A Fuzzy Inventory Model Based on Different Defuzzification Techniques of Various Fuzzy Numbers

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Abstract -- *In this paper a fuzzy inventory model with constant demand and allowable shortage is considered. Our aim is to minimize the total cost in the proposed inventory model. To achieve this various defuzzification techniques are used for Triangular fuzzy numbers, Trapezoidal fuzzy numbers, Pentagonal fuzzy numbers and hexagonal fuzzy numbers. Numerical example is given to analyze the various defuzzification techniques. Finally sensitivity analysis is given.*

Keywords -- *defuzzification, pentagonal and hexagonal fuzzy numbers*

1 INTRODUCTION

An inventory consists of raw materials, work in progress or finished goods. Effective inventory control is essential for manufacturing organizations for many reasons. The objective of many inventory problems is to deal with minimization of carrying cost. Thus it is essential to determine a suitable inventory model to meet the future demand. The most widely used inventory model is the Economic Order Quantity (EOQ) model, in which the successive operations are classified as supply and demand. The first quantitative treatment of inventory was the simple EOQ model. In EOQ model we can find the order quantity so as to minimize the total cost of inventory. Thus EOQ model is very useful in real life situation. This model was developed by Harris et al [1]. Wilson [2] showed interest in developing EOQ model in academics and industries. Hadely et al[3] analyzed many inventory models.

Uncertainties and imprecision is inherent in real inventory problems. This can be approached by probabilistic methods in earlier days. But there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solution, fuzzy set theory is considered as more convenient than probability theory.

Dutta and Kumar [4] developed fuzzy inventory model without shortages using fuzzy trapezoidal number and used Signed distance method for defuzzification. Jaggi et al [5] developed fuzzy inventory model with deterioration where demand was taken as time-varying. Kumar and Rajput [6] developed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Huey –Ming Lee and Jing-shing [7] developed Economic order quantity in Fuzzy sense for Inventory without backorder Fuzzy sets. Harish Nagar and Priyanka Surana [8] worked on fuzzy inventory model for deteriorating items and use the parameters in pentagonal fuzzy number. Dutta and Pavan Kumar[9] also developed inventory model without shortages by considering holding cost, ordering cost and demand in fuzzy environment. Jershan Chiang-Shing Yao., and Huey-Ming Lee [10] deal fuzzy inventory with backorder. S.K. Indirajit Sinha, P.N. Samantha and U.K. Mishra [11] deals fuzzy inventory model with shortages under fully backlogged using signed distance method. G.Michael Rosario and R.M.Rajalakshmi [12] investigated an inventory model with allowable shortage using different fuzzy numbers and defuzzified using signed distance method.

This paper is organized as follows: section 2 presented some basic definition of various fuzzy numbers. Section 3 explains the assumptions and notations used in the proposed inventory model. Section 4 includes the proposed inventory model in crisp sense. Section 5 explains various defuzzification techniques in triangular , Trapezoidal, pentagonal and hexagonal fuzzy numbers. Section 6 provides numerical example and the results are analyzed using defuzzification methods and concluded in Section 7.

2 PRELIMINARIES

A. Definition (Fuzzy set)

Let X be a nonempty set. Then a fuzzy set A in X (ie., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0,1]$. Such a function μ_A is called the membership function and for each $x \in X$, $\mu_{\tilde{A}}(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A.

In other words, A fuzzy set $\tilde{A} = \{(x, \mu_A(x)) / x \varepsilon X\}$ where $\mu_A : X \to [0,1]$. F(X) denotes the collection of all fuzzy sets in X, called the fuzzy power set of X.

B. Definition (Fuzzy number)

A fuzzy subset of real number with membership function $\mu_{\tilde{A}} : R \to [0,1]$ is called a fuzzy number if

1. $\tilde{A}$ is normal, that is there exists an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$;
2. $\tilde{A}$ is convex that is $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)\ \forall$ $x_1, x_2 \in R\ \&\ \lambda \in [0,1]$
3. $\mu_{\tilde{A}}$ is upper semi continuous;
4. Supp($\tilde{A}$) is bounded, here supp($\tilde{A}$) = supp{$x \in R : \mu_{\tilde{A}}(x) > 0$}.

C. Definition (Triangular fuzzy number)

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$ is triangular if its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & when \quad a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & when \quad a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

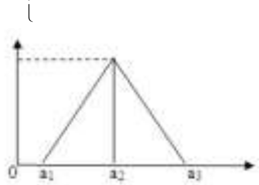

Fig. (1)

D. Definition (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A}$ =(a,b,c,d) is represented with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x - a}{b - a}, & when \quad a \leq x \leq b \\ 1 & when \quad b \leq x \leq c \\ R(x) = \dfrac{d - x}{d - c} & when \quad c \leq x \leq d \\ 0 & otherwise \end{cases}$$

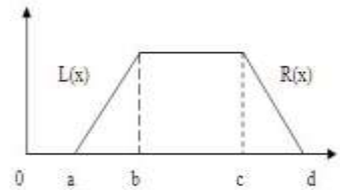

Fig.(2)

