A Method for Joint Compressing and Recovering Destructed Signals in Wireless Multimedia Sensor Networks

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Abstract

The WSNs are developed to sense, gather, process and transmit the real-world information and so in recent years, solving numerous challenges of wireless sensor networking are considered intensely. In addition, in Wireless Multimedia Sensor Networks intra- and inter-signal correlations can be exploited in the theory of distributed source coding and similarly in distributed compressive sensing to compress signals as much as possible. These cases may be occurred in applications and services in which work with smart spaces and context aware networks. In this paper based on compressive sensing a framework denoted asECMis proposed to compress and reconstruct the signals of the sensors even for networks which the data transmission is imperfect.ECM uses the concepts of distributed compressive sensing and the shared information between sensor's signals to compress the signals more.

Keywords – Wireless Sensor Network, Distributed Compressive Sensing, Signal Compression

I. INTRODUCTION

The Wireless sensor networks (WSN) provide a flexible way to monitor the physical parameters of the environment through the deployment of a large number of sensor nodes. There are three main challenges in WSN, i.e., network lifetime, computational ability and bandwidth constraints. With the availability of low-energy and low-cost multimedia devices, such as microphones and cameras, which may capture multimedia contents from the field, the next generation WSNs, Wireless Multimedia Sensor Networks (WMSN) have been proposed and drawn the immediate attention of the research community [1].

Suppose there are *J*sensors in the region, measuring a phenomenon in spatio-temporal manner. A Fusion Center (FC) gets all the measurements and runs an algorithm which jointly decodes the signals of the sensors and reconstructs the phenomenon at the sensor positions. In cases that there is a significant correlation between the sensors' signals, the joint decoding based on Distributed Source Coding (DSC) [2] could be used in the FC to decompress the transmitted sensors' signals. Using DSC methods can increase compression rate so that data can be transmitted by consuming less power and bandwidth.

On the other hand, Compressive Sensing (CS) is a sampling theory [3], which leverages the compressibility of the signal to reduce the number of samples required for reconstruction and significantly reduces costs of sampling and computation at a sensor with restricted capabilities. CS theory exhibits that a signal with a sparse transform representation can be reconstructed from a small set of irrelevant random projections. CS technique as the data acquisition approach in a WSN can significantly reduce the energy consumed in the process of sampling and transmission through the network, and also lower the wireless bandwidth required for communication.

CS can also be applied in distributed scenario similar to distributed source coding (DSC). The theory of Distributed Compressive Sensing (DCS) [4] uses inter -and intra- signal correlations, for proposing new distributed source coding and compression algorithms for multi-signal ensembles based on CS theory. In a typical DCS setting and Joint Sparsity Model (JSM) [4], each sensor compresses its signal individually by means of projecting it on an incoherent basis and transmits the compressed information (actually sensed) to the Fusion Center (FC). If the right conditions exist, FC can reconstruct jointly all the signals considering that the measured signal of each sensor is individually sparse in some basis.

Current paper proposes amethod for compressing the signals of a WMSN and reducing the amount of the transmitted data. In the regarded WMSN, the captured signal of each sensor should be transmitted to a central unit (or FC) through a wireless interface. In addition signals of the sensors are sparse in a dictionary or basis. The proposed method denoted as *Enhanced Common Model (ECM)*and can be basically used in WMSN's services in which there is an intrinsic shared part between the captured signals of the sensors. This means that *ECM* is an algorithm which is context aware and relies on the captured signals' properties. Another aim of the proposed method is that they be able to reconstruct the signals' of sensors robustly if some perturbations are occurred in the transmission procedure. To this end ECM exploits the following strategies: 1)ECM models the perturbation of the transmission system by using disturbance filters between each sensor node and the FC. The reconstruction formulas of the mentioned methods consist of the estimated version of these filters and provides some compensation against the errors occurred. 2) Similar to other DSC methods, ECM benefits the shared common part between the ensembles of the signals for compressing the signals further. The proposed method basically can be used to develop a framework for WSN's services in which there is an intrinsic sparse shared part between the sensors' captured signals and also this shared common component is almost fix for a while large enough with respect to the sensing time interval. Such cases can occur in multi-view imaging [5], large camera arrays imaging of a scene [6] and distributed compressive video sensing (DCVS) [7].

