

# Workspace Optimization of 8 UPS Spatial Parallel Manipulator

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**Abstract** - In this paper the workspace optimization of 8 UPS spatial parallel manipulator under joint limitations is performed. The workspace is parametrized utilizing a few plan parameters that traverse an extensive scope of values. In this paper both the unconstrained and the obliged workspaces (i.e., workspace with joint points of confinement) are utilized. For the workspace of each outline arrangement two execution records are figured utilizing a Monte Carlo technique: a) Transmission Index; b) Manipulability index. Plots both the indices as a capacity of the outline parameters are created and ideal esteems for these plan parameters are resolved. For the ideal plan, it is demonstrated that by presenting joint limits, the global isotropy of the parallel stages is enhanced at the cost of workspace diminishment.

## I. INTRODUCTION

G.Sutherland and B.Roth [1] developed a general index of the quality of motion transmission for spatial; mechanism using the theory of screws. This index is related to the mechanical error possible in a linkage. A method for synthesizing spatial linkages with desirable motion transmission and mechanical error characteristics is developed. The problem of determining a RGGR function generator with optimum transmission and the error sensitivity characteristic is dealt with.

Salisbury and Craig [2] are the first to apply the condition number concept to mechanisms. They have used the condition number of the Jacobian Matrix as an optimization criterion to obtain ideal dimensions for the mechanisms with the two revolute joints and Stanford JPL Articulated hand. The concept of manipulability measure was proposed by Tsuneo Yoshikawa [3]. This is a measure of manipulating ability of robotic mechanism in positioning and orienting the end effectors. Properties of the manipulability measure are also established by Yoshikawa. The utilization of this measure for determining the best postures of planar two links, PUMA and SCARA Robots were discussed by him. Hollerbach and Suh [4] examined the methods for resolving kinematic redundancies of manipulators by the effect on joint torques. Kazuhiro Kosuge and Katsuhisa Furuta [5] used the inverse of the condition number of Jacobian Matrix as a measure of kinematic controllability. For dynamic controllability another measure based on condition number concept in the Jacobian matrix and the matrix concerned with the inertia term is considered by them. Khalil, W. Kleinfinger [6] presented a new geometric notation for the description of the kinematic of open-loop, tree and closed-loop structure robots. The method is derived from the well-known Denavit and Hartenberg (D-H) notation, which is powerful for serial robots but leads to ambiguities in the case of tree and closed-loop structure robots. The given method has all the advantages of D-H notation in the case of open-loop robots. Klein and Blaho [7] presented several dexterity measures for an optimizing posture for a given end-effector position and for intriguing the optimum link lengths of an arm. The four measures Determinant, Condition number, Minimum singular value and Joint range availability are determined for the entire reach of the planar three link revolute jointed manipulator Clement Gosselin [8] developed a design aid to provide Transmission and conditioning maps for parallel manipulators. The proposed method is based on the description of the workspace of a three and six-degree-of-freedom parallel manipulators. The Transmission maps obtained can be used to study the Transmission of manipulators in the Cartesian directions or to investigate the minimum Transmission by using the eigenvalues of the Transmission matrix. J. Angeles and Patel [9] proposed a kinematic design of redundant seven-axis manipulator. The focus is on the optimization of the kinematic conditioning of the manipulator. The research aimed at the optimization of a performance index of the manipulator from both kinematic and static viewpoints based on the condition number. The discussion is about the manipulability index in their work. Therefore the manipulability index fails to give a reliable measure of the kinetostatic performance of the manipulator because it is independent of the operation point of the end effector. Pond and Carretero [10] introduced a dimensionally homogeneous Jacobian matrix, which is used to determine the dexterity of parallel mechanisms regardless of the number and type of degrees of freedom of the mechanism. A 3-PRS manipulator is analyzed by using the new concept. Kucuk and Bingul [11] studied manipulability measure and condition number for the optimal robot design. The structures of the robot manipulators were compared based on the structural length index and global conditioning index. Ozgoren [12] reviewed the mathematical properties associated with the exponential rotation matrices. By means of two typical mechanism examples, it is demonstrated that these properties constitute a versatile analytical tool, which can be used effectively in kinematic studies on spatial mechanical systems