E. Definition (Pentagonal Fuzy Number)

A pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}(x)$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a}{b - a}, & a \leq x \leq b \\ \dfrac{x - b}{c - b}, & b \leq x \leq c \\ 1 & , x = c \\ \dfrac{d - x}{d - c}, & c \leq x \leq d \\ \dfrac{e - x}{e - d}, & d \leq x \leq e \\ 0 & , otherwise \end{cases}$$

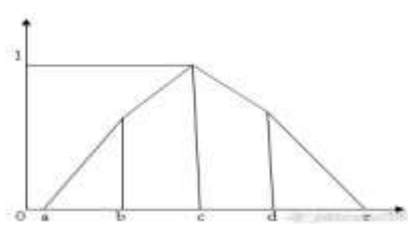

Fig.(3)

F. Definition (Hexagonal Fuzzy Number)

A hexagonal fuzzy number $\tilde{A} = (a, b, c, d, e, f)$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{2(b-a)} & , a \le x \le b \\ \dfrac{1}{2} + \dfrac{x-b}{2(c-d)} & , b \le x \le c \\ 1 & , c \le x \le d \\ 1 - \dfrac{x-d}{2(e-d)} & , d \le x \le e \\ \dfrac{f-x}{2(f-c)} & , e \le x \le f \\ 0 & , otherwise \end{cases}$$

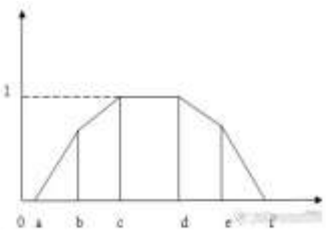

Fig.(4)

G. Definition (Arithmetic Operations in TFN)

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers then the arithmetical operations are defined as

1. $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$
3. $\tilde{A} \Theta \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = \left(\dfrac{a_1}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_1} \right)$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3), \alpha \ge 0 \\ (\alpha a_3, \alpha a_2, \alpha a_1) \alpha < 0 \end{cases}$

similarly we can define arithmetic operations for trapezoidal, pentagonal and hexagonal fuzzy numbers.

H. Definition (Graded Mean Integration Representation)

Graded mean integration representation for defuzzifying

(i) Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is defined as $d_F(\tilde{A}) = \dfrac{a_1 + 4a_2 + a_3}{6}$.

(ii) Trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is defined as $d_F(\tilde{A}) = \dfrac{a_1 + 2a_2 + 2a_3 + a_4}{6}$.

(iii) Pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ is defined as $d_F(\tilde{A}) = \dfrac{a_1 + 3a_2 + 4a_3 + 3a_4 + a_5}{12}$.

(iv) Hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ is defined as $d_F(\tilde{A}) = \dfrac{a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6}{12}$.

I. Definition (Signed distance method)

Let $\tilde{A}$ be a fuzzy set defined on R. then the signed distance of $\tilde{A}$ is defined as

$d_F(\tilde{A}, 0) = \dfrac{A_1 + 2A_2 + A_3}{4}$ for defuzzifying triangular fuzzy number $\tilde{A} = (A_1, A_2, A_3)$.

$d_F(\tilde{A}, 0) = \dfrac{A_1 + A_2 + A_3 + A_4}{4}$ for defuzzifying trapezoidal fuzzy number $\tilde{A} = (A_1, A_2, A_3, A_4)$

The signed distance method for defuzzifying pentagonal fuzzy number is defined as $d_F(\tilde{A}, 0) = \dfrac{A_1 + 2A_2 + 2A_3 + 2A_4 + A_5}{8}$.

The signed distance method for defuzzifying hexagonal fuzzy number is defined as $d_F(\tilde{A}, 0) = \dfrac{A_1 + 2A_2 + A_3 + A_4 + 2A_5 + A_6}{8}$.

3 ASSUMPTIONS AND NOTATIONS

A. Assumptions

The following assumptions are considered in this paper

1. Total demand is constant
2. Time plan is constant
3. Shortage is allowed

B. Notations

T - length of plan.
a- storage cost per unit quantity per unit time.
$\tilde{a}$ - fuzzy storage cost
b- backorder cost per unit quantity per unit time.
$\tilde{b}$ - fuzzy backorder cost
c- setup cost per unit quantity per unit time.
$\tilde{c}$ - fuzzy setup cost
d- total demand over the planning time period

[0,T].
t_q - length of a cycle.
q - order quantity per cycle.
s -shortage quantity per cycle

4 PROPOSED INVENTORY MODEL IN CRISP SENSE

The crisp total cost on the planning period [0,T] is given by

$$F(q,s) = \frac{a(q-s)^2 T}{2q} + \frac{bs^2 T}{2q} + \frac{cd}{q} \quad (0< s < q) \text{ [7]} \qquad (1)$$

The crisp optimal solution is

Optimal order quantity $q^* = \sqrt{\frac{2(a+b)cd}{abT}}$ ___ (2)

Optimal backorder quantity

$$s^* = \sqrt{\frac{2acd}{b(a+b)T}} \qquad (3)$$

Minimum total cost $F(q^*, s^*) = \sqrt{\frac{2abcdT}{a+b}}$ ____(4)

