Fuzzy $\gamma*$ Generalized Continuous Mappings

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Abstract: *In this paper we have introduced fuzzy $\gamma*$ generalized continuous mappings and investigated some of their properties.*

Keywords: *fuzzy topology, fuzzy $\gamma*T_{1/2}$ space, fuzzy $\gamma*_cT_{1/2}$ space, fuzzy $\gamma*_pT_{1/2}$ space, fuzzy $\gamma*$ generalized continuous mappings.*

1. Introduction

The concept of fuzzy set and fuzzy set operations were introduced by Zadeh [10]. The fuzzy topological space using the concept of fuzzy sets was introduced by Chang [3]. In this paper we have introduced fuzzy $\gamma*$ generalized continuous mappings and investigated some of their properties.

2. Preliminaries

Definition 2.1: [10] Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \rightarrow [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of member of element x in a fuzzy set A, for each $x \in X$. It is clear that A is determined by the set of tuples of $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2.2: [10] Let A and B be two fuzzy sets $A = \{(x, \mu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x)) : x \in X\}$.Then, their union $A \vee B$, intersection $A \wