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Transient Analysis of Rotating Beams with Varying Parameters Simulating the Foreign Object Damages

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Abstract - *In the present work, it is proposed to carry out a Transient analysis of rotating beams with varying parameters simulating the foreign object damages using ANSYS. Transient analysis is used to design structures subjected to shock loads, such as automobile doors and bumpers, building frames, and suspension systems. The vibration characteristics of rotating structures such as natural frequencies and mode shapes should be well-identified compared to the vibration characteristics of non-rotating structures. The variation of results from the stretching induced by the centrifugal inertia force due to the blades' rotational motion causes the structure's bending stiffness increment. This results in the variation of natural frequencies and mode shapes. The analysis has been carried out by idealizing the compressor blade as a cantilever beam for the parameters keeping the notch height constant. The notch radius is varied 2 to 6mm in steps of 2mm. It is analyzed for various notch heights of 20mm, 50mm, 80mm, 120mm, and 150mm for the said notch radii. To make sure that a given design can withstand Impact loads at different forcing frequencies.*

Keywords—*Transient Analysis, Natural Frequencies, Mode Shapes, Notch Radius, etc.*

I. INTRODUCTION

Foreign object damage (FOD) is a major source of fatigue crack nucleation in aircraft and jet engines. It can range from a scratch or dent to a deep gouge. Fan and early-stage compressor blades are prone to HCF failure initiating from FOD on or near the leading edges. FOD is usually distributed along the blades' concave side, ranging from the platform toward the tip, with a higher concentration of FOD near the higher velocity tip. Foreign object damage is an object or article alien to an aircraft that can cause damage. FOD stands for Foreign Object Damage. According to the National Aerospace Standard 412, maintained by the National Association of FOD Prevention, Inc, Foreign Object Debris is a substance, debris, or article alien to the vehicle or system that would potentially damage.

The term indicates damage from bird strikes and hard body impacts, such as stones, striking the turbine engine fan blades primarily when ingested with the airflow. Depending on the impact conditions, FOD can result in the immediate separation of a blade or can cause sufficient microstructural damage, stress raising notches, or even cracks, which induce the early initiation of fatigue cracks. Since the fan and compressor blades can experience in service transient airflow dynamics from resonant conditions of the engine, in the form of low-amplitude aerofoil excitations in the KHz regime (and, depending on the blade span location, very high mean-stress levels), such premature cracking can result in essentially unpredictable failures due to fatigue crack growth in very short periods. A majority of FOD involves damage sizes that are less than (0.080in) in depth. Fan and the compressor blades at the front end of jet engines are the components that receive the majority of the damage, particularly at the leading edge of the airfoil. Due to high-frequency vibratory stresses in the compressor sections associated with normal engine operation, it is not uncommon for cracks to initiate from FOD defects and grow catastrophically within minutes to hours running. HCF caused by steady-state or transient vibrations of the component is of the leading causes of in-service failures of blades that have been subjected to FOD. Therefore, it is important to know the fatigue strength of materials and airfoil geometries subjected to FOD of various types, sizes, velocities, and incident impact angles.FOD is a prime reason for maintenance and repair. In particular, the damage induced by small hard objects of mm size, in association with the typical load spectra experienced by airfoils, i.e., low-cycle fatigue (LCF) cycling due to normal start/flight/landing cycles superimposed with high-cycle fatigue (HCF) cycles due to vibrations and resonant loads, can lead to non-conservative life prediction and unexpected high-cycle fatigue failures.

It has been shown that fatigue failure from FOD arises as a result of three main aspects:

a. The geometric stress concentration in a V-notch or similar geometry;
b. Microstructural damage and void nucleation
c. Residual stress effects due to the plasticity generated in the zone surrounding the damage.

II. MODELING ANALYSIS

The plate geometry shown in the figure is modeled and meshed by using ANSYS macros. The plate dimensions are length L=200mm, width D=50mm, and thickness t=5mm and modeled using the RECTNG or BLC4/ BLC5 macro commands under /PREP7 pre-processor giving the equivalent dimensions of the plate in the respective working plane coordinates.

A. Material Properties

The material taken for the blade is steel, have properties

Young's Modulus = 2.1e5 MPa.

Density = 7850 Kg/m^3

Poisson's ratio = 0.3

The material is assumed to be in linear isotropic elastic condition.

The blade is assumed to be rotating at a speed of 15000 rpm, and the angular velocity of the blade is calculated from the available data as

$$\omega = \frac{2\pi N}{60} = \frac{2\pi * 15000}{60} = 1570.796 rad/\sec$$

Centrifugal Force of the Blade is given by

$F = mr\omega^2$

F = 435.690 KN.

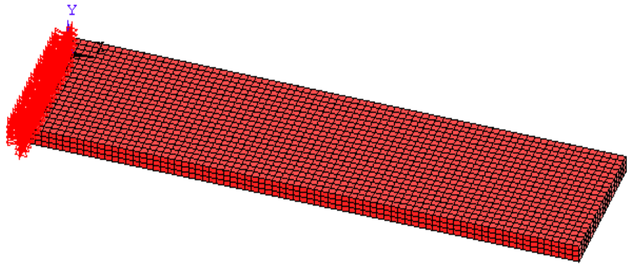


Fig.1: Finite Element Model with applied boundary conditions

This model's element type is PLANE42 and SOLID45 and is written in macro commands using the ET-for element type. The model is fine-meshed and coarse meshed the LESIZE-line element size and divisions and AMESH-area mesh, as shown in the figure. After meshing the plane 42 element, then the plane 42 element type is extruded to SOLID45 by using the EXT command. After extruding, the PLANE42 elements are deleted, and the nodes along Z=0 are selected. The model is arrested in X, Y, and Z direction by selecting the nodes fixed with D. the NSEL makes the node selection with S or R command. The solution phase begins with /SOLU command, and the modal analysis type is switch on by writing ANTYPE, 1. The problem is solved by using the SOLVE macro command. The sequence of /POST1 can carry the Post-processing of results. The model consists of 4800 SOLID45 elements.

III. RESULTS

The resulting modal frequencies for 1F, 1T, 2F, and 2T are found experimentally, and the results obtained from the finite element method are tabulated. The mode shaped for 1F, 1T, 2F, and 2T are as shown.

A. Convergence Results (Enhanced Mesh Density)

Case 1: In the first case, the plate mesh density is increased from 1000 elements in the vertical direction to 4800. The values of frequency obtained are found converging to the experimental results, as shown in the table.