Rest of the paper includes following parts. Section II introduces the mathematical preliminaries of the used theorems such as Compressive Sensing and Joint Sparsity Model. The system model of the wireless sensor network is exhibited in section III. Section IV represents the mathematical criteria used for developing the *ECM* method such as estimating the disturbance filters, decompressing the signals jointly. The corresponding experimental results are reported and discussed in section [V]. Finally, the paper is concluded in section [VI].

II. MATHEMATICAL BACKGROUND

Interpret the linear system $x = D\alpha$ as a way of constructing a signal $x \in \mathbb{R}^N$ where $D \in \mathbb{R}^{N \times K}$ and $\alpha \in \mathbb{R}^K$. The matrix D is referred as a dictionary of atoms and each of the K columns of Dis a possible basis signal in R^N and can be called as atomic signals or atoms [8]. The signal x can be represented sparsely if x can be generated by the multiplication of D by a sparse vector α with $k_0 \ll K$ non-zeros which produces a linear combination of k_0 atoms with varying weights. In compressed sensing (CS) research, the interest is the inverse problem of recovering a signal x from the noisy linear measurements $y = \Phi x + n$ ($y, n \in \mathbb{R}^w$). The focus is on under-determined problems where the forward operator $\Phi \in \mathbb{R}^{w \times N}$ has unit norm rows and forms an incomplete basis with $w \ll N$. The resulting ill-posed inverse problem is regularized by assuming that 1) the unknown signal x is compressible with $k_0 \ll K$ significant coefficients in a dictionary D and 2) the noise process is bounded by ε . The theory of CS states that, for most full-rank matrices Φ that are incoherent to D, if α is sparse with respect to its dimension N, there is the unique solution of a regularized l_1 -minimization program: $\hat{\alpha} = \min_{\dot{\alpha}} \| \dot{\alpha} \|_1 \text{ subject to } \| y - \Phi D \dot{\alpha} \|_2 \le \varepsilon(1)$

(1) is a convex optimization problem and the literature of convex optimization has provided a long list of solvers for this task. Basis Pursuit DeNoising

(BPDN) [9] refers to the solution of (1) as using relaxation

$$\hat{\alpha} = \min_{\dot{\alpha}} \frac{1}{2} \|y - \Phi D \dot{\alpha}\|_{2}^{2} + \lambda \|\dot{\alpha}\|_{1}$$
(2)

and so the solution $\hat{\alpha}$ is a function of the parameter λ . This solution contains signal-plus-residual which the size of the residual is controlled by λ . If $\lambda \to 0$, the solution behaves exactly like BP applied to y and the the residual goes to zero. As $\lambda \to \infty$, the residual gets large and $\hat{\alpha} = 0$.

For the reconstruction in the theory of DCS, the required measurements could be decreased by using intra- and inter-signal correlations. In a typical DCS setting, the sensors measure signals that are each individually sparse in some basis. DCS assumes that each signal in a signal ensemble $x_i \in \mathbb{R}^N, j \in$ $\{1, 2, ..., J\}$ is generated as a combination of two parts: the common part x_c , and the innovation part x_{inn_i} is $x_i = x_c + x_{inn_i}$ Different sparsity That assumptions for the common and the innovation parts lead to three different models and authors in [4] proposed some solving methods to deal with JSMs. In JSM-1 both common and innovation parts are sparse, in JSM-2, both common and innovation parts are sparse and have same supports, in JSM-3, the common part is not sparse but the innovation part is sparse.