involving position, velocity, acceleration, and singularity analyses using lie algebra. The mechanism in the first example allows analytical solution for its joint variables, whereas the joint variables of the mechanism in the second can be obtained only by a semi analytical solution. However, the scope of these examples is limited with the position, velocity, acceleration, and singularity analyses. Chao Chen and Jorge Angeles [13] proposes a generalized transmission index (GTI) for spatial investigation based on the virtual coefficient between the transmission torque screw and the output twist screw. The GTI sums up Sutherland and Roth's transmission index (TI). The pressure angle and the transmission angle are special cases of the GTI in their attainable range. The GTI gives the force transmission quality accurately where Sutherland and Roth's Transmission index fails. Yangmin Li and Qingsong Xu [14] presents the Transmission characteristics of a 3-PUU translational parallel kinematic machine (PKM). The Transmission matrix is derived by considering actuations and constraints, and the compliances subjected to both actuators and legs. The Transmission performance of the manipulator is evaluated by utilizing the extremum Transmission values, and the influences of design parameters on the Transmission properties Daxing Zeng et al. [15] analyzed a 3DOF 3-PRUR PM . It is found that the PM can behave like a conventional X-Y-Z Cartesian machine and is completely decoupled on the initial position. After obtaining the corresponding atlases to these performance indices it is concluded that the mechanism's performance will not be influenced even if all links have the same length .Sergiu-Dan Stan et al. [16]outlined a mono-objective optimum design procedure for parallel robot by using optimality criterion of workspace and numerical aspects. A kinematic optimization was also performed to maximize the workspace of the 6 degrees-of-freedom micro parallel robot. After maximizing the workspace they also concluded that the optimal design method is useful for the design of a six degree of freedom micro parallel robot with translation actuators.

**A. Manipulability**

It is important to formulate a quantitative measure of manipulation capability of the mechanical system.

Yoshikawa have introduced the concept of kinematics manipulability is quantified as

$$M = \sqrt{\det(JJ^T)} \tag{1.1}$$

Where J is the jacobian matrix and depends on the instantaneous configuration of the manipulator defined by a joint vector, when m=n (that is when consider non-redundant manipulator) the manipulability M reduces to

$$M = |\det J(q)| \tag{1.2}$$

**B. Transmission Index**

A parallel manipulator must transmit the joint forces and torques to the output platform, resisting the external loads, through its mechanical structure. Amid the procedure of force transmission, the rising internal wrenches, particularly, the transmission wrenches, can be imparted by the TWS. It is understood that a TWS must be integral to the twist screws

Permitted by the segregated joints in the relating leg, when the dynamic joints are bolted. If all the parallel manipulators are considered as exactly constrained systems, then

$$EF=W \tag{1.3}$$

Where W is the unit wrench,  $F=[f_1 f_2 \dots f_n]$  ,and E is termed as transmission matrix.

$$E = \begin{bmatrix} e_1 & e_2 & \dots & e_n \\ (c_1 \times e_1) + h_1 e_1 & (c_2 \times e_2) + h_2 e_2 & \dots & (c_n \times e_n) + h_n e_n \end{bmatrix} \tag{1.4}$$

where  $e_i$  is the unit directional vector of the TWS,  $c_i$  is the vector pointing from the center of the platform to the characteristic point of the I th leg,  $h_i$  is the pitch of the TWS,  $f_i$  is the magnitude of the TWS in terms of force and  $\sigma_i$  are the singular values of E . The transmission index (TI) is given by

$$TI = \sum 1/\sigma_i^2$$

**C. KINEMATIC MODELING**

$$p_i = s + Qp'_i \quad i = 1,2,3 \dots 8 \tag{1.5}$$

Where  $p'_i = [p'_{ix}, p'_{iy}, p'_{iz}]^T$ . Subtracting vector  $b_i$  from both sides of eq. (1.5), one obtains

$$p_i - b_i = s + Qp'_i - b_i \quad i = 1,2,3 \dots 8 \tag{1.6}$$

Where the left-hand side represents, in fact, a vector connecting point  $B_i$  to point  $A_i$  , along the  $i_{th}$  leg. Hence, taking the Euclidean norm of both sides of this equation leads to

$$\rho^2 = \|b_i - p_i\|^2$$

$$\rho^2 = (s + O v' - b_i)^T (s + O v' - b_i) \tag{1.7}$$

**1. Inverse kinematics & Jacobian analysis**

When eq. (1.7) is differentiated with respect to time, a set of linear equations relating the joint rates to the Cartesian velocities is obtained. Following the formalism proposed in Gosselin and Angeles (1990) for parallel manipulators, two Jacobian matrices A and B are obtained and the velocity equations can be written as

$$A t = B \rho, \tag{1.8}$$

where t is the six-dimensional twist of the platform and ρ is the vector of joint velocities. These vectors are defined as

$$t = [s \cdot^T, \omega^T]^T, \quad \rho = [\rho_1, \dots, \rho_8]^T, \tag{1.9}$$

in which the angular velocity of the platform is defined as ω and s=[ẋ, ẏ, ż]<sup>T</sup> is the velocity of point O'. The aforementioned Jacobian matrices can then be written as

$$B = \text{diag}(\rho_1, \dots, \rho_8) \tag{1.10}$$

and

$$A = \begin{bmatrix} c_1^T \\ \vdots \\ c_8^T \end{bmatrix}, \tag{1.11}$$

with

$$c_i = \begin{bmatrix} d_i \\ (Q p_{a_i}) \times d_i \end{bmatrix}, \quad i = 1, \dots, 8 \tag{1.12}$$

where d<sub>i</sub> is the vector connecting point B<sub>i</sub> to point P<sub>i</sub>, i.e.,

$$d_i = p_i - b_i, \quad i = 1, \dots, 8. \tag{1.13}$$

Finally, the rotation matrix Q representing the orientation of the platform with respect to the base can be written using Euler angles (Hughes 1986) as

$$Q = \begin{bmatrix} \cos \theta \cos \psi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \cos \theta \sin \psi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \tag{1.14}$$

Where ψ, θ, φ are three Euler angles defined according to the convention (Qz, Qy, Qx). Other representations of the rotation could also be used (e.g., quaternions, linear invariants, dual representations), which would not affect the algorithm presented

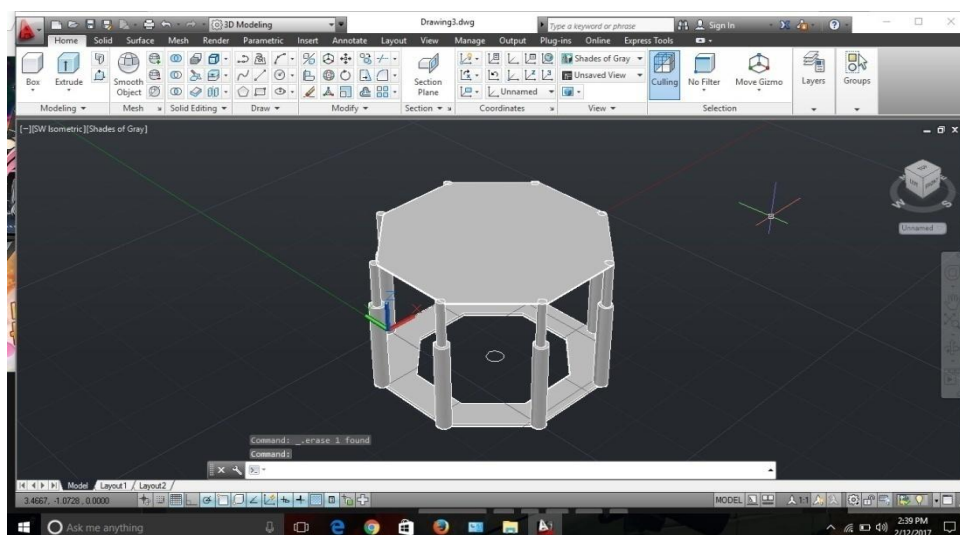


Fig 1 CAD model of 8-UPS manipulator

## II. RESULTS AND DISCUSSION

The results obtained by using the MATLAB code for the performance indices: Manipulability and Transmission index, are discussed in the following sub sections. The graphs for Transmission index are obtained by considering a specified workspace of 8-UPSspatial manipulator. For the selected structures, the manipulability index values lie outside the specified workspace considered for other indices.