5 PROPOSED INVENTORY MODEL IN FUZZY SENSE

A. *Triangular fuzzy number (graded mean)*

Now consider triangular fuzzy number to the parameter a , b , c . let $\tilde{a} = (a_1, a_2, a_3)$ $\tilde{b} = (b_1, b_2, b_3)$ and $\tilde{c} = (c_1, c_2, c_3)$

$$T\tilde{C} = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b} s^2 T}{2q} + \frac{cd}{q}.$$

$$T\tilde{C} = \frac{(a_1, a_2, a_3) \otimes (q-s)^2 \otimes T}{2q} \oplus \frac{(b_1, b_2, b_3) \otimes s^2 \otimes T}{2q} \oplus \frac{(c_1, c_2, c_3) \otimes d}{q}$$

$$= \left[\frac{a_1(q-s)^2 T}{2q}, \frac{a_2(q-s)^2 T}{2q}, \frac{a_3(q-s)^2 T}{2q}\right] \oplus \left[\frac{b_1 s^2 T}{2q}, \frac{b_2 s^2 T}{2q}, \frac{b_3 s^2 T}{2q}\right] \oplus \left[\frac{c_1 d}{q}, \frac{c_2 d}{q}, \frac{c_3 d}{q}\right]$$

$$= \left(\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 d}{q}, \frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 d}{q}, \frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 d}{q}\right)$$

$$= (TC_1, TC_2, TC_3)$$

Where $TC_i = \frac{a_i(q-s)^2 T}{2q} + \frac{b_i s^2 T}{2q} + \frac{c_i d}{q}$, i = 1,2,3

$$\frac{d}{dq}(TC_i) = \frac{a_i T}{2}\left(\frac{q\,2(q-s) - (q-s)^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2},$$

i =1,2,3

$$= \frac{a_i T}{2}\left(1 - \frac{s^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2}, \text{ i = 1,2,3}$$

$$\frac{d^2}{dq^2}(TC_i) = \frac{a_i T}{2}\left(1 + \frac{2s^2}{q^3}\right) + \frac{b_i s^2 T}{q^3} + \frac{2c_i d}{q^3}, \text{ i = 1,2,3} \qquad (5)$$

To defuzzify the value of total cost, we use Graded mean integration method

$$d_F T\tilde{C} = \frac{1}{6}(TC_1 + 4TC_2 + TC_3)$$

$$= \frac{1}{6}\left(\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 d}{q} + 4\left(\frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 d}{q}\right) + \frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 d}{q}\right)$$

$$= \frac{1}{6}\left(\frac{(a_1 + 4a_2 + a_3)(q-s)^2 T}{2q} + \frac{(b_1 + 4b_2 + b_3)s^2 T}{2q} + \frac{(c_1 + 4c_2 + c_3)d}{q}\right)$$

$$= F(q,s)$$

To find the optimum value differentiate the above equation and equate to zero

$$F'(q,s) = \frac{(a_1 + 4a_2 + a_3)T}{2}\left(1 - \frac{s^2}{q^2}\right) - \frac{(b_1 + 4b_2 + b_3)s^2 T}{2q^2} - \frac{(c_1 + 4c_2 + c_3)d}{q^2} = 0 \qquad (6)$$

After simplifying equation (6) we get

$$q^* = \sqrt{\frac{2d(c_1 + 4c_2 + c_3)(a_1 + 4a_2 + a_3 + b_1 + 4b_2 + b_3)}{T(a_1 + 4a_2 + a_3)(b_1 + 4b_2 + b_3)}}$$

$$s^* = \sqrt{\frac{2d(c_1 + 4c_2 + c_3)(a_1 + 4a_2 + a_3)}{(b_1 + 4b_2 + b_3)(a_1 + 4a_2 + a_3 + b_1 + 4b_2 + b_3)T}}$$

Total cost =

$$\frac{1}{6}\left(\frac{(a_1 + 4a_2 + a_3)(q-s)^2 T}{2q} + \frac{(b_1 + 4b_2 + b_3)s^2 T}{2q} + \frac{(c_1 + 4c_2 + c_3)d}{q}\right)$$

Substitute in (5), we get $F''(q,s) > 0$ for all values .Therefore the above obtained solution is optimal.

B. Triangular Fuzzy number (Signed distance method)

Defuzzification using signed distance method gives the optimum solution as

$$q^* = \sqrt{\frac{2d(c_1+2c_2+c_3)(a_1+2a_2+a_3+b_1+2b_2+b_3)}{T(a_1+2a_2+a_3)(b_1+2b_2+b_3)}}$$

$$s^* = \sqrt{\frac{2d(c_1+4c_2+c_3)(a_1+4a_2+a_3)}{(b_1+4b_2+b_3)(a_1+4a_2+a_3+b_1+4b_2+b_3)T}}$$

Total cost =

$$\frac{1}{4}\left(\frac{(a_1+2a_2+a_3)(q-s)^2T}{2q}+\frac{(b_1+2b_2+b_3)s^2T}{2q}+\frac{(c_1+2c_2+c_3)d}{q}\right)$$