Table I: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.96	104.4925
2F	661.473	654.8433
1T	827.971	801.915
2T	2553.4	2405.74

Case 2: In the second, the plate mesh density is increased to 12000 elements, and the solution still converged towards the experimental results.

Table II: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.93	103.6247
2F	661.473	649.4045
1T	827.971	795.2544
2T	2553.2	2385.763

Case 3: In the third, the plate mesh density is increased to 30000 elements, and the solution still converged towards the experimental results.

Table III: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.9	103.6247
2F	661.473	649.4045
1T	827.971	795.2544
2T	2553.23	2385.763

B. Mode Shapes Of The Rectangular Plate Without Rotation And Rotation

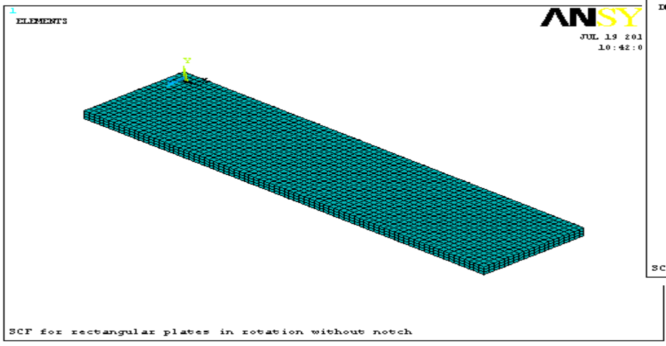


Fig 2: The plate mesh density model is increased from 1000 elements in a vertical direction to 4800.

