Evaluation of Various Reliability Measures of Three Unit Standby System Consisting of One Standby Unit and One Generator

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Abstract

The paper presents reliability measures of compressor standby system which consist of three compressor units and one generator. Initially two compressor units are in functioning state while third compressor unit and generator are in standby state. For system operation working of at least two compressor units is must. System can interrupt due to failure of compressor unit and halt in electricity. These constraints can be overcome by using standby compressor unit and generator. For practical utility many years real failure and repair data has been collected from Verka milk plant to evaluate various measures of reliability effectiveness such as MTSF, availability, busy period and profit. All these measures have been computed numerically as well as graphically by using semi Markov process and regenerative point technique.

Keywords — Regenerative technique, semi-Markov process, refrigeration system

INTRODUCTION

In field of reliability, standby units are commonly used for uninterrupted functioning of system to increase its efficiency. Many researchers [1-7] have contributed a lot through their research by keeping in their mind the role of standby unit in system functioning. There are many practical situations available in real life where system functioning can be interrupted due to failure in the operating unit or by other factors needed to keep system in functioning state such as continuous electricity supply. Going on these lines in this paper we have considered unit failure and electricity cut as major issues of the concern in proper functioning of the system.

The present study is the sincere effort of evaluating reliability measures of compressor standby system which consist of three compressor units and one generator. Initially two compressor units are in functioning state while third compressor unit and generator are in standby state. For system operation working of at least two compressor units is must.

It has also been assumed that any recent failure in the compressor unit will be given priority over the previously failed compressor unit. System functioning can be interrupted due to failure of compressor unit and halt in electricity. These interruptions can be overcome with the help of standby compressor unit and generator respectively. For practical utility many years real failure and repair data has been collected from Verka milk plant to evaluate various measures of reliability effectiveness such as MTSF, availability, busy period and profit. All these measures have been computed numerically as well as graphically by using semi Markov process and regenerative point technique.

Notations

\(\lambda_1\)  Constant failure rate of compressor unit 1
\(\lambda_2\)  Constant failure rate of compressor unit 2
\(\lambda_3\)  Constant failure rate of compressor unit 3
\(\lambda\)  Constant failure rate of electricity
\(S_i\)  State number, \(i=0,1,2,3,4,5,6,7,8,9,10\)
\(\beta\)  Operating rate of generator
\(G_{i}(t), g_{i}(t)\)  c.d.f and p.d.f of the repair time of compressor unit 1
\(G_2(t), g_2(t)\)  c.d.f and p.d.f of the repair time of compressor unit 2
\(G_3(t), g_3(t)\)  c.d.f and p.d.f of the repair time of compressor unit 3
\(O_1, O_2, O_3\)  Compressor units 1, 2 and 3 are in operating state
\(S_{III}\)  Compressor unit 3 is in standby state
\(F_{RI}, F_{RH}, F_{RHI}\)  Compressor units 1, 2 and 3 are under repair respectively
\(F_{RI}, F_{RH}\)  Compressor unit 1 and 2 are under repair from previous state respectively
\(F_{WR}, F_{WHR}\)  Compressor unit 1 and 2 are waiting for repair respectively
\(G_w\)  Generator is in operating state
\(G\)  Generator is in standby state

Model Description and Assumptions

1) All the random variables are independent.
2) Failure times are assumed to have exponential distribution whereas repair times have general distribution.
3) The system has single repairman facility.
4) The repairman comes immediately as soon as unit fails.
5) After each repair, the system works as good as new one.

**Transition Probabilities and Mean Sojourn Times**

All the 10 states are regenerative states in the Fig 1 are regenerative states. States $S_1, S_3, S_5$ and $S_9$ are down states, state $S_2$ and $S_7$ are halt states. The non zero elements $p_{kj}$ can be represent as below

\[ p_{00} = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \quad p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \quad p_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2}, \quad p_{10} = \frac{\lambda_2}{\lambda_1 + \lambda_3} \]

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of given system, we regard the failed states of the system as absorbing states and the mean time to system failure (MTSF) when the system starts from state $S_0$ is

\[ MTSF = T_0 = \lim_{s \to \infty} \frac{1 - \phi_0''(s)}{s} \]

Using L’Hospital rule and putting the value of $\phi_0''(s)$ we have

\[ T_0 = \frac{N}{D} \]

where

\[ N = \mu_0 + \mu_1, P_{01} \]

\[ D = 1 - P_{00}P_{01} - P_{02}P_{02} \]

**Availability Analysis**

Using the arguments of the theory of regenerative processes in steady state availability of the system is

\[ A_0 = \lim_{s \to \infty} sA_0'(s) = \frac{N_1}{D_1} \]

where

\[ N_1 = -p_{00}(-1 + p_{01} + p_{02} + p_{04} + p_{13} + p_{14} + p_{15}) \]

\[ D_1 = -p_{01} - p_{02} + p_{03} + p_{04} + p_{13} + p_{14} + p_{15} \]

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

\[ m_{ij} = \int_0^\infty dQ_i(t) = -q_i' (0) \]

\[ m_{00} + m_{02} = \mu_0, m_{01} + m_{12} + m_{14} + m_{15} = \mu_1, \]

\[ m_{03} + m_{04} = \mu_0, m_{01} + m_{12} + m_{14} + m_{15} = \mu_1, m_{02} + m_{13} + m_{14} + m_{15} = \mu_1 \]

\[ m_{20} = k_2, m_{22} = k_2, m_{24} = k_2, m_{23} = k_2, m_{41} = k_2, m_{40} = k_2, m_{42} = k_2 \]