C. Trapezoidal Fuzzy number (Graded mean)

Now consider trapezoidal fuzzy number to the parameter a , b , c . let $\tilde{a}=(a_1,a_2,a_3,a_4)$ $\tilde{b}=(b_1,b_2,b_3,b_4)$ and $\tilde{c}=(c_1,c_2,c_3,c_4)$.

$$T\tilde{C}=\frac{\tilde{a}(q-s)^2T}{2q}+\frac{\tilde{b}s^2T}{2q}+\frac{\tilde{c}d}{q}.$$

$$T\tilde{C}=\frac{(a_1,a_2,a_3,a_4)\otimes(q-s)^2\otimes T}{2q}\oplus\frac{(b_1,b_2,b_3,b_4)\otimes s^2\otimes T}{2q}\oplus\frac{(c_1,c_2,c_3,c_4)\otimes d}{q}$$

$$=\left[\frac{a_1(q-s)^2T}{2q},\frac{a_2(q-s)^2T}{2q},\frac{a_3(q-s)^2T}{2q},\frac{a_4(q-s)^2T}{2q}\right]\oplus\left[\frac{b_1s^2T}{2q},\frac{b_2s^2T}{2q},\frac{b_3s^2T}{2q},\frac{b_4s^2T}{2q}\right]\oplus\left[\frac{c_1d}{q},\frac{c_2d}{q},\frac{c_3d}{q},\frac{c_4d}{q}\right]$$

$$=\begin{pmatrix}\frac{a_1(q-s)^2T}{2q}+\frac{b_1s^2T}{2q}+\frac{c_1d}{q},\\ \frac{a_2(q-s)^2T}{2q}+\frac{b_2s^2T}{2q}+\frac{c_2d}{q},\\ \frac{a_3(q-s)^2T}{2q}+\frac{b_3s^2T}{2q}+\frac{c_3d}{q},\\ \frac{a_4(q-s)^2T}{2q}+\frac{b_4s^2T}{2q}+\frac{c_4d}{q}\end{pmatrix}=(TC_1,TC_2,TC_3,TC_4)$$

Where $TC_i=\frac{a_i(q-s)^2T}{2q}+\frac{b_is^2T}{2q}+\frac{c_id}{q}$, i = 1,2,3,4

$$\frac{d}{dq}(TC_i)=\frac{a_iT}{2}\left(\frac{q\,2(q-s)-(q-s)^2}{q^2}\right)-\frac{b_is^2T}{2q^2}-\frac{c_id}{q^2},$$

i =1,2,3,4

$$=\frac{a_iT}{2}\left(1-\frac{s^2}{q^2}\right)-\frac{b_is^2T}{2q^2}-\frac{c_id}{q^2},\ i=1,2,3,4$$

$$\frac{d^2}{dq^2}(TC_i)=\frac{a_iT}{2}\left(1+\frac{2s^2}{q^3}\right)+\frac{b_is^2T}{q^3}+\frac{2c_id}{q^3},$$

i =1, 2 ,3,4 _______ (7)

To defuzzify the value of total cost, we use Graded mean integration

$$d_F\,T\tilde{C}=\frac{1}{6}\left(TC_1+2TC_2+2TC_3+TC_4\right)$$

$$=\frac{1}{6}\begin{pmatrix}\frac{a_1(q-s)^2T}{2q}+\frac{b_1s^2T}{2q}+\frac{c_1d}{q}+\\ 2\left(\frac{a_2(q-s)^2T}{2q}+\frac{b_2s^2T}{2q}+\frac{c_2d}{q}\right)+\\ 2\left(\frac{a_3(q-s)^2T}{2q}+\frac{b_3s^2T}{2q}+\frac{c_3d}{q}\right)+\\ \frac{a_4(q-s)^2T}{2q}+\frac{b_4s^2T}{2q}+\frac{c_4d}{q}\end{pmatrix}$$

$$=\frac{1}{6}\begin{pmatrix}\frac{(a_1+2a_2+2a_3+a_4)(q-s)^2T}{2q}+\\ \frac{(b_1+2b_2+2b_3+b_4)s^2T}{2q}+\\ \frac{(c_1+2c_2+2c_3+c_4)d}{q}\end{pmatrix}=F(q,s)$$

To find the optimum value differentiate the above equation and equate to zero, we get

F'(q,s) =

$$\frac{(a_1+2a_2+2a_3+a_4)T}{2}\left(1-\frac{s^2}{q^2}\right)-\frac{(b_1+2b_2+2b_3+b_4)s^2T}{2q^2}-\frac{(c_1+2c_2+2c_3+c_4)d}{q^2}=0\ (8)$$