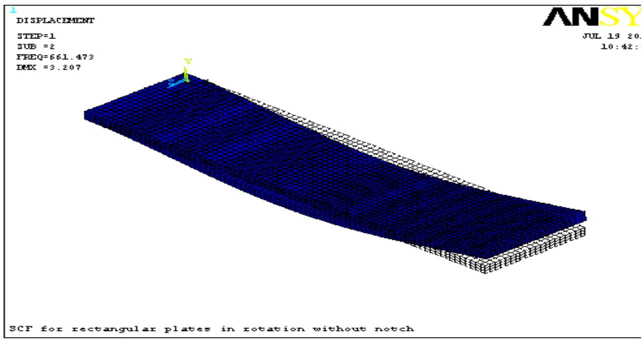


Fig 3: 2F mode shape of the rectangular plate in rotation without Notch

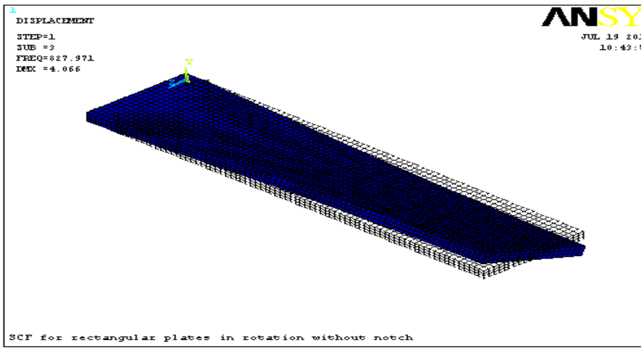


Fig 4: 1T mode shape of the rectangular plate in rotation without Notch

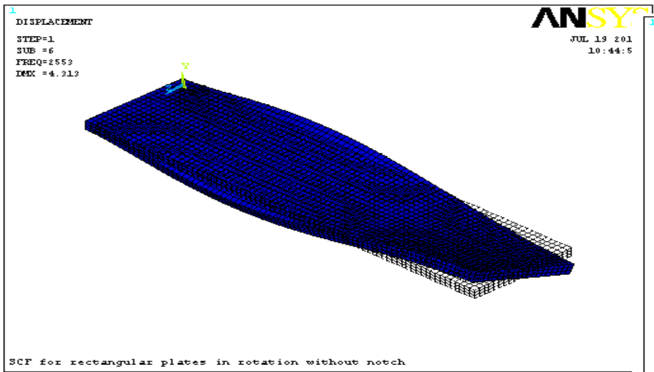


Fig 5: 2T mode shape of the rectangular plate in rotation without Notch

Case 2: In the second, the plate mesh density is increased to 12000 elements

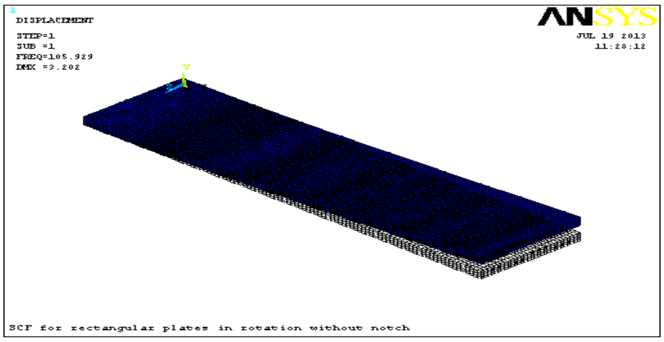


Fig 6: 1F mode shape of the rectangular plate in rotation without Notch

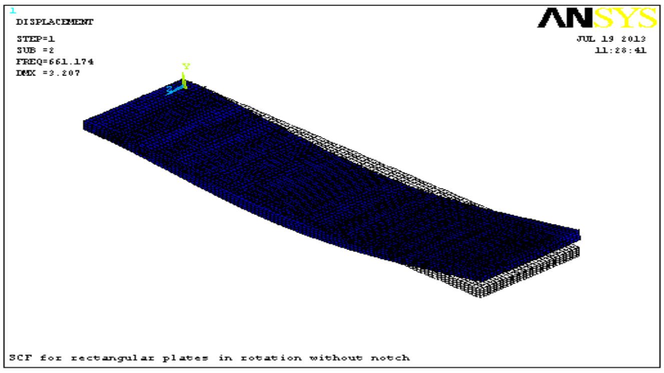


Fig 7: 2F mode shape of the rectangular plate in rotation without Notch

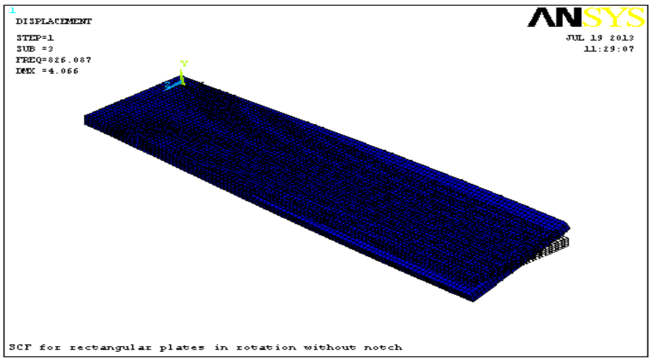


Fig 8: 1T mode shape of the rectangular plate in rotation without Notch

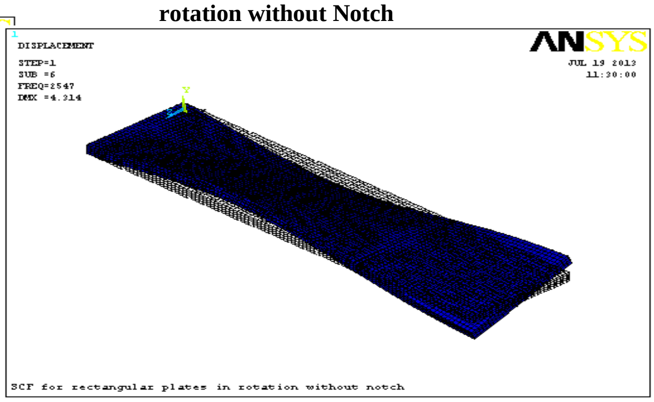


Fig 9: 2T mode shape of the rectangular plate in rotation without Notch

C. Theoretical Calculations

For flexural modes:

1st flexural mode

$\Theta = 1.875$

$\Theta_2 = 3.516$

$E = 2.10\text{x}1011 \ N/m^2$

D. The Rectangular Plate With The Semicircular Notch

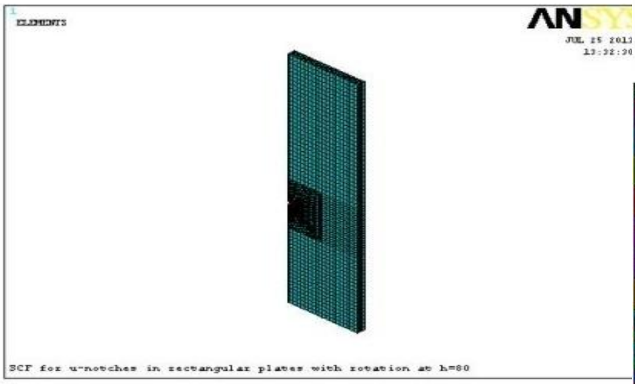


Fig: 10: Cantilever Beam with a semicircular notch and applied Transient Load

For the harmonic analysis of the gas turbine engine compressor blade idealized as a cantilever beam and modeled using Ansys, the centrifugal force F = 435.690 kN, calculated using angular velocity, need to be applied in the horizontal direction from the root to the tip of the beam along its leading edge as shown in the figure. The beam is modeled with the notches at different locations, which simulates the foreign object damages. The harmonic analysis requires the forcing frequency range to apply the calculated centrifugal force. We need to carry out modal analysis first, then using the obtained forcing frequency range from the modal analysis, the harmonic analysis is continued. The harmonic analysis can generate plots of displacement amplitudes at given points in the structure as a function of forcing frequency, so here we are selecting the specific nodes as the response points in the blade. Since the points of interest are to analyze the harmonic response of the beam at its tip and the Notch, the response points chosen for the review of results is at the tip of the beam and mid of the Notch.

E. The Response Points Chosen For The Review Of Transient Response

The response points chosen for the review of results is at the tip of the beam and mid of the Notch.

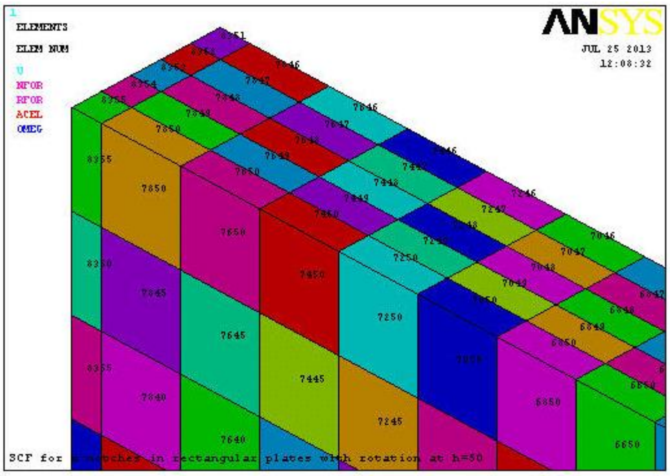


Fig 11: The response point taken at the tip of the blade

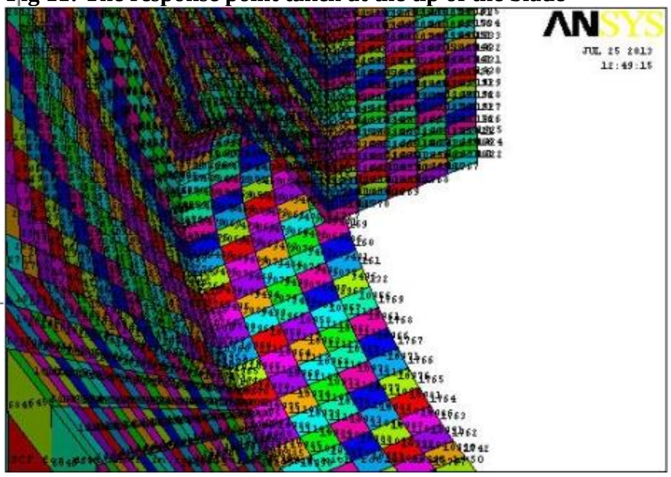


Fig 12: The response point taken at the mid of the semicircular Notch.

IV. RESULTS AND DISCUSSION

A. Convergence Results

The modal frequencies for 1F, 1T, 2F, 2T are found using ANSYS, and the same was compared with the theoretically obtained modal frequencies. Both the results were found to be converged. The convergent solution of the rotating beam's modal frequencies was carried out for three specific cases by increasing the mesh density, and we obtained the results as follows.

Case 1: In the first case, the plate mesh density is increased from 1000 elements in the vertical direction to 4800. The values of frequency obtained are found converging to the experimental results as shown in the table

Table IV: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.96	104.4925
2F	661.473	654.8433
1T	827.971	801.915
2T	2553.4	2405.74

Case 2: In the second, the plate mesh density is increased to 12000 elements, and the solution still converged towards the experimental results.

Table V: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.93	103.6247
2F	661.473	649.4045
1T	827.971	795.2544
2T	2553.2	2385.763

Case 3: In the third, the plate mesh density is increased to 30000 elements, and the solution still converged towards the experimental results.

