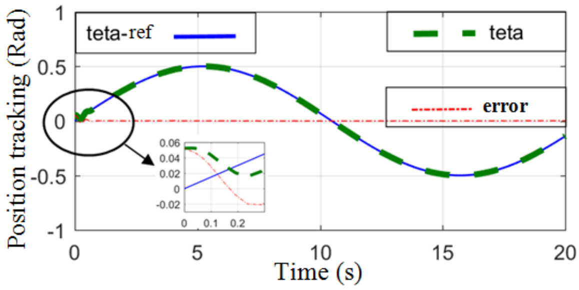

Fig 5: The motion path of the inverted pendulum

Case 2:

$$\theta = 0.5sin\left(0.3t\right) + 0.3sin\left(0.8t\right);\ \ d = 0.05cos\left(1 - 0.5t\right)$$

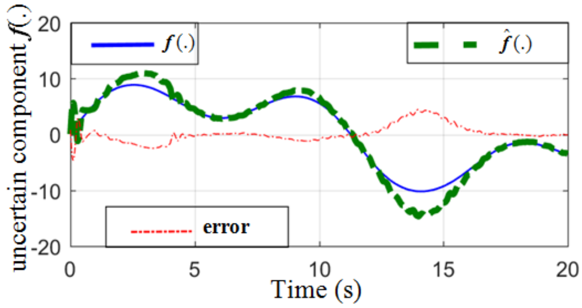

Fig 6: The estimation of uncertain term *f(.)*

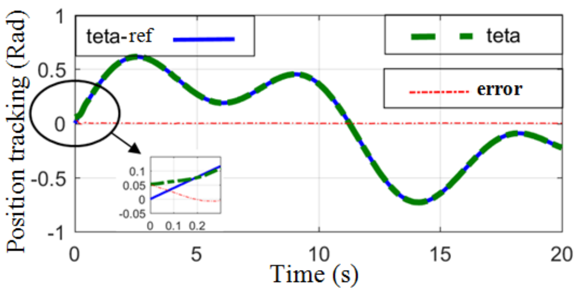

Fig 7: The motion path of the inverted pendulum

B. *Remarks*

Fig. 4 and Fig. 6 illustrate that the uncertain term *f(.)* can be evaluated based on Neural RBF network, the weights updating law using adaptive structure AB maintains the robust stability of system. In addition, Fig. 5 and Fig. 7 clearly show that the output motion of the inverted pendulum tightly follows the reference. By changing the input references and disturbances, it can be seen that our proposed algorithm shows its satisfactory. Based on that, the effectiveness and feasibility of our proposed method for anti-disturbance and compensation have been verified.

V. CONCLUSION

This paper proposed a method of synthesizing control system that ensuring the robustness and anti-disturbance for a class of uncertain nonlinear objects based on Neural RBF network and Sliding mode. The sliding mode control algorithm and the weights updating law has been obtained to make sure the existence of sliding mode and system stability. The effectiveness of proposed method has been verified by an illustrative example and numerical investigation employing Matlab-Simulink.

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