On Nwg-Normal and Nwg-Regular spaces

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Abstract— *In this paper we introduced a class of space called Nwg-Normal space and analyzed few of its properties. We have observed some preservation theorem. Further Nwg-regular space is defined and investigated some of its characteristics.*

Keywords— *Nano topological space, Nano continuous function, Nwg-continuous function, Nwg-irresolute function, Nwg-closed map, Nwg Normal and Nwg Regular spaces.*

I. INTRODUCTION

In 1971 Viglino[1] generalized Normal space by defining Semi Normal space. Singal and Arya [2] introduced almost normal space and proved that a space is Normal if and only if it is both a semi Normal and an almost Normal space. In 1987, Ganulay et al [3] generalized the usual notion of regularity and normality by replacing closed with g-closed set and obtained g- regularity and g- notmality. Ganster et al [4] studied semi g-regularity and semi g-normality. Paul and Bhattacharya [5] discussed P-normal space. Maheswari and Prasad [6,7] introduced and investigated some properties of S-normal spaces and S-regular spaces. Many authors have investigated different form of normality [8-17]

Lellis Thivagar [18] observed that universal set, lower and upper approximations of a subset of universal set forms a basis for topology and named it as Nano topology because of its small size. He [19] also studied the Nano continuity in terms of Nano interior and Nano closure of a set. Dhanis Arul Mary A et al [20] studied some Characterizations of mildly nano gb-normal spaces. S.B.Shalini [21] introduced Nano Generalized b Regular Spaces and Nano Generalized b Normal Spaces in Nano Topological Spaces.

Nagaveni and Bhuvaneswari [22-24] applied the concept of weakly generalization in Nano topology and introduced Nwg-closed sets and continued their work to Nwg-continuous and Nwg-closed map.

Throughout this paper $(U, \tau_R(X))$ is a Nano Topological space with respect to X Where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. $(V, \tau_{R'}(Y))$ is a Nano Topological space with respect to Y Where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence classes of V by R'. In this paper we introduced Nwg-Normal and Nwg regular spaces and investigated some of these properties.

II. PRELIMINARIES

This section is to recall some definitions and properties which are useful in this study.

Definition:2.1[18]

Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another.The pair (U,R) is said to be the approximation space. Let $X \subseteq U$,

1. The Lower approximation of X with respect to R is the set of all objects,which can be for certain classified as X with respect to R and is defined by $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$.
   Where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects,which can be possibly classified as X with respect to R and is defined by $u_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects,which can be classified neither as X nor as not- X with respect to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Definition: 2.3[18]

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

Definition:2.4 [18]

If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B=\{U, \; L_R(X), \; U_R(X)\}$ is the basis for $\tau_R(X)$.

Definition:2.5 [18]

If $(U,\tau_R(X))$ is a Nano Topological space with respect to X where $X\subseteq U$ and if $A\subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A).Nano interior is the largest Nano open subset of A.

Definition:2.6 [18]

The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A).It is the smallest Nano closed set containing A.

Definition:2.7 [18]

Let $(U,\tau_R(X))$ be a Nano Topological space with respect to X and $A\subseteq U$. Then A is said to be

(i) Nano semi- open if $A\subseteq NCl(NInt(A))$

(ii) Nano pre- open if $A\subseteq NInt(NCl(A))$

(iii)Nanoα -open if $A\subseteq NInt(NCl(NInt(A)))$

(iv) Nano Regular open if $A = NInt(NCl(A))$

Definition:2.8 [21]

Let $(U,\tau_R(X))$ be a Nano Topological space. A subset A of $(U,\tau_R(X))$ is called Nano weakly generalized closed (briefly Nwg-closed) set if $Ncl(N\operatorname{int}(A))\subseteq V$ where $A\subseteq V$ and V is Nano open. The complement of Nano weakly generalized closed set is Nano weakly generalized open set.

Definition: 2.9

The map $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is called

(i) Nano-continuous [19] on U if the inverse image of every Nano closed set in V is Nano closed in U.

(ii) Nano closed (open) on U if the image of every Nano closed set in U is Nano closed(open) set in V.

(iii) Nwg-continuous [22] on U if the inverse image of every Nano closed set in V is Nano weakly generalized closed in U.