Table VI: Convergent solution of Modal Frequency of the Rectangular plate

Mode Shape	FEM results (Hz)	Theoretical results (Hz)
1F	105.90	103.6247
2F	661.473	649.4045
1T	827.971	795.2544
2T	2553.23	2385.763

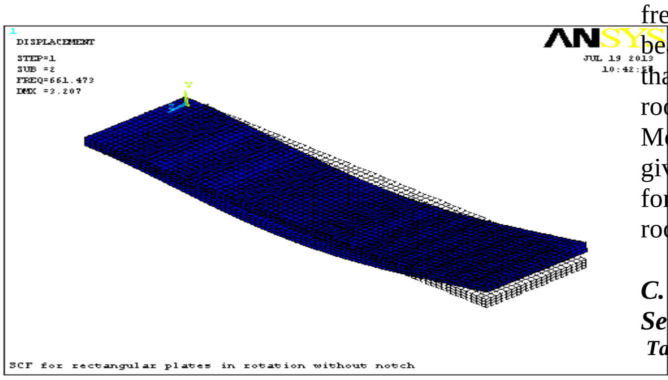


Fig 13: 2F mode shape of the rectangular in a plate in rotation without Notch

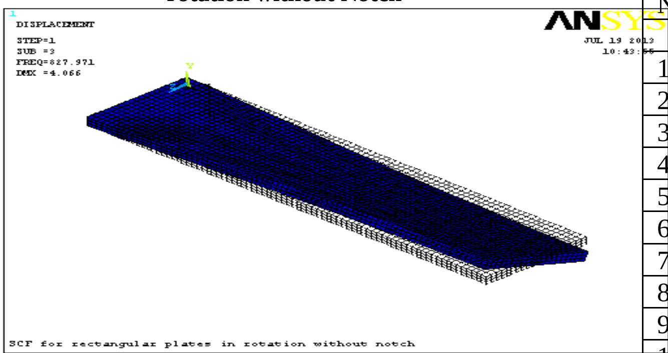


Fig 14: 1T mode shape of the rectangular plate in rotation without Notch

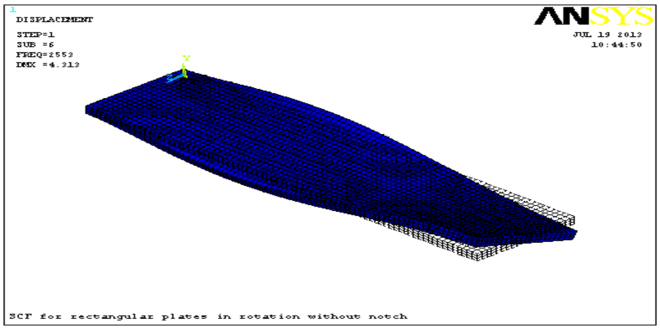


Fig 15: 2T mode shape of the rectangular plate in rotation without Notch

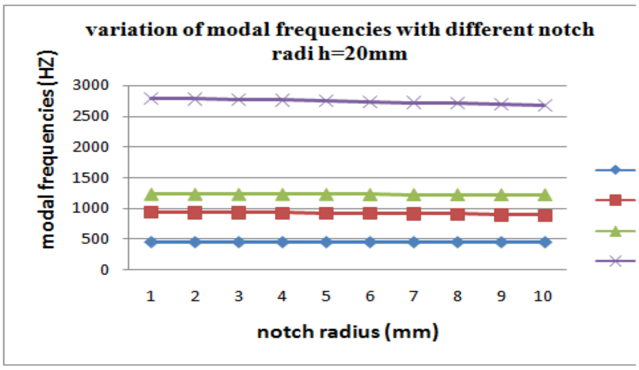
B. Free Vibration Analysis

The Free Vibration Analysis of Rectangular Cantilever beams were done for various notch parameters to obtain the Natural frequencies of the beam. The average natural frequency for First Flexural (1F) and First Torsion(1T) for notch dimension h=20mm, 50mm,80mm,120mm, 150mm, and for Notch height 20mm and radius ranging from 1 to 10mm are 448.84Hz and 937.74Hz respectively as the notch radius is increased from 1 to 10mm in steps of 1mm. It is observed that the 1 F frequency decreases. When the Notch is at the beam's tip, the Natural frequency is slightly greater than those obtained when the Notch is at the beam's root. For Transient Analysis, the above obtained Modal analysis results and the centrifugal force are given. The Natural Frequencies of the beam increase for the notch location far from the cantilever beam's root.

C. Modal Frequencies For Rotating Beam With Semicurcular Notches

Table VII: Variation of modal frequencies with different notch radius at h=20mm

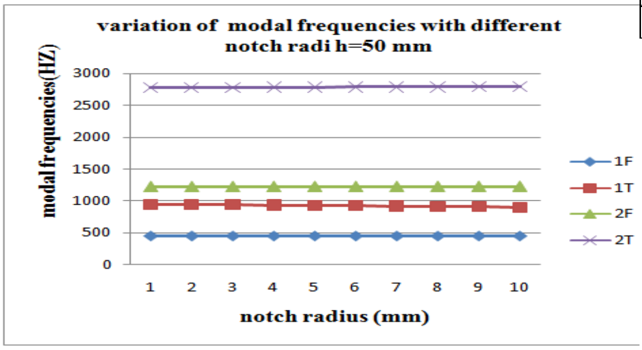
The radius of Notch(mm)	Modal Frequencies (Hz)			
	1F	**1T**	**2F**	**2T**
1	448.84	937.74	1227	2788
2	448.68	935.21	1226	2781
3	448.48	931.94	1225	2773
4	448.24	927.92	1225	2763
5	447.97	923.16	1224	2752
6	447.66	917.63	1223	2739
7	447.32	911.34	1222	2724
8	447.08	905.09	1221	2711
9	446.74	897.32	1220	2695
10	446.37	888.66	1219	2679