The Transmission index of the manipulator is dependent on the end effector ‘s position and orientation with in the specified workspace of the manipulator. For the optimal design, the parameters: fixed platform size (b),moving platform size (p), twist angle( $\Psi$ ), tilt angle about Y axis( $\theta$ ) and tilt angle about X axis( $\hat{O}$ ) are considered; it is implemented by evaluating the influence of these parameters on Transmission index and Manipulability.

As a part of Optimal design the graphs showing the variation of dexterity measures: Transmission index and Manipulability for different structures of the manipulator are developed. Only three structures are finally selected based on the optimum values of the dexterity measures. In each structure by fixing the twist and tilt angles the variation of the measures in a plane parallel to the base platform at different vertical reaches of the moving platform are developed.

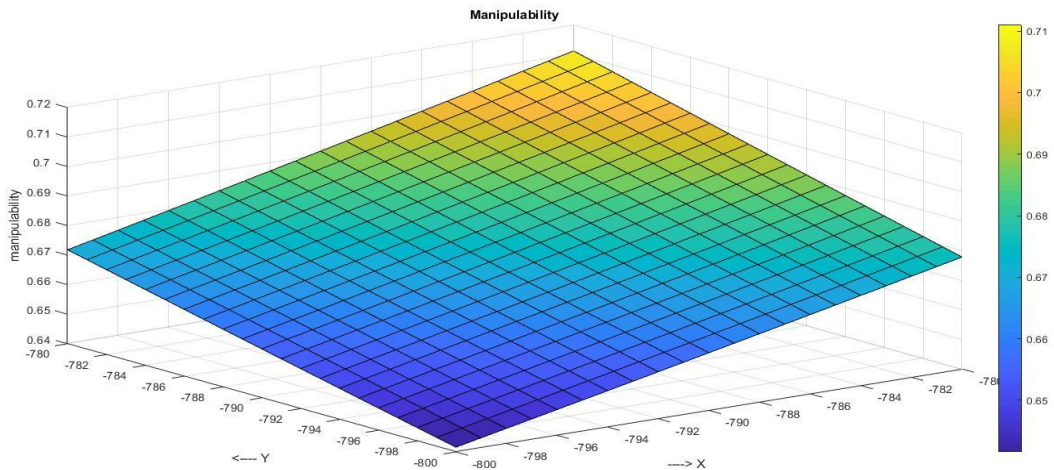


Fig.2 for  $R_b=60, R_p=30$  &  $z=20$ , Twist angle  $\Psi=10^0$ , Tilt angles  $\theta=10^0, \hat{O}=10^0$

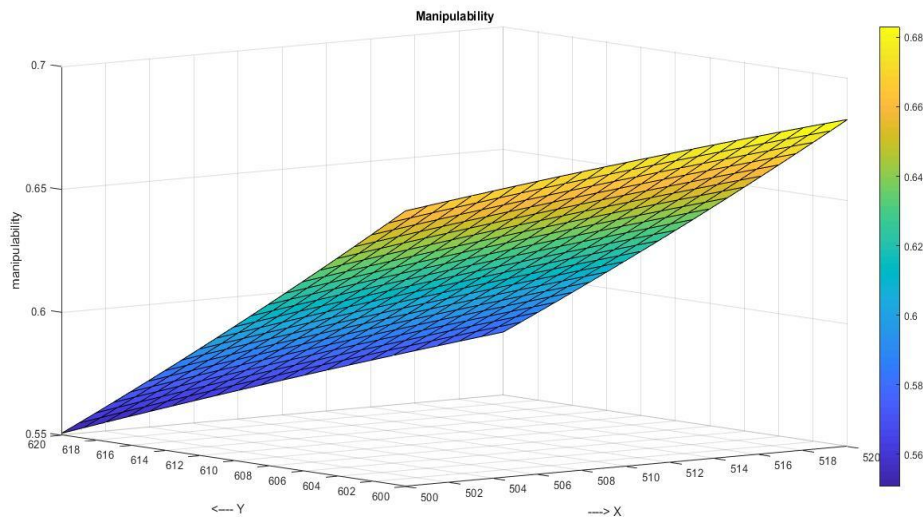


Fig.3 for  $R_b=70, R_p=30$  &  $z=35$ , Twist angle  $\Psi=8^0$ , Tilt angles  $\theta=9^0, \hat{O}=9^0$