(iv) Nwg-irresolute on [22] U if the inverse image of every Nwg closed set in V is Nano weakly generalized closed in U.

(vii) Nwg-closed (open) [23] on U if the image of every Nano-closed set in U is Nwg-closed(open) set in V.

Definition: 2.10

A Nano Topological Space $(U,\tau_R(X))$ is said to be Nano-normal space if for any pair of disjoint nano closed sets A and B, there exists disjoint Nano-open sets M and N such that $A\subset M$ *and* $B\subset N$.

Definition: 2.11

A Nano topological space $(U,\tau_R(X))$ is said to be Nano-regular space, if for each Nano closed set F and each point $x\notin F$, there exists disjoint Nano open sets G and H such that $x\in G$ and $F\subset H$.

III NWG-NORMAL SPACE

In this section we define a new space called Nwg-normal space and study some of its properties.

Definition :3.1

A Nano Topological Space $(U,\tau_R(X))$ is said to be Nwg-normal space if for any pair of disjoint nano closed sets A and B, there exists disjoint Nwg-open sets M and N such that $A\subset M$ *and* $B\subset N$.

Theorem: 3.2

Every Nano normal space is Nwg-normal space.

Proof: Let $(U,\tau_R(X))$ is Nano normal space and A and B are two disjoint pair of nano closed sets. Since $(U,\tau_R(X))$ is Nano normal there exists disjoint Nano open sets M and N such that $A\subset M$ *and* $B\subset N$. since every nano open set is Nwg-open M and N are Nwg open sets.Henece $(U,\tau_R(X))$ is Nwg-normal space.

Remark:3.3

The following example shows that the converse of the above theorem need not be true.

Example: 3.4

Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,b\},\{c\},\{d\}\}$ and $X=\{a,b\}$. Then the Nano topology is $\tau_R(X)=\{U,\phi,\{a,b\}\}$. Then

$(U,\tau_R(X))$ is Nwg-normal space but not Nano normal space.

Remark:3.5

Let U be the universe, $X \subseteq U$, and if $U_R(X) = X$ and $L_R(X) = \phi$ then $(U,\tau_R(X))$ is Nwg-normal space.

Remark:3.6

Let U be the universe, $X \subseteq U$, and if $U_R(X) = X$ and $L_R(X) = U$ then $(U,\tau_R(X))$ is not Nwg-normal space.

Theorem: 3.7

If A Nano topological space U is Nwg-normal then for every pair of nano open M and N whose union is U, there exist Nwg closed sets A and B such that $A \subset M$, $B \subset N$ and $A \cup B = U$.

Proof:

Let M and N be a pair of Nano open sets in a Nwg-normal space U such that $M \cup N = U$. Then $U - M$, $U - N$ are disjoint Nano closed sets. Since U is Nwg-normal space, there exists two Nwg-open sets M_1 and N_1 such that $U - M \subset M_1$ and $U - N \subset N_1$. Let $A = U - M_1$ and $B = U - N_1$. Then A and B are Nwg-closed sets such that $A \subset M$, $B \subset N$ and $A \cup B = U$.

Theorem 3.8

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nano continuous injective, Nwg open function and U is Nano normal space then V is Nwg-normal.

Proof:

Let E and F be disjoint Nano closed set in V Since f is Nano continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint Nano closed in U. Now U is Nano normal space, there exist disjoint Nano open sets G and H such that $f^{-1}(E) \subset G$ and $f^{-1}(F) \subset H$. That is $E \subset f(G)$ and $F \subset f(H)$ since f is Nwg-open function, $f(G)$, $f(H)$ are Nwg-open sets in V and f is injective, $f(G) \cap f(H) = f(G \cap H)$ $= f(\phi) = \phi$. Therefore V is Nwg-normal space.

Remark:3.9

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nano continuous injective, Nano open function and U is Nano normal space then V is Nwg-normal.

Theorem 3.10

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nwg-continuous, Nano closed injection and V is a Nano normal space then U is Nwg-normal.

Proof:

Let E and F be disjoint Nano closed set in U Since f is Nano closed injection, $f(E)$ and $f(F)$ are disjoint Nano closed in V. Now V is Nano normal space, there exist disjoint Nano open sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Nwg-continuous function, $f^{-1}(G)$ and $f^{-1}(H)$ are Nwg-open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore U is Nwg-normal space.