Graph 1: Variation of modal frequencies with different notch radius at h=20mm

Table VIII: Variation of modal frequencies with different notch radius at h=50mm

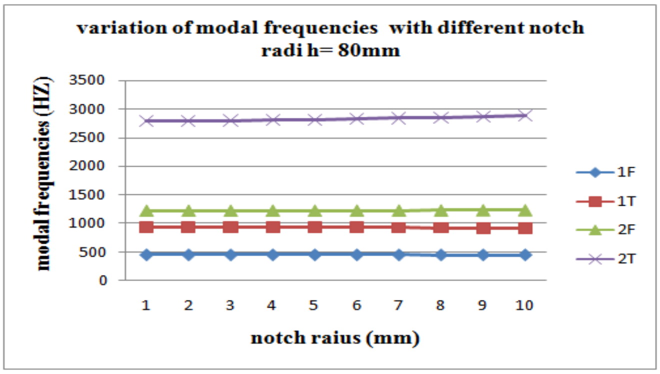
Notch rad. (mm)	Modal Frequencies (Hz)			
	1F	1T	2F	2T
1	448.31	936.85	1224.4	2787.2
2	448.28	935.11	1224.5	2787.9
3	448.25	932.61	1224.7	2788.9
4	448.20	929.35	1225.0	2790.0
5	448.13	925.34	1225.3	2791.2
6	448.06	920.55	1225.7	2792.5
7	447.97	915.00	1226.1	2793.7
8	447.88	908.66	1226.6	2794.8
9	447.76	901.51	1227.1	2795.9
10	447.64	893.56	1227.7	2796.8



Graph 2: Variation of modal frequencies with different notch radius at h=50mm

Table IX: Variation of modal frequencies with different notch radius at h=80mm

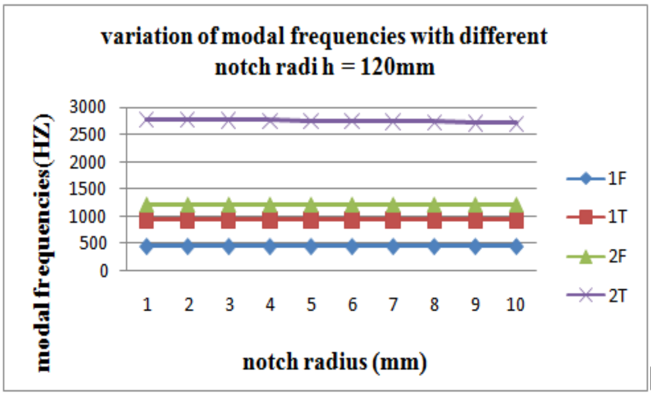
Rad Notch (mm)	Modal Frequencies (Hz)			
	1F	1T	2F	2T
1	448.29	936.15	1225	2793
2	448.29	936.15	1225	2793
3	448.25	934.53	1225	2799
4	448.21	932.37	1226	2806
5	448.15	929.67	1226	2815
6	448.07	926.39	1227	2826
7	447.99	922.51	1228	2838
8	447.90	917.99	1229	2851
9	447.80	912.83	1231	2866
10	447.68	906.98	1232	2882



Graph 3: Variation of modal frequencies with different notch radius at h=80mm

Table X: Variation of modal frequencies with different notch radius at h=120mm

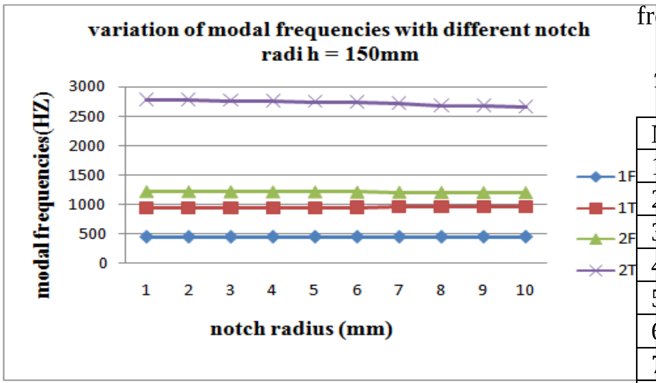
Rad. of Notch (mm)	Modal Frequencies (Hz)			
	1F	1T	2F	2T
1	448.34	938.09	1224.1	2784.9
2	448.33	938.20	1223.8	2781.5
3	448.32	938.29	1223.6	2776.7
4	448.29	938.33	1223.3	2770.5
5	448.27	938.25	1223.1	2763.1
6	448.24	938.01	1223.0	2754.4
7	448.20	937.57	1222.9	2744.9
8	448.16	936.86	1222.9	2734.6
9	448.11	935.83	1223.1	2723.9
10	448.06	934.44	1223.4	2713.2



Graph 4: Variation of modal frequencies with different notch radius at h=120mm

Table XI: Variation of modal frequencies with different notch radius at h=150mm

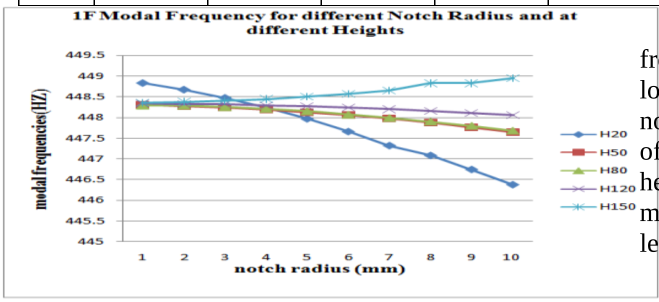
The radius of Notch (mm)	Modal Frequencies (Hz)			
	1F	1T	2F	2T
1	448.36	938.63	1223.9	2783.9
2	448.38	939.62	1223.1	2778.9
3	448.41	940.97	1222.1	2771.6
4	448.45	942.63	1220.8	2762.0
5	448.51	944.57	1219.3	2750.1
6	448.57	946.72	1217.4	2735.9
7	448.65	949.04	1215.3	2719.6
8	448.83	954.04	1210.4	2680.6
9	448.83	954.04	1210.4	2680.6
10	448.95	956.64	1207.5	2658.3



Graph 5: Variation of modal frequencies with different notch radius at h=150mm

Table XII: 1F Modal Frequency for different Notch Radius and at different Heights

	H20	H50	H80	H120	H150
1	448.84	448.31	448.29	448.34	448.36
2	448.68	448.28	448.29	448.33	448.38
3	448.48	448.25	448.25	448.32	448.41
4	448.24	448.20	448.21	448.29	448.45
5	447.97	448.13	448.15	448.27	448.51
6	447.66	448.06	448.07	448.24	448.57
7	447.32	447.97	447.99	448.20	448.65
8	447.08	447.88	447.90	448.16	448.83
9	446.74	447.76	447.80	448.11	448.83
10	446.37	447.64	447.68	448.06	448.95