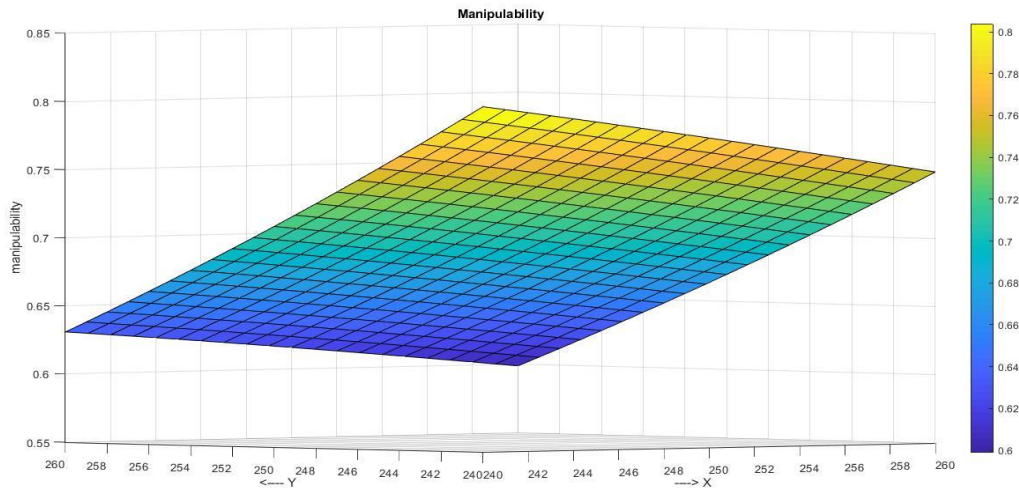


Fig.4 for  $R_b=30, R_p=20$  &  $z=28$ , Twist angle  $\Psi=9^\circ$ , Tilt angles  $\theta=7^\circ, \dot{\theta}=9^\circ$

The value of manipulability index lies between zero to one. The workspace in which the manipulability index values exists is entirely different from other indices.

For structure 1 specified by base platform and moving platform radius as 60mm and 30mm respectively with twist angle ( $\Psi$ )= $10^\circ$ , tilt angle about Y axis( $\theta$ )= $10^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $10^\circ$  the value of Manipulability increases in the vertical reach range of 10mm to 20mm and the maximum value of Manipulability is 0.71105 which occurs at  $x=-780$  mm,  $y=-780$  mm and vertical reach of 20mm as shown in fig.2 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 2 specified by base platform and moving platform radius as 70mm and 30mm respectively with twist angle ( $\Psi$ )= $8^\circ$ , tilt angle about Y axis( $\theta$ )= $9^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $9^\circ$  the value of Manipulability increases in the vertical reach range of 30mm and 35mm and the maximum value of Manipulability is 0.68307 which occurs at  $x= 520$  mm,  $y= 600$  mm and vertical reach of 35mm as shown in fig.3 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 3 specified by base platform and moving platform radius as 30mm and 20mm respectively with twist angle ( $\Psi$ )= $9^\circ$ , tilt angle about Y axis( $\theta$ )= $7^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $9^\circ$  the value of Manipulability increases in the vertical reach range of 20mm to 28mm and the maximum value of Manipulability is 0.80389 which occurs at  $x=240$ mm,  $y=240$ mm and vertical reach of 28mm as shown in fig.4 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