Remark 3.11

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nano-continuous, Nano closed injection and V is a Nano normal space then U is Nwg-normal.

Theorem 3.12

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nwg irresolute, Nano closed injection and V is a Nwg-normal then U is Nwg-normal.

Proof:

Let E and F be disjoint Nano closed set in U. Since f is Nano closed injection, $f(E)$ and $f(F)$ are disjoint Nano closed in V. Now V is Nwg-normal space, there exist disjoint Nwg-open sets G and H such that $f(E) \subset G$ and $f(G) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$, Since f is Nwg irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are Nwg open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore U is Nwg-normal space.

Remark 3.13

If $f:(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is Nwg irresolute, Nano closed injection and V is a Nwg-normal then U is Nwg-normal.

IV NWG-REGULAR SPACE

In this section we introduce a new space called Nwg-regular space and investigate some of its characteristics.

Definition 4.1.

A Nano topological space $(U, \tau_R(X))$ is said to be Nwg- regular space, if for each Nano closed set F and each point $x \notin F$, there exists disjoint Nwg open sets G and H such that $x \in G$ and $F \subset H$.

Theorem 4.2.

Every Nano regular space is Nwg- regular space.

Proof:

Let F be a Nano closed set and $x \notin F$ be a point of a Nano regular space $(U, \tau_R(X))$. Since U is Nano regular space there exist two disjoint Nano open sets G and H such that $x \in G$ and $F \subset H$. since every Nano open set is Nwg open set, G and H are Nwg open sets such that $x \in G$ and $F \subset H$. Hence $(U, \tau_R(X))$ is Nwg-regular space.

Remark 4.3.

Every Nwg-regular space need not be Nano regular as given in the following example.

Example 4.4

Let $U = \{a, b, c, d\}$ with $U / R = \{\{a, c\}, \{b\}, \{d\}\}$ and $X = \{a, c\}$. Then the Nano topology is $\tau_R(X) = \{U, \phi, \{a, c\}\}$. Then $(U, \tau_R(X))$ is Nwg-regular space but not Nano regular space.

Theorem 4.5.

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nano continuous bijective, Nwg open function and U is a Nano regular space then V is Nwg regular.

Proof:

Let F be a Nano closed set in V and $y \notin F$ Let $y = f(x)$ for some $x \in U$. Since f is Nano continuous, $f^{-1}(F)$ is Nano closed in U such that $x \notin f^{-1}(F)$. Now U is Nano regular space, there exist disjoint Nano open sets G and H such that $x \in G$ and $f^{-1}(F) \subset H$. That is $y = f(x) \in f(G)$ and $F \in f(H)$. Since f is Nwg open function, $f(G)$ and $f(H)$ are Nwg open sets in V. $f(G) \cap f(H) = f(G \cap H) = \phi$. Therefore V is Nwg regular space.

Remark 4.6

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nano continuous bijective, Nwg open function and U is a Nano regular space then V is Nwg regular.

Theorem 4.7

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nwg continuous, Nano closed injection and V is a Nano regular space then U is Nwg regular.

Proof:

Let F be a Nano closed set in U and $x \notin F$. Since f is Nano closed injection, $f(F)$ is Nano closed set in V such that $f(x) \notin f(F)$. Now V is Nano regular, there exist disjoint Nano open sets G and H such that $f(x) \subset G$ and $f(F) \subset H$. Thus $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Nwg continuous function $f^{-1}(G)$ and $f^{-1}(H)$ are Nwg open sets in U. $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence U is Nwg regular space.

Remark4.8

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nano continuous, Nano closed injection and V is a Nano regular space then U is Nwg regular.

Theorem 4.9

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nwg irresolute, Nano closed injection and V is a Nwg-regular then U is also Nwg regular.

Proof:

Let F be a Nano closed set in U and $x \notin F$. $f(F)$ is Nano closed set in V since f is Nano closed. But V is Nwg regular, hence there exist disjoint Nwg open sets G and H *in* V such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Where $f^{-1}(G)$ and $f^{-1}(H)$ are Nwg open sets in U since f is Nwg irresolute. And $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint sets. Hence U is Nwg regular space.