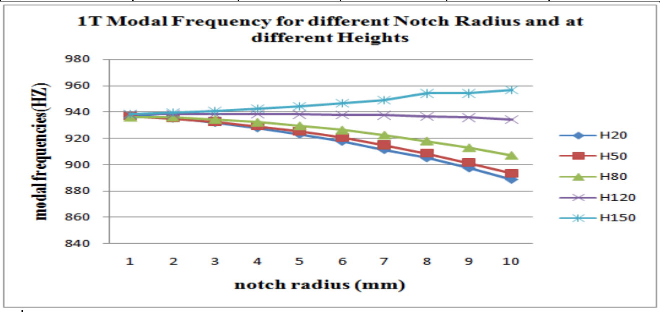


Graph 6: 1F Modal Frequency for different Notch Radius and at different Heights

Comparison of First Flexural (IF) frequencies for different notch radius with the varying location of notch height is given in Table-6.9. The notch radius is varied from 2 mm to 6 mm in steps of 2 mm with the various location heights of 20 mm, 50mm, 80 mm, 120 mm, and 150 mm from the root to tip of the blade along the leading edge. It is observed that for 2 mm, the radius of Notch and the locations mentioned, the frequency decreases from the root to the blade's tip. The percentage decrease in 1F frequency for a 2 mm notch radius, from the initial height of 20 mm to the final height of 150 mm, is 0.096%. For further increase of the notch radius in steps of 2 mm for the locations mentioned, frequency increases from the root to the blade's tip. The percentage increase in 1F frequency for a 4 mm notch radius, from the initial height of 20 mm to the final height of 150 mm, is 0.054%. For further increase of the notch radius for 6 mm and mentioned height, the percentage increase in frequency for 1F frequency is 0.182%.

Table XIII: 1T Modal Frequency for different Notch Radius and at different Heights

Notch rad	H20	H50	H80	H120	H150
1	937.74	936.85	936.15	938.09	938.63
2	935.21	935.11	936.15	938.20	939.62
3	931.94	932.61	934.53	938.29	940.97
4	927.92	929.35	932.37	938.33	942.63
5	923.16	925.34	929.67	938.25	944.57
6	917.63	920.55	926.39	938.01	946.72
7	911.34	915.00	922.51	937.57	949.04
8	905.09	908.66	917.99	936.86	954.04
9	897.32	901.51	912.83	935.83	954.04
10	888.66	893.56	906.98	934.44	956.64



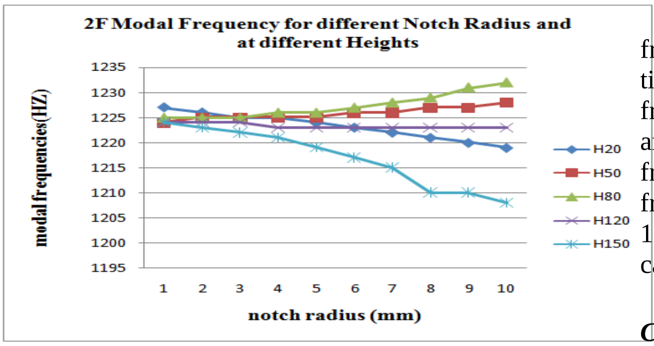
Graph 7: 1T Modal Frequency for different Notch Radius and at different Heights

Comparison of First Torsional (1T) frequency for different notch radius with the varying location of notch height is given in Table-6.10. The notch radius is varied from 2 mm to 6 mm in steps of 2 mm with the varying location of the notch height of 20 mm, 50mm, 80 mm, 120 mm, and 150 mm from the root to the tip of the blade along the leading edge.

The 1F and 1T frequencies are obtained for different notch radius and notch heights from the above modal analysis. As mentioned above, this frequency range, i.e., 449-940 Hz, is further used to carry out the cantilever beam's transient analysis.

Table XIV: 2F Modal Frequency for different Notch Radius and at different Heights

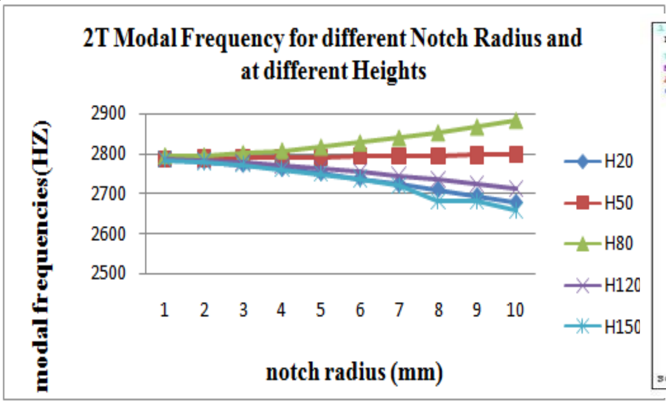
Radius of Notch	H20	H50	H80	H120	H150
1	1227	1224	1225	1224	1224
2	1226	1225	1225	1224	1223
3	1225	1225	1225	1224	1222
4	1225	1225	1226	1223	1221
5	1224	1225	1226	1223	1219
6	1223	1226	1227	1223	1217
7	1222	1226	1228	1223	1215
8	1221	1227	1229	1223	1210
9	1220	1227	1231	1223	1210
10	1219	1228	1232	1223	1208



Graph 8: 2F Modal Frequency for different Notch Radius and at different Heights

Table XV: 2T Modal Frequency for different Notch Radius and at different Heights

Radius of Notch	H20	H50	H80	H120	H150
1	2788	2787	2793	2785	2784
2	2781	2788	2793	2782	2779
3	2773	2789	2799	2777	2772
4	2763	2790	2806	2771	2762
5	2752	2791	2815	2763	2750
6	2739	2793	2826	2754	2736
7	2724	2794	2838	2745	2720
8	2711	2795	2851	2735	2681
9	2695	2796	2866	2724	2681
10	2679	2797	2882	2713	2658



Graph 9: 2T Modal Frequency for different Notch Radius and at different Heights

Comparison of second Flexural (2F) frequency for different notch radius with the varying location of notch height is given in Table-6.8. The notch radius is varied from 2 mm to 6 mm in steps of 2 mm with the varying location of the notch height of 20 mm, 50mm, 80 mm, 120 mm, and 150 mm from the root to the tip of the blade along the leading edge. It is observed that for 2 mm, the radius of Notch, and the locations mentioned, the frequency decreases from the root to the blade's tip. The percentage decreases in 1F frequency for 2 mm notch radius from the blade's initial height tip because the stiffness is also decreased.

In the same manner, second Torsional (2T) frequency also decreases from the root to the blade's tip. From the above modal analysis, the 1F and 1T frequencies are obtained for different notch radius, and notch heights as mentioned above, this frequency range, i.e., 449-940 Hz and 2F, 2T frequencies are obtained, and that frequency range is 1227-2658 Hz. Further, the transient analysis is carried out for the cantilever beam.