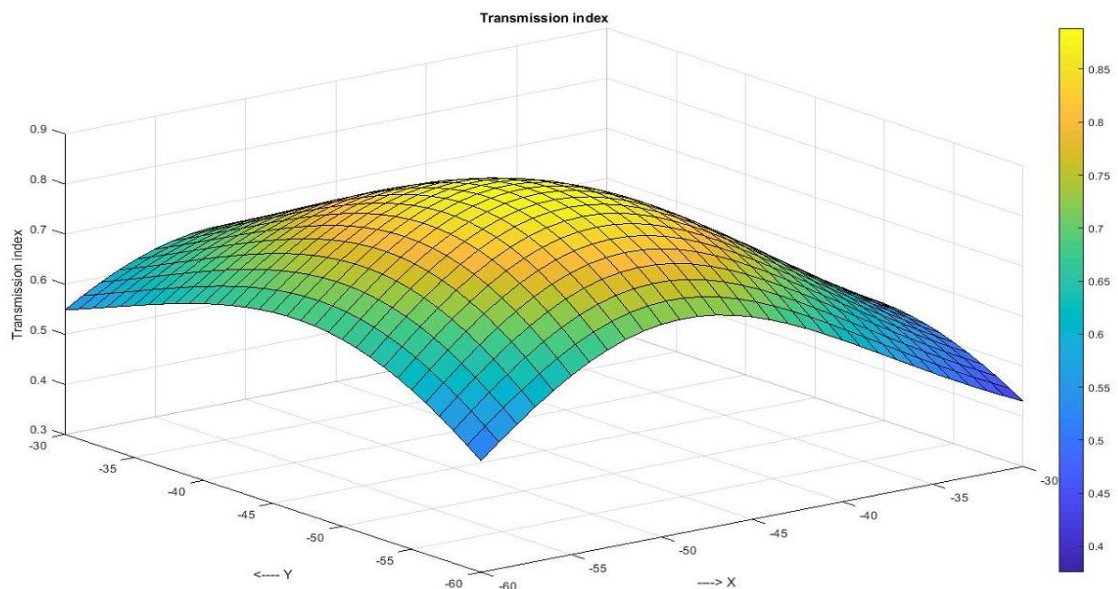


Fig.4.5 for  $R_b=70, R_p=30$  &  $z=20$ , Twist angle  $\Psi=10^\circ$ , Tilt angles  $\theta=10^\circ, \dot{\theta}=10^\circ$

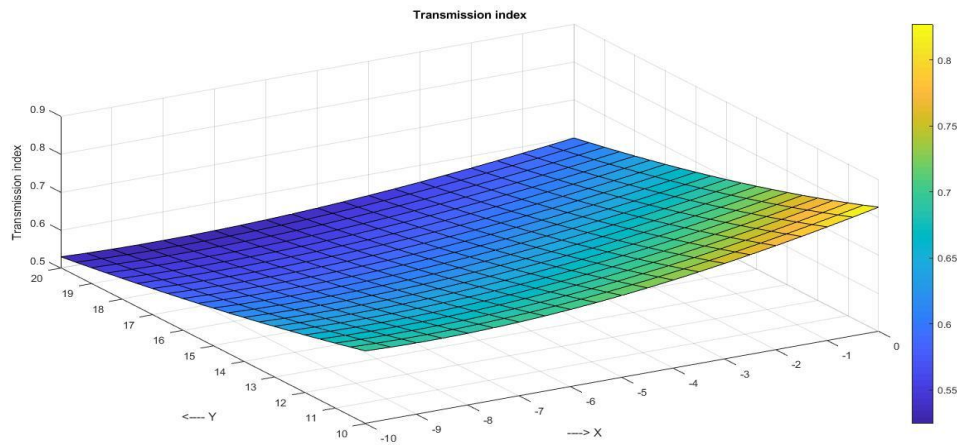


Fig.6 for  $R_b=30, R_p=20$  &  $z=35$ , Twist angle  $\Psi=8^\circ$ , Tilt angles  $\theta=9^\circ, \dot{\theta}=9^\circ$

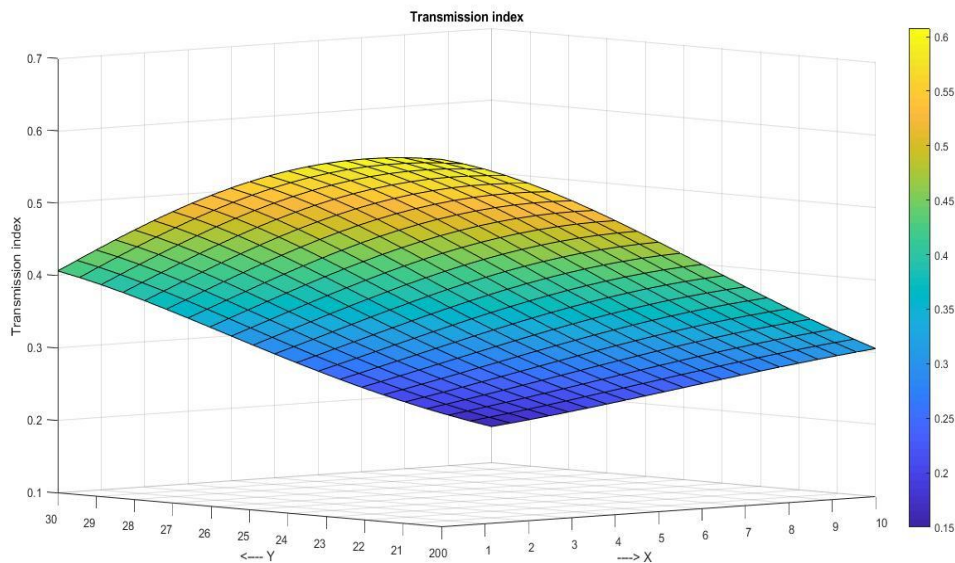


Fig.7 for  $R_b=60, R_p=30$  &  $z=28$ , Twist angle  $\Psi=9^\circ$ , Tilt angles  $\theta=7^\circ, \dot{\theta}=9^\circ$

The variation of Transmission index for three different structures of the 8-UPSspatial parallel manipulator is shown in fig.5 to 7. It is observed that, in every structure the Transmission index is increasing with increase in vertical reach of the moving platform. Optimum values of the Transmission index are obtained when the ratio of radius of base platform to moving platform is greater than or equal to 2.0.

For structure 1 specified by base platform and moving platform radius as 70mm and 30mm respectively with twist angle ( $\Psi$ )= $10^\circ$ , tilt angle about Y axis( $\theta$ )= $10^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $10^\circ$  the value of transmission index increases in the vertical reach range of 5mm to 10mm and the maximum value of Transmission index is 0.88853 which occurs at  $x=-49\text{mm}$ ,  $y=-49\text{mm}$  and vertical reach of 10mm. At other values of vertical reach for this structure the obtained results show that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 2 specified by base platform and moving platform radius as 60mm and 30mm respectively with twist angle ( $\Psi$ )= $8^\circ$ , tilt angle about Y axis( $\theta$ )= $9^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $9^\circ$  the value of transmission index increases in the vertical reach range of 30mm to 35mm and the maximum value of Transmission index is 0.82721 which occurs at  $x=0\text{mm}$ ,  $y=10\text{mm}$  and vertical reach of 35mm. At other values of vertical reach for this structure the obtained results show that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 3 specified by base platform and moving platform radius as 30mm and 20mm respectively with twist angle ( $\Psi$ )= $9^\circ$ , tilt angle about Y axis( $\theta$ )= $7^\circ$  and tilt angle about X axis( $\dot{\theta}$ )= $9^\circ$  the value of transmission index increases in the vertical reach range of 20mm to 28mm and the maximum value of Transmission index is

0.60782 which occurs at  $x=0\text{mm}$ ,  $y=20\text{mm}$  and vertical reach of 28mm. At other values of vertical reach for this structure the obtained results show that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

### III.CONCLUSIONS

Present work has been focused on investigation of 8-UPS spatial parallel manipulators kinematics and performance evaluation based on the indices: Transmission index and Manipulability. The following conclusions were drawn from the results.

- Using the kinematic model the loop closure equations are developed for 8-UPS manipulators.
- Jacobian matrices are derived for both 8-UPS and manipulators.
- All the performance measures considered in the present work are functions of the jacobian matrix.
- Graphs showing the variation of the performance measures in a specified workspace are developed.
- Maximum value of Transmission index i.e. is 0.88853 is obtained for the 8-UPS manipulator with structure having base radius to moving platform radius of 2.3, the orientation of the moving platform specified by  $\Psi=10^0$ ,  $\theta=10^0$  &  $\phi=10^0$  and the position in the workspace:  $x=10\text{mm}$ ,  $y=10\text{mm}$  and  $z=20\text{mm}$ . This posture will allow minimum deformation and maximum load carrying capacity.
- Maximum value of Manipulability index i.e. 0.80389 is obtained for the 8-UPS manipulator with structure having base radius to moving platform radius of 1.5, the orientation of the moving platform specified by  $\Psi=9^0$ ,  $\theta=7^0$  &  $\phi=9^0$  and the position in the workspace:  $x=240\text{mm}$ ,  $y=240\text{mm}$  and  $z=28\text{mm}$ . This structure with the configuration specified above is useful for arbitrarily changing the position and orientation of the end effector. The structure with configurations having manipulability index value nearer to 1.0 are generally used in fast packaging applications.

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