Remark 4.10

If $f : (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is Nwg irresolute, Nano closed injection and V is a Nano-regular then U is also Nwg regular.

References

[1] G. Viglino, Semi-normal and C-compact spaces, Duke J. Math., 38(1971), 57-61.

[2] M. K. Singal and S. P. Arya, On almost normal and almost completely regular spaces, Glasnik Mat., 5(5)(1970), 141-152.

[3] G A. Ganguly and R. S. Chandel, Some results on general topology, J. Indian Acad. Math. 9(2)(1987), 87–91.

[4] M. Ganster, S. Jafari and G. B. Navalagi, On semi-g-regular and semi-g-normal spaces, Demonstratio Math 2002, 35(2), 415-421.

[5] Paul and Bhattacharyya, On p-normal spaces, Soochow J. Math., 21(3)(1995), 273-289.

[6] S. N. Maheshwari and R. Prasad, On s-regular spaces, Glasnik, Mat.Ser.III, 10(1975), 347-350.

[7] S. N. Maheshwari and R. Prasad, On s-normal spaces, Bull. Math. Soc. Sci. Math. R. S. Roumanie (N. S.), 22(68)(1978), 27-29.

[8] S.P. Arya and M.P. Bhamini, A generalization of normal spaces, Mat. Vesnik 35 (1983), 1-10.

[9] S. P. Arya and T. M. Nour, Characterizations of s-normal spaces, Indian J. Pure Appl. Math., 21(1990), 717-719.

[10] M. T M E Abd El-Monsef, A N Geaisa, and R A Mahmoud. β-regular spaces. Proceedings of the Mathematical and Physical Society of Egypt, 60:47–52, 1985.

[11] A. K. Das , Δ -normal spaces and decompositions of normality, Applied General Topology, Volume 10, No. 2, 2009, pp. 197-206.

[12] Erdal Ekici, On γ-normal spaces, Bull. Math. Soc. Sci. Math. Roumanie Tome 50(98) No. 3, 2007, 259–272

[13] M. Ganster, S. Jafari and G. B. Navalagi, On semi-g-regular and semi-g-normal spaces, Demonstratio Math 2002, 35(2), 415-421.

[14] J. K. Kohli and A. K. Das, New Normality Axioms and Decompositions of Normality, Glasnik Mat, Vol. 37(57), 2002, 165 – 175.

[15] J.K. Kohli and D. Singh, Weak normality properties and factorizations of normality, Acta. Math. Hungar. 110 (2006), no. 1-2, 67–80.

[16] J.Mack, Countable paracompactness and weak normality properties, Trans. Amer.Math. Soc. 148 (1970), 265–272.

[17] T. Noiri, Semi-normal spaces and some functions, Acta Math. Hungar., 65(3)(1994), 305-311.

[18] M.Lellis Thivagar, Carmel Richard, "on Nano forms of weakly open sets", International Journal of Mathematics and Statistics Invention, volume 1, Issue 1, August 2013, pp-31-37.

[19] M.Lellis Thivagar, Carmel Richard, "on Nano continuity , Mathematical theory and Modelling , Voi.3, No 7, 2013, 32-37.

[20] Dhanis Arul Mary A, Arockiarani I, Characterizations of mildly nano gb-normal spaces, International journal of Applied Research, 1 (9), 587-591, 2015.

[21] S.B.Shalini, G. Sindhu and K.Indirani, On Nano Generalized b Regular Spaces and Nano Generalized b Normal Spaces in Nano Topological Spaces, Annals of Pure and Applied Mathematics, Vol. 14, No. 2, 2017, 225-229.

[22] N.Nagaveni and M.Bhuvaneswari"On Nano weakly generalized closed sets", International Journal of Pure and Applied Mathematics ,Volume 106, No.7 , 2016,129-137.

[23] N.Nagaveni and M.Bhuvaneswari, On Nano weakly generalized continuous functions", International Journal of Emerging Research in Management &Technology, Volume-6, Issue-4, 2017,95-100.

[24] M. Bhuvaneswari & N. Nagaveni, A Weaker form of a closed map in Nano Topological space, International Journal of Innovation in Science and Mathematics Volume 5, Issue 3, 2017, 77-82.