After simplifying we get the optimum solution for q , s and TC as

$$q^*=\sqrt{\frac{2d(c_1+2c_2+2c_3+c_4)(a_1+2a_2+2a_3+a_4+b_1+2b_2+2b_3+b_4)}{T(a_1+2a_2+2a_3+a_4)(b_1+2b_2+2b_3+b_4)}}$$

$$s^* = \sqrt{\frac{\dfrac{2d(c_1+2c_2+2c_3+c_4)}{(a_1+2a_2+2a_3+a_4)}}{\dfrac{(b_1+2b_2+2b_3+b_4)}{(a_1+2a_2+2a_3+a_4+b_1+2b_2+2b_3+b_4)T}}}$$

$$\text{Total cost} = \frac{1}{6}\left(\begin{array}{l}\dfrac{(a_1+2a_2+2a_3+a_4)(q-s)^2 T}{2q}+\\ \dfrac{(b_1+2b_2+2b_3+b_4)s^2 T}{2q}\\ +\dfrac{(c_1+2c_2+2c_3+c_4)d}{q}\end{array}\right)$$

Sub in (7), we get F'' (q,s) > 0 for all values. Therefore the above obtained solution is optimal.

D. Trapezoidal Fuzzy number (Signed distance method)

Defuzzification using signed distance method

$$d_F T\tilde{C} = \frac{1}{4}\left(TC_1 + TC_2 + TC_3 + TC_4\right)$$

gives the optimum solution as

$$q^* = \sqrt{\frac{2d(c_1+c_2+c_3+c_4)(a_1+a_2+a_3+a_4+b_1+b_2+b_3+b_4)}{T(a_1+a_2+a_3+a_4)(b_1+b_2+b_3+b_4)}}$$

$$s^* = \sqrt{\frac{2d(c_1+c_2+c_3+c_4)(a_1+a_2+a_3+a_4)}{(b_1+b_2+b_3+b_4)(a_1+a_2+a_3+a_4+b_1+b_2+b_3+b_4)T}}$$

Total cost =

$$\frac{1}{4}\left(\begin{array}{l}\dfrac{(a_1+a_2+a_3+a_4)(q-s)^2 T}{2q}+\dfrac{(b_1+b_2+b_3+b_4)s^2 T}{2q}+\\ \dfrac{(c_1+c_2+c_3+c_4)d}{q}\end{array}\right)$$

E. Pentagonal Fuzzy number(Graded mean)

Consider the parameter a , b , c as pentagonal fuzzy number . let $\tilde{a} = (a_1, a_2, a_3, a_4, a_5)$ $\tilde{b} = (b_1, b_2, b_3, b_4, b_5)$ and $\tilde{c} = (c_1, c_2, c_3, c_4, c_5)$.

$$T\tilde{C} = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b}s^2 T}{2q} + \frac{cd}{q}.$$

$$T\tilde{C} = \frac{(a_1,a_2,a_3,a_4,a_5)\otimes(q-s)^2\otimes T}{2q} \oplus \frac{(b_1,b_2,b_3,b_4,b_5)\otimes s^2\otimes T}{2q} \oplus \frac{(c_1,c_2,c_3,c_4,c_5)\otimes d}{q}$$

.

$$= \left[\frac{a_1(q-s)^2 T}{2q}, \frac{a_2(q-s)^2 T}{2q}, \frac{a_3(q-s)^2 T}{2q}, \frac{a_4(q-s)^2 T}{2q}, \frac{a_5(q-s)^2 T}{2q}\right]$$
$$\oplus \left[\frac{b_1 s^2 T}{2q}, \frac{b_2 s^2 T}{2q}, \frac{b_3 s^2 T}{2q}, \frac{b_4 s^2 T}{2q}, \frac{b_5 s^2 T}{2q}\right]$$
$$\oplus \left[\frac{c_1 d}{q}, \frac{c_2 d}{q}, \frac{c_3 d}{q}, \frac{c_4 d}{q}, \frac{c_5 d}{q}\right]$$

$$= \left(\begin{array}{l}\dfrac{a_1(q-s)^2 T}{2q}+\dfrac{b_1 s^2 T}{2q}+\dfrac{c_1 d}{q}, \dfrac{a_2(q-s)^2 T}{2q}+\dfrac{b_2 s^2 T}{2q}+\dfrac{c_2 d}{q},\\ \dfrac{a_3(q-s)^2 T}{2q}+\dfrac{b_3 s^2 T}{2q}+\dfrac{c_3 d}{q},\\ \dfrac{a_4(q-s)^2 T}{2q}+\dfrac{b_4 s^2 T}{2q}+\dfrac{c_4 d}{q}, \dfrac{a_5(q-s)^2 T}{2q}+\dfrac{b_5 s^2 T}{2q}+\dfrac{c_5 d}{q}\end{array}\right)$$

= (TC_1 , TC_2 , TC_3, TC_4, TC_5)

Where $TC_i = \dfrac{a_i(q-s)^2 T}{2q} + \dfrac{b_i s^2 T}{2q} + \dfrac{c_i d}{q}$, i = 1,2,3,4,5

$$\frac{d}{dq}(TC_i) = \frac{a_i T}{2}\left(\frac{q\,2(q-s)-(q-s)^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2},$$

i =1,2,3,4,5

$$= \frac{a_i T}{2}\left(1-\frac{s^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2}, i = 1,2,3,4,5$$

$$\frac{d^2}{dq^2}(TC_i) = \frac{a_i T}{2}\left(1+\frac{2s^2}{q^3}\right) + \frac{b_i s^2 T}{q^3} + \frac{2c_i d}{q^3},$$

i = 1, 2 ,3,4,5____________ (9)

To defuzzify the value of total cost, we use Graded mean integration

$$d_F T\tilde{C} = \frac{1}{12}\left(TC_1 + 3TC_2 + 4TC_3 + 3TC_4 + TC_5\right)$$

$$= \frac{1}{12}\left(\begin{array}{l}\dfrac{a_1(q-s)^2 T}{2q}+\dfrac{b_1 s^2 T}{2q}+\dfrac{c_1 d}{q}+3\left(\dfrac{a_2(q-s)^2 T}{2q}+\dfrac{b_2 s^2 T}{2q}+\dfrac{c_2 d}{q}\right)\\ +4\left(\dfrac{a_3(q-s)^2 T}{2q}+\dfrac{b_3 s^2 T}{2q}+\dfrac{c_3 d}{q}\right)+\\ 3\left(\dfrac{a_4(q-s)^2 T}{2q}+\dfrac{b_4 s^2 T}{2q}+\dfrac{c_4 d}{q}\right)+\dfrac{a_5(q-s)^2 T}{2q}+\dfrac{b_5 s^2 T}{2q}+\dfrac{c_5 d}{q}\end{array}\right)$$

$$= \frac{1}{12}\left(\frac{(a_1+3a_2+4a_3+3a_4+a_5)(q-s)^2 T}{2q} + \frac{(b_1+3b_2+4b_3+3b_4+b_5)s^2 T}{2q} + \frac{(c_1+3c_2+4c_3+3c_4+c_5)d}{q} \right) = F(q,s)$$

To find the optimum value differentiate the above equation and equate to zero

F' (q,s) =

$$\frac{(a_1+3a_2+4a_3+3a_4+a_5)T}{2}\left(1-\frac{s^2}{q^2}\right) - \frac{(b_1+3b_2+4b_3+3b_4+b_5)s^2 T}{2q^2} - \frac{(c_1+3c_2+4c_3+3c_4+c_5)d}{q^2} = 0 \qquad (10)$$

After simplifying we get the optimum solution for q , s and TC as

$$q^* = \sqrt{\frac{2d(c_1+3c_2+4c_3+3c_4+c_5)(a_1+3a_2+4a_3+3a_4+a_5+b_1+3b_2+4b_3+3b_4+b_5)}{T(a_1+3a_2+4a_3+3a_4+a_5)(b_1+3b_2+4b_3+3b_4+b_5)}}$$

$$s^* = \sqrt{\frac{2d(c_1+3c_2+4c_3+3c_4+c_5)(a_1+3a_2+4a_3+3a_4+a_5)}{(b_1+3b_2+4b_3+3b_4+b_5)(a_1+3a_2+4a_3+3a_4+a_5+b_1+3b_2+4b_3+3b_4+b_5)T}}$$

Total cost =

$$\frac{1}{12}\left(\frac{(a_1+3a_2+4a_3+3a_4+a_5)(q-s)^2 T}{2q} + \frac{(b_1+3b_2+4b_3+3b_4+b_5)s^2 T}{2q} + \frac{(c_1+3c_2+4c_3+3c_4+c_5)d}{q} \right)$$

Sub in (9), we get F'' (q,s) > 0 for all values .Therefore the above obtained solution is optimal.

F. Pentagonal Fuzzy number(Signed distance method)

The signed distance method for defuzzifying pentagonal fuzzy number is

$$d_F T\tilde{C} = \frac{1}{8}(TC_1 + 2TC_2 + 2TC_3 + 2TC_4 + TC_5)$$

The optimum value is

$$q^* = \sqrt{\frac{2d(c_1+2c_2+2c_3+2c_4+c_5)(a_1+2a_2+2a_3+2a_4+a_5+b_1+2b_2+2b_3+2b_4+b_5)}{T(a_1+2a_2+2a_3+2a_4+a_5)(b_1+2b_2+2b_3+2b_4+b_5)}}$$

$$s^* = \sqrt{\frac{2d(c_1+2c_2+2c_3+2c_4+c_5)(a_1+2a_2+2a_3+2a_4+a_5)}{(b_1+2b_2+2b_3+2b_4+b_5)(a_1+2a_2+2a_3+2a_4+a_5+b_1+2b_2+2b_3+2b_4+b_5)T}}$$

Total cost =

$$\frac{1}{8}\left(\frac{(a_1+2a_2+2a_3+2a_4+a_5)(q-s)^2 T}{2q} + \frac{(b_1+2b_2+2b_3+2b_4+b_5)s^2 T}{2q} + \frac{(c_1+2c_2+2c_3+2c_4+c_5)d}{q} \right)$$

G. Hexagonal fuzzy number (Graded mean)

Consider the parameter a , b , c as hexagonal fuzzy number . let $\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6)$

$\tilde{b} = (b_1, b_2, b_3, b_4, b_5, b_6)$ and $\tilde{c} = (c_1, c_2, c_3, c_4, c_5, c_6)$.