III. SYSTEM MODEL

Assume that J sensors are distributed in the region and each of them captures a signal $x_j \in \mathbb{R}^N$, $(j \in \{1, 2, ..., J\})$. A common component $x_c \in \mathbb{R}^N$ is shared by each signal such that $x_j = x_c + x_{inn_j}$ where $x_{inn_j} \in \mathbb{R}^N$ is the innovation part of each signals can be represented sparsely as a linear combination of the atoms (columns) of the dictionary $(x_j = D\alpha_j)$. Clearly, this dictionary is able to exhibit signals sparsely as $x_j = D(\alpha_c + \alpha_{inn_j})$ in which α_c and α_{inn_j} s belong to space \mathbb{R}^K with different sparsity levels.

The signal of each sensor is compressed (actually sensed) with $\mathbf{y}_j = \Phi_j \mathbf{x}_j$ by using an individual measurement matrix $\Phi_j \in \mathbb{R}^{w_j \times N}$. Each sensor's sensed signal $\mathbf{y}_j \in \mathbb{R}^{w_j}$ should be sent to the FC and consequently detected in FC via a conventional digital communication transceiver module. The received detected signals by the FC are denoted by $\mathbf{r}_j \in \mathbb{R}^{w_j}$.

Clearly, \mathbf{y}_j consists of two parts: The common part $\mathbf{y}_{c_j} \in \mathbb{R}^{w_j}$ and the innovation part $\mathbf{y}_{inn_j} \in \mathbb{R}^{w_j}$ which can be represented as $\mathbf{y}_c = \Phi_j \mathbf{x}_c$ and $\mathbf{y}_{inn_j} = \Phi_j \mathbf{x}_{inn_j}$. Therefore, it is possible to write that $\mathbf{r}_j = \mathbf{r}_c + \mathbf{r}_{inn_j}$ where \mathbf{r}_c and \mathbf{r}_{inn_j} are the

received detected signals corresponded to the common and innovation parts.

In our proposed method, the y_{c_j} will be assumed known in FC and will be used for estimating some disturbance filters which we'll introduce later. Therefore, not only the two common and innovation parts $y_{c_j} \in R^{w_j} \operatorname{and} y_{inn_j}$) are transmitted simultaneously to the FC but also these two components should be separable in the FC. For instance signals can be transmitted simultaneously by using multi-user and multiple access communication techniques such as CDMA with different codes.

IV. RECOVERING THE SIGNALS

The first and the simplest idea to reconstruct the PSD \mathbf{x}_j is to compute $\hat{\mathbf{x}}_j = \mathbf{D}\hat{\mathbf{a}}_j$ where $\hat{\mathbf{a}}_j$ is attained by solving problem (3).

$$\widehat{\boldsymbol{\alpha}}_{j} = \min_{\boldsymbol{\alpha}} \left\| \boldsymbol{\alpha}_{j} \right\|_{1} \text{ subject to } \left\| \boldsymbol{r}_{j} - \boldsymbol{\Phi}_{j} \boldsymbol{D} \boldsymbol{\alpha}_{j} \right\|_{2} \le \varepsilon(3)$$

However it is obvious that solving equation (3) to recover the samples can only be useful when there is no difference between r_j and y_j . Inaccurate compressive sensing based reconstruction can be caused by this destructive difference and this fact makes this recovery method impractical especially for scenarios which the symbol errors have occurred. In order to recover the uncompressed signals x_j more robustly, we propose here to use the shared common component x_c based on the JSM concepts. In mentioned models, since the disturbance filters β_j are embedded in the reconstruction criteria, recovering the signals is straight and also more robust against bit and symbol error rates.

A. Estimating the Disturbance Filters

For simplicity, we simulate the effect of disturbance filter by circular convolution $\mathbf{r}_j = \mathbf{y}_j \odot \boldsymbol{\beta}_j + \mathbf{n}_j$ where \odot shows the circular convolution operator. The disturbance filter $\boldsymbol{\beta}_j \in R^{w_j}$ and additive noise $\mathbf{n}_j \in R^{w_j}$ are used to model the deviations between the values of the original sent and the received signals. First of all, we should estimate the disturbance filter $\boldsymbol{\beta}_j$. Similarly, we can model the received signal of the common part \mathbf{r}_{c_j} as $\mathbf{r}_{c_j} = \mathbf{y}_{c_j} \odot \boldsymbol{\beta}_j + \mathbf{n}_j$ or in the matrix multiplication form as $\mathbf{r}_{c_i} = \mathbf{Y}_{c_i}^0 \boldsymbol{\beta}_j + \mathbf{n}_j (4)$

where $\mathbf{Y}_{c_j}^0 \in \mathbb{R}^{w_j \times w_j}$ is the circulant matrix of $\mathbf{y}_{c_j} \in \mathbb{R}^{w_j}$ [10]. So if we assume that \mathbf{y}_{c_j} signals are known by the FC or equivalently FC knows the $\boldsymbol{\alpha}_c$ and Φ_j s, the estimated impulse response of the destructive filter $\boldsymbol{\beta}_j$ can be achieved by solving the simple optimization problem:

$$\widehat{\boldsymbol{\beta}}_{j} = min_{\hat{\boldsymbol{\beta}}_{j}} \left\| \boldsymbol{r}_{c_{j}} - \boldsymbol{Y}_{c_{j}}^{0} \hat{\boldsymbol{\beta}}_{j} \right\|_{2}^{2} .(5)$$

Now, after estimating the destructive filters β_j s, which are related to the communication paths

between the *j*th sensor and the FC, we can construct the circulant matrix of the $\hat{\beta}_i$ as $B_i^0 \in R^{w_j \times w_j}$.

B. Individual Model

Remember that the received signal of each sensor to the FC were modeled in the equation (4). By using the estimated disturbance filters and the matrix form of the circular convolution, we simply define the *Individual Model (IndM)*in equation (6) to represent the received signal of each sensor.

$$\boldsymbol{r}_j = \boldsymbol{B}_j^{\boldsymbol{O}} \boldsymbol{y}_j + \boldsymbol{n}_j = \boldsymbol{B}_j^{\boldsymbol{O}} \boldsymbol{\Phi}_j \boldsymbol{D} \boldsymbol{\alpha}_j + \boldsymbol{n}_j (6)$$

By using the proposed *IndM*, therecovered signals can be computed by $\hat{x}_j = D\hat{a}_j$ where \hat{a}_j is obtained after solving the optimization problem in equation (7).

$$\widehat{\boldsymbol{\alpha}}_{j} = \min_{\boldsymbol{\alpha}} \left\| \boldsymbol{\alpha}_{j} \right\|_{1} \text{ sub. to } \left\| \boldsymbol{r}_{j} - \boldsymbol{B}_{j}^{\boldsymbol{0}} \boldsymbol{\Phi}_{j} \boldsymbol{D} \boldsymbol{\alpha}_{j} \right\|_{2} \le \varepsilon(7)$$

C. Common Model

But in order to reconstruct the uncompressed signals x_j more efficiently, it is logical to use the shared common component x_c between neighbor sensors based on the JSM and therefore, reconstruct the data in lower measuring rate. Inspired from JSM equations (8) to (12) are defined to model, reconstruct and decompress the signals for a J neighbor.

$$\mathbf{r} = \overline{\mathbf{B}} \boldsymbol{\Phi} \boldsymbol{\Psi} \boldsymbol{\alpha} + \mathbf{n}$$
(8)

$$\mathbf{r} = [\mathbf{r}_{1}^{T} \mathbf{r}_{2}^{T} \cdots \mathbf{r}_{j}^{T}]^{T}$$
(9)

$$\mathbf{\alpha} = [\mathbf{\alpha}_{c}^{T} \mathbf{\alpha}_{inn_{1}}^{T} \cdots \mathbf{\alpha}_{inn_{j}}^{T}]^{T}$$
(9)

$$\mathbf{\alpha} = \begin{bmatrix} \Phi_{1} & 0 & \cdots & 0 \\ 0 & \Phi_{2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \Phi_{j} \end{bmatrix}$$
(10)

$$\Psi = \begin{bmatrix} \mathbf{D} & \mathbf{D} & 0 & \cdots & 0 \\ \mathbf{D} & 0 & \mathbf{D} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \mathbf{D} & 0 & \cdots & 0 & \mathbf{D} \end{bmatrix}$$
(11)

$$\overline{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_{1}^{O} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{2}^{O} & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{j}^{O} \end{bmatrix}$$
(12)

where $\mathbf{r}, \mathbf{n} \in \mathbb{R}^W$, $\boldsymbol{\alpha} \in \mathbb{R}^{K(J+1)}$, $\Phi \in \mathbb{R}^{W \times NJ}$, $\Psi \in \mathbb{R}^{JN \times K(J+1)}$ and $W = \sum_{j=1}^{J} w_j$. Therefore, the desired signals can be yielded by $\hat{\boldsymbol{\alpha}}_j = \boldsymbol{D}(\hat{\boldsymbol{\alpha}}_c + \hat{\boldsymbol{\alpha}}_{inn_j})$ where $\hat{\boldsymbol{\alpha}}_c$ and $\hat{\boldsymbol{\alpha}}_{inn_j}$ s are located in the found vector $\hat{\boldsymbol{\alpha}}$ by solving the optimization problem (13). $\hat{\boldsymbol{\alpha}} = \min_{\hat{\boldsymbol{\alpha}}} ||\hat{\boldsymbol{\alpha}}||_1$ sub. to $||\mathbf{r} - \overline{\mathbf{B}} \boldsymbol{\Phi} \boldsymbol{\Psi} \hat{\boldsymbol{\alpha}}||_2 \le \varepsilon(13)$

D. Enhanced Common Model

Till now, was have proposed to model the signals by eq. (8). But remember that, we assume the common part α_c is known by the FC. This item can be used in the receiver (FC) to enhance the reconstruction performance and system's

efficiency.Consequently, the signal of each sensor \hat{x}_j will be reconstructed by (15) where $\hat{\alpha}_{inn_j}$ s are located in the computed $\hat{\alpha}_l$ vector from (14). $\hat{\alpha} = \min_{\hat{\alpha}} ||\hat{\alpha}||_1$ sub. to $||\mathbf{r} - \overline{\mathbf{B}} \boldsymbol{\Phi} \boldsymbol{\Psi} \hat{\alpha}||_2 \le \varepsilon (14)$

 $\widehat{\boldsymbol{x}}_{j} = \boldsymbol{D}(\boldsymbol{\alpha}_{c} + \widehat{\boldsymbol{\alpha}}_{inn_{j}})(15)$

V. SIMULATION RESULTS

Some experimental results of the proposed models are reported here in order to show their capabilities. Five signals $x_i \in$ $R^{400}, (j \in \{1, 2, ..., 5\})$ from J = 5 sensors were generated in these experiments such that there is a shared common component x_c between them and also each one of them is sparse in a random dictionary $\boldsymbol{D} \in R^{400 \times 512}$ with different sparsity levels (Maximum 50-sparse). Consequently, five different measurement matrices $\Phi_i \in \mathbb{R}^{w_j \times 400}$ with Gaussian random set of projections sense the The samples $y_i \in R^{w_j}, (j \in$ signals. sensed $\{1,2,\ldots,5\}$) are sent to the FC through a digital transceiver system. Binary Phase Shift Keying modulating (BPSK), 1/2 channel encoding, and DS-CDMA with 4-chip's length are the specifications of the used transceiver system. Simulations are experimented for 100 frames with different x_is and the achieved mean results are reported. The average ratio between the power of innovation part and common part was such that $\frac{|\alpha_c|_2}{|\alpha_j|_2}$ within these frames. The configuration of the used system is Intel Core2Duo, 2.53GHz, P8700, 4GB RAM in 64Bit-

Matlab R20011b platform. CVX [11] and SparseLab [12] matlab toolboxes are used to solve the mentioned problems.