Proceeding in the similar fashion as above following measures in steady state have also been obtained.
Busy period Analysis of Repairman

\[ B_t = N_t / D_t \]

Expected Number of Visits by Repairman

\[ V_t = N_t / D_t \]

where

\[ N_t = \mu_t \left( 1 - p_0 \right) \left( 1 - p_2 \right) \left( 1 - p_3 \right) \left( 1 - p_4 \right) \left( 1 - p_5 \right) \]

\[ - \mu_t \left( 1 - p_0 \right) \left( 1 - p_2 \right) \left( 1 - p_3 \right) \left( 1 - p_4 \right) \left( 1 - p_5 \right) \]

\[ \left( 1 - p_0 \right) \left( 1 - p_2 \right) \left( 1 - p_3 \right) \left( 1 - p_4 \right) \left( 1 - p_5 \right) \]

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Cost Benefit Analysis

The expected total profit incurred to the system in steady state is given by

\[ P = C_0 A - C_1 B_t - C_2 V_t \]

Where

\[ C_0 = \text{Revenue per unit up time of the system} \]

\[ C_1 = \text{Cost per unit time for which the repairman is busy} \]

\[ C_2 = \text{Cost per visit of the repairman} \]

Particular Case

For graphical representation, let us suppose that

\[ g_1(t) = \alpha_1 e^{-\lambda_1 t}, g_2(t) = \alpha_2 e^{-\lambda_2 t}, g_3(t) = \alpha_3 e^{-\lambda_3 t} \]

Using the particular case, the following values are estimated as

\[ \lambda_1 = 0.000042935, \lambda_2 = 0.0000477627 \]

\[ \lambda_3 = 0.001020147, \alpha_1 = 0.01434491 \]

\[ \alpha_2 = 0.045222199, \alpha_3 = 0.003294625 \]

\[ \beta = 0.010597367 \]

\[ C_0 = 22090, C_1 = 20007.71428, C_2 = 800 \]

Graph Between MTSF and \( \lambda_3 \) (variation in \( \lambda_3 \)) fig 2

Fig. 2 represents the behaviour of the Mean Time to System Failure (MTSF) with respect to the failure rate \( \lambda_3 \). It can be concluded from the graph that the MTSF decreases as the failure rate \( \lambda_3 \) and variation in \( \lambda_3 \) increases.

Graph Between MTSF and \( \alpha_1 \) (variation in \( \lambda_3 \)) fig 3

Graph in fig 3 represents the behaviour of MTSF and repair rate \( \alpha_1 \) with variation in \( \lambda_3 \). It is clear that as repair rate \( \alpha_1 \) increases MTSF decreases. As variation is taken in failure rate \( \lambda_3 \) for MTSF, it can be concluded as the failure rate \( \lambda_3 \) increases MTSF decreases.

Graph Between MTSF and \( \alpha_1 \) (variation in \( \lambda \)) fig 4

Graph in fig 4 reveals the behaviour of MTSF and repair rate \( \alpha_1 \) for different values of \( \lambda \). It can be interpreted MTSF decreases as repair rate \( \alpha_1 \) increases. It is clear from fig4 MTSF decreases as failure rate \( \lambda \) increases.

Graph Between Profit and Revenue \( C_0 \) (variation in \( C_1 \)) fig 5

It can be interpreted from graph(fig 5) that profit increases with increase in values of revenue per unit up time (\( C_1 \)). It can also be noticed that if \( C_1 = 20007.142 \), then \( P > 0 \) or \( \alpha < 2259 \) accordingly as \( C_0 > \alpha \). So for \( C_1 = 20007.142 \), revenue per unit up time should be fixed greater than 2259. Similarly for \( C_1 = 25007.142 \) and 30007.142, the revenue per unit up time should be greater than 2824 and 3388 respectively.
CONCLUSION

Three compressors standby system in refrigeration system of verka milk plant have been studied where two units must be in operative state for functioning of system and working of standby unit depend upon the requirement of the system operation. In present paper measures of system effectiveness have been obtained by using semi Markov process and regenerative point techniques. These measures are:
1. Steady state probabilities
2. Mean sojourn time
3. Mean time to system failure
4. Availability
5. Busy period of repairman
6. Expected number of visits by repairman
7. Profit analysis
8. Graphs concerning to MTSF with respect to failure rate/repair rate have been plotted
9. Profit graph has been plotted for better understanding of system behaviour. Also by using cut off points one can easily decide about the values of the various parameters for which profit is positive.

In brief, conclusion has basically drawn on the original data from Verka milk plant. Our model can be utilized by anyone using similar system by putting the values of his/her interest in the general expressions calculated by us. Profit Analysis for system has been obtained to increase uptime and reduce the cost occurred in the system.

MTSF = 24798.625 hrs.
Availability = 0.798864
Busy period of repairman = 0.090188 hrs
Expected number of visits = 0.000063 hrs

REFERENCES
State Transition Diagram Fig 1

Where

- Operating State
- Halt State
- Regeneration point
- Failure state