$$T\tilde{C} = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b}s^2 T}{2q} + \frac{cd}{q} .$$

$$T\tilde{C} = \frac{(a_1,a_2,a_3,a_4,a_5,a_6)\otimes(q-s)^2\otimes T}{2q} \oplus \frac{(b_1,b_2,b_3,b_4,b_5,b_6)\otimes s^2\otimes T}{2q} \oplus \frac{(c_1,c_2,c_3,c_4,c_5,c_6)\otimes d}{q} .$$

$$= \left[\frac{a_1(q-s)^2 T}{2q}, \frac{a_2(q-s)^2 T}{2q}, \frac{a_3(q-s)^2 T}{2q}, \frac{a_4(q-s)^2 T}{2q}, \frac{a_5(q-s)^2 T}{2q}, \frac{a_6(q-s)^2 T}{2q}\right]$$

$$\oplus \left[\frac{b_1 s^2 T}{2q}, \frac{b_2 s^2 T}{2q}, \frac{b_3 s^2 T}{2q}, \frac{b_4 s^2 T}{2q}, \frac{b_5 s^2 T}{2q}, \frac{b_6 s^2 T}{2q}\right]$$

$$\oplus \left[\frac{c_1 d}{q}, \frac{c_2 d}{q}, \frac{c_3 d}{q}, \frac{c_4 d}{q}, \frac{c_5 d}{q}, \frac{c_6 d}{q}\right]$$

$$= \begin{pmatrix} \frac{a_1(q-s)^2T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 d}{q}, \frac{a_2(q-s)^2T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 d}{q}, \\ \frac{a_3(q-s)^2T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 d}{q}, \frac{a_4(q-s)^2T}{2q} + \frac{b_4 s^2 T}{2q} + \frac{c_4 d}{q}, \\ \frac{a_5(q-s)^2T}{2q} + \frac{b_5 s^2 T}{2q} + \frac{c_5 d}{q}, \frac{a_6(q-s)^2T}{2q} + \frac{b_6 s^2 T}{2q} + \frac{c_6 d}{q} \end{pmatrix}$$

$= (TC_1 , TC_2 , TC_3, TC_4, TC_5,TC_6)$

Where $TC_i = \frac{a_i(q-s)^2 T}{2q} + \frac{b_i s^2 T}{2q} + \frac{c_i d}{q}$, $i = 1,2,3,4,5,6$

$$\frac{d}{dq}(TC_i) = \frac{a_i T}{2}\left(\frac{q\,2(q-s) - (q-s)^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2},$$

$i = 1,2,3,4,5,6$

$$= \frac{a_i T}{2}\left(1 - \frac{s^2}{q^2}\right) - \frac{b_i s^2 T}{2q^2} - \frac{c_i d}{q^2}, \ i = 1,2,3,4,5,6$$

$$\frac{d^2}{dq^2}(TC_i) = \frac{a_i T}{2}\left(1 + \frac{2s^2}{q^3}\right) + \frac{b_i s^2 T}{q^3} + \frac{2c_i d}{q^3},$$

$i = 1, 2 ,3,4,5,6$______ (11)

To defuzzify the value of total cost, we use Graded mean integration

$$d_F T\tilde{C} = \frac{1}{12}(TC_1 + 3TC_2 + 2TC_3 + 2TC_4 + 3TC_5 + TC_6)$$

$$= \frac{1}{12}\begin{pmatrix} \frac{a_1(q-s)^2T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 d}{q} \\ + 3\left(\frac{a_2(q-s)^2T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 d}{q}\right) \\ + 2\left(\frac{a_3(q-s)^2T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 d}{q}\right) \\ + 2\left(\frac{a_4(q-s)^2T}{2q} + \frac{b_4 s^2 T}{2q} + \frac{c_4 d}{q}\right) \\ + 3\left(\frac{a_5(q-s)^2T}{2q} + \frac{b_5 s^2 T}{2q} + \frac{c_5 d}{q}\right) \\ + \frac{a_6(q-s)^2T}{2q} + \frac{b_6 s^2 T}{2q} + \frac{c_6 d}{q} \end{pmatrix}$$

$$= \frac{1}{12}\begin{pmatrix} \frac{(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6)(q-s)^2 T}{2q} + \\ \frac{(b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)s^2 T}{2q} \\ + \frac{(c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6)d}{q} \end{pmatrix}$$

$= F(q,s)$

To find the optimum value differentiate the above equation and equate to zero

$$F'(q,s) = \frac{(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6)T}{2}\left(1 - \frac{s^2}{q^2}\right) - \frac{(b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)s^2 T}{2q^2} - \frac{(c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6)d}{q^2} = 0$$

After simplifying we get the optimum solution for q , s and TC as

follows
$$q^* = \sqrt{\frac{2d(c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6)(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6 + b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)}{T(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6)(b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)}}$$

$$s^* = \sqrt{\frac{\frac{2d(c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6)}{(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6)}}{\frac{(b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)}{(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6 + b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)T}}}$$

Total cost

$$= \frac{1}{12}\begin{pmatrix} \frac{(a_1 + 3a_2 + 2a_3 + 2a_4 + 3a_5 + a_6)(q-s)^2 T}{2q} + \\ \frac{(b_1 + 3b_2 + 2b_3 + 2b_4 + 3b_5 + b_6)s^2 T}{2q} \\ + \frac{(c_1 + 3c_2 + 2c_3 + 2c_4 + 3c_5 + c_6)d}{q} \end{pmatrix}$$

Sub in (9), we get $F''(q,s) > 0$ for all values. Therefore the above obtained solution is optimal.

H. Hexagonal fuzzy number (Signed distance method)

The signed distance method for defuzzifying hexagonal fuzzy number is

$$d_F \tilde{TC} = \frac{1}{8}(TC_1 + 2TC_2 + TC_3 + TC_4 + 2TC_5 + TC_6)$$

The optimum solution is

$$q^* = \sqrt{\frac{\dfrac{2d(c_1+2c_2+c_3+c_4+2c_5+c_6)}{(a_1+2a_2+a_3+a_4+2a_5+a_6+b_1+2b_2+b_3+b_4+2b_5+b_6)}}{\dfrac{T(a_1+2a_2+a_3+a_4+2a_5+a_6)}{(b_1+2b_2+b_3+b_4+2b_5+b_6)}}}$$

$$s^* = \sqrt{\frac{\dfrac{2d(c_1+2c_2+c_3+c_4+2c_5+c_6)}{(a_1+2a_2+a_3+a_4+2a_5+a_6)}}{\dfrac{(b_1+2b_2+b_3+b_4+2b_5+b_6)}{(a_1+2a_2+a_3+a_4+a_5+a_6+b_1+2b_2+b_3+b_4+2b_5+b_6)T}}}$$

$$\text{Total cost} = \frac{1}{8}\left(\frac{(a_1+2a_2+a_3+a_4+2a_5+a_6)(q-s)^2T}{2q} + \frac{(b_1+2b_2+b_3+b_4+2b_5+b_6)s^2T}{2q} + \frac{(c_1+2c_2+c_3+c_4+2c_5+c_6)d}{q}\right)$$

6.ILLUSTRATION

A. Crisp model

T =5,a =4 , b = 14 , c = 20

S.No	Demand (d)	q*	s*	TC
1	1000	50.709	11.268	788.81
2	1200	55.549	12.344	864.0999
3	1400	60	13.333	933.3333
4	1600	64.142	14.253	997.7753
5	1800	68.033	15.118	1058.300

B. Fuzzy model: **Triangular Fuzzy number**

T=5 a = (2,4,6), b = (7,14,21,28) , c = (10,20,30)

S. No	Demand (d)	q*	s*	TC	
				Graded mean	Signed distance
1	1000	50.71	11.27	788.81	788.81
2	1200	55.55	12.34	864.10	864.10
3	1400	59.99	13.33	933.33	933.33
4	1600	64.14	14.26	997.78	997.78
5	1800	68.03	15.12	1058.31	1058.31

Trapezoidal Fuzzy number: T=5 a = (2,4,6,8), b = (7,14,21,28,35) , c = (10,20,30,40)

S.No	Demand (d)	q*	s*	TC	
				Graded mean	Signed distance
1	1000	50.71	11.27	986.01	986.01
2	1200	55.55	12.34	1080.12	1080.13
3	1400	59.99	13.33	1166.67	1166.67
4	1600	64.14	14.25	1247.22	1247.22
5	1800	68.03	15.12	1322.88	1322.89

Pentagonal Fuzzy number: T = 5, a = (2,4,6,8,10), b = (7,14,21,28,35); C=(10,20,30,40,50)

S.No	Demand (d)	q*	s*	TC	
				Graded mean	Signed distance
1	1000	50.71	11.27	1183.22	1183.22
2	1200	55.55	12.34	1296.15	1296.15
3	1400	59.99	13.33	1400.00	1400.00
4	1600	64.14	14.25	1496.66	1496.66
5	1800	68.03	15.12	1587.46	1587.46

Hexagonal Fuzzy number: T=5, a=(2,4,6,8,10,12) b = (7,14,21,28,35,42), C =(10,20,30,40,50,60)

S.No	Demand (d)	q*	s*	TC	
				Graded mean	Signed distance
1	1000	50.71	11.27	1380.42	1380.42
2	1200	55.55	12.34	1512.18	1512.18
3	1400	59.99	13.33	1633.33	1633.33
4	1600	64.14	14.25	1746.11	1746.11
5	1800	68.03	15.12	1852.04	1852.04

7 CONCLUSIONS

From the table we observe that out of all the fuzzy numbers, triangular fuzzy number gives the optimum solution. We get the minimum total cost when we defuzzify the triangular fuzzy number using graded mean integration method and signed distance method. The trapezoidal fuzzy number gives the minimum total cost next to triangular fuzzy number. Hexagonal fuzzy number gives maximum total cost when compared to other fuzzy numbers. Also the defuzzification using graded mean method and signed distance method gives the same optimum total cost in all types of fuzzy numbers. The order quantity and shortage cost are same in all the cases.

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