First of all, the simulation results of experiments in a loss-less scenario are considered here. In the loss-less scenario the channels are assumed ideal and there is not any bit errors. So in using BPDN we can set $\lambda = 0$ or equivalently use BP. Fig. 1 shows the reconstruction performance of the methods for different sensing rates (w_j) and evaluated by normalized mean squared error (NMSE). The Sensing Rate percentage (and the Compression Rate) can be computed as $100 \times \frac{w_j}{N}$ (and $100 \times (1 - \frac{w_j}{N})$). NMSE which we used for our experiments is

$$NMSE = \frac{1}{M^2} \sum_{j=1}^{M^2} \sum_{n=1}^{N} \left(\frac{\hat{s}_j(n)}{\|\hat{s}_j\|_2} - \frac{s_j(n)}{\|s_j\|_2} \right)^2 (17)$$

where M² and N are the number of sensors and the number of PSDs' samples, respectively.

It can be inferred from Fig. 1 that 1) Since Common Model exploits the shared common component of the signals as like as DSC schemes, makes the reconstruction more efficient than (*Individual Model*)*IndM* and the signals can be



Fig. 1. Comparison between compression ability of the proposed models.

reconstructed in a lower sensing rate. 2) Using *Enhanced Common Model(ECM)*makes the reconstruction even more efficient than Common Model and the signals can be reconstructed in a lower sensing rate.

We can report that the elapsed time for *IndM* is significantly lower than common models and this phenomenon is caused by the higher solution complexity in *Common Models* and the larger length of the variables vector. Therefore, when the scenario of the case study is such that α_c can be used, the best method to reach our goals is *ECM*, otherwise *IndM* is more preferable and efficient (it needs much lower solving time).

Here a destructive system will be considered too. Because of this, the sensed samples are transmitted to the FC through Additive White Gaussian (AWGN) Channels which cause symbol errors. In Fig. 2 the Semi Normalized Mean Squared Error of the signal reconstruction for different Bit Error Rates is shown. In this figure the reconstruction accuracy without using destructive filters (eq. (3)) and with using destructive filters (*IndM* and *ECM*) can be compared. In this experiment, the sensing rate is set $w_j = 90$. Fig. 2 indicates that higher BERs brings more errors for all the methods (as expected) but both *IndM* and *ECM* methods can compensate this destruction significantly and reconstruct signals with lower errors.

Fig. 2 shows two interesting phenomena. One is that *IndM* intersects the perfect reconstruction line in almost 2 times worse bit error rates in comparison to using no destructive filters. This fact happens for *ECM* method even for 5 times worse bit error rates which is a great and notable result for proposed *IndM* and *ECM* methods. The other phenomenon is that since *ECM* method uses the shared component between the signals, by equal sensing rate *ECM* reconstructs signals with lower



Fig. 3. The accuracy of the signal reconstruction for different bit error rates.

errors even for bit error rates which all of the methods achieve their final and best results. This phenomenon was seen before in Fig. 1.

VI. CONCLUSION

In this paper amethod called Enhanced Common Method (ECM)is proposed to compress the signals of the sensors which should be transmitted to the FC and also reconstruct the signals more robust when the data transmissions to the FC is imperfect. We model the perturbation of the transmission system by using disturbance filters between each sensor node and the FC. We also suggest an scheme to estimate the destructive filters. The proposed methodis based on compressive sensing. In addition, ECM uses the shared common part between the ensembles of the signals to compress the signals further. ECM basically can be used in WMSN's services which there are an intrinsic shared part between the sensors' captured signals and the signals can be represented sparsely in some basis. This issue makes that ECM be a context aware method and will be useful in many multimedia applications.

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