C.TRANSIENT ANALYSIS

The impulse force is applied along the tip of the blade. The impulse force is obtained from the angular velocity, calculated from the constant engine speed of 15000 rpm. Further, for Transient Analysis, the notch radius is varied from 2mm to 6mm in steps of 2mm.

To simulate the blade attachment on the compressor in the finite element model, the beam is constrained in X, Y& Z translation DOF. The impulse force is applied for transient analysis, as shown in Fig.6.4, For the Notch location at the height 20mm, 50mm, 80mm, and for the notch radius 2mm, 4mm, 6mm, respectively. The transient analysis was carried out, and the results obtained were the peak amplitude and the period to reach a steady state.

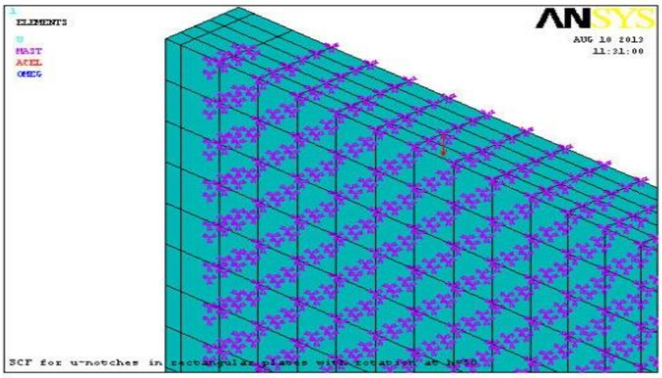


Fig. 16: Finite element model of beam constrained all DOF with Applied Impulse load

D. Deformed Shape Plot

Nodal Displacement Plot

Height 80mm: Notch radius 2mm

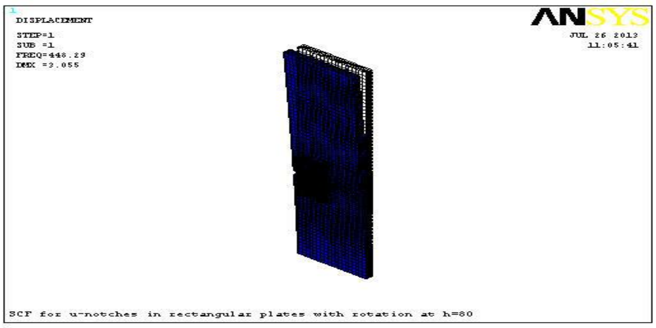


Fig. 17: Deformed and Undeformed Shape

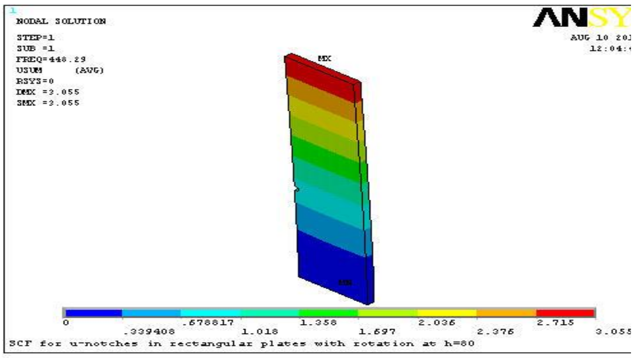
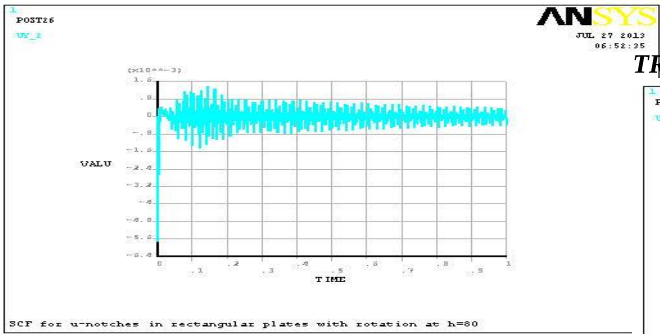


Fig. 18: Nodal displacement of the beam

TRANSIENT RESPONSE PLOT



Graph 10: The Time v/s. amplitude plot at Height 80mm and Notch radius 2mm.

Transient response plot at the response point, i.e., node number 7306 at the tip of the blade. The peak amplitude occurs at the period of 0.14s

A similar analysis is carried out by varying notch radius from 2mm to 6mm in steps of 2mm for height 80 mm, and the results are as shown.

DISPLACEMENT PLOTS

NODAL DISPLACEMENT PLOT

Height 80mm: Radius 4mm

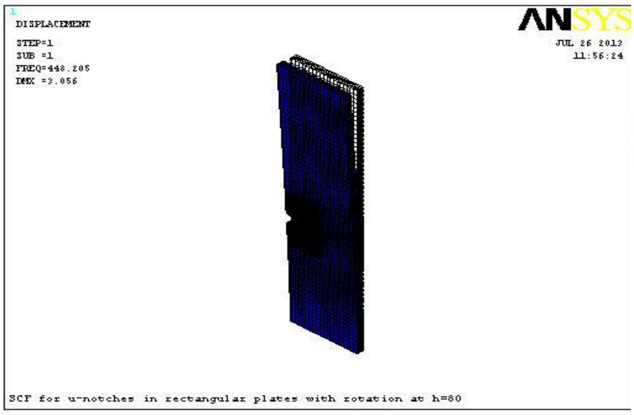


Fig.19: Deformed shape& Undeformed Shape

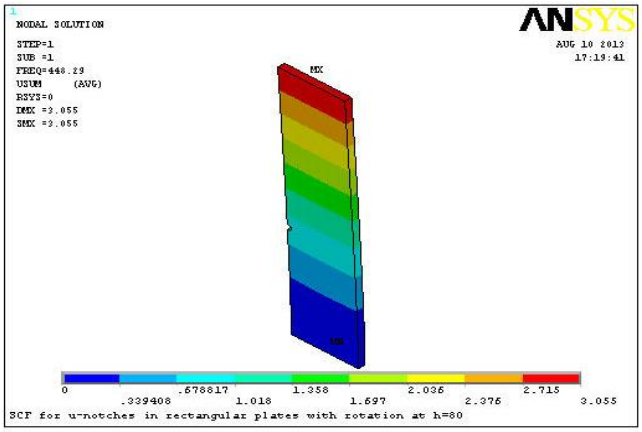
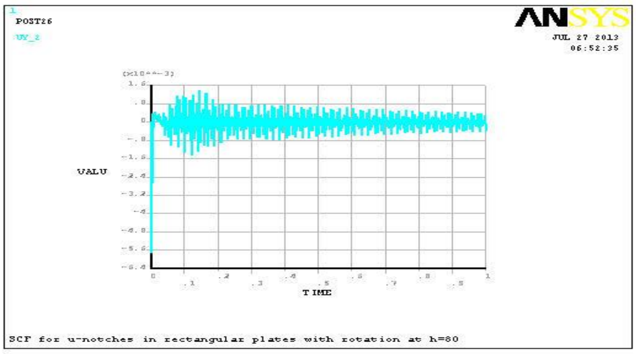


Fig. 20: Nodal displacement

TRANSIENT RESPONSE PLOT



Graph 11: Transient responses Plot at node 7290 Height 80mm: Radius 6mm

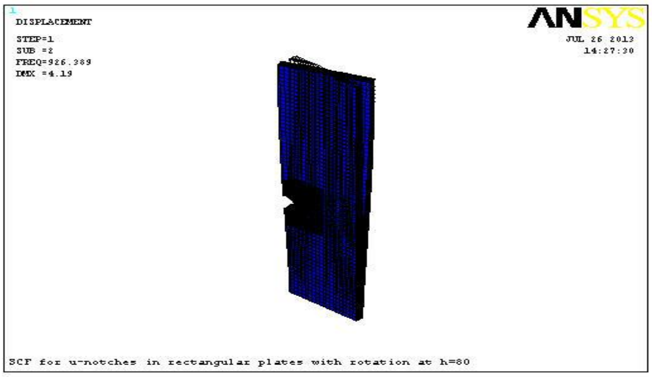


Fig. 21: Deformed shapes & Undeformed Shape

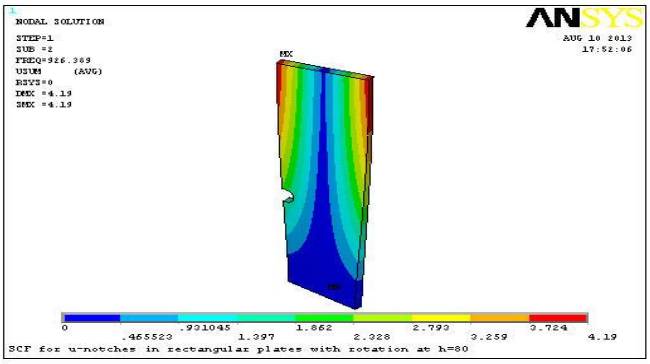
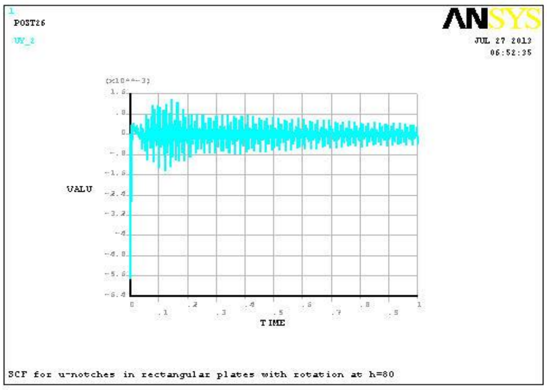


Fig. 22: Nodal displacement

TRANSIENT RESPONSE PLOT



Graph 12: Transient responses Plot at node 7300

The same analysis procedure is carried out by considering different notch locations by varying the parameter notch heights at 80mm, 100mm, 120mm, and 150mm from root to the tip of the beam and by varying the parameter of semicircular Notch from 2mm to 6mm radius in steps of 2mm. The detailed results obtained from each analysis are given in the Appendix.

Table XVI: Peak amplitude values at different Notch height 20mm with varying Notch radius

Notch Radius (mm)	Peak Amplitude (X10-3)	Time Period(secs)
2	0.8	0.01
4	1.2	0.1
6	1.4	0.1

Table XVII: Peak amplitude values at different Notch height 50mm with varying Notch radius

Notch Radius (mm)	Peak Amplitude (X10-3)	Time Period(secs)
2	0.20	0.02
4	0.25	0.02
6	0.40	0.01

Table XVIII: Peak amplitude values at different Notch height 80mm with varying Notch radius

Notch Radius (mm)	Peak Amplitude (X10-3)	Time Period(secs)
2	1.2	0.12
4	1.4	0.14
6	1.6	0.16

Table XIX: Steady-State amplitude values at different Notch height 20mm with varying Notch radius

Notch Radius (mm)	Steady State Amplitude (X10-3)	Period (secs)
2	0.2	0.3
4	0.2	0.8
6	0.4	0.8

Table XX: Steady-State amplitude values at different Notch height 50mm with varying Notch radius

Notch Radius (mm)	Steady State Amplitude (X10-^{3})	Period (secs)
2	0.10	0.40
4	0.12	0.40
6	0.14	0.45

Table XXI: Steady-State amplitude values at different Notch height 80mm with varying Notch radius

Notch Radius (mm)	Steady State Amplitude (X10-^{3})	Period (secs)
2	0.20	0.8
4	0.25	0.9
6	0.30	0.9

V. CONCLUSIONS

Results obtained through this study can be summarized as follows

a. The harmonic response for semicircular notches at the root of the beam is slightly greater than that for the notches at the beam's tip.
b. Harmonic analysis has generated plots of displacement amplitudes at a response point in the cantilever beam structure as a function of forcing frequency.
c. The dominant harmonic response is found to be in the range of 499.60Hz to 514.34Hz for various height and notch parameters.
d. The harmonic response frequency decreases as the notches' heights increase compared to the harmonic response frequency for the notches at the height nearer to the root of the cantilever beam.
e. Compare the results of this study with the working models.
f. Using the results, the blade with FOD's presence is safer for continuation or needs to be replaced. This can be identified by looking into the FOD location and its dimension.
g. This study can be used as a working chart to identify risk levels.
h. When the notch location is constant, but the notch radius is varied, the beam's natural frequency decreases.
i. The Natural frequency for the Notch at the root is greater than for the Notch at the tip.
j. Harmonic response increases with the notch radius.

VI. SCOPE FOR FUTURE WORK

a. A transient analysis can be carried out.
b. Spectrum or random vibration analyses can be carried out.
c. Phase angle Ψ allows multiple out of phase loads. Analysis can be carried out for multiple load cases and also for complex load cases.
d. The current analysis is solved using the full method. It can also be solved by using the reduced method and mode superposition method.
e. Harmonic analysis can be carried out by considering superalloy materials.

f. The sharp cracks can be considered instead of semicircular notches.
g. The above analysis can be carried out for the actual turbine blade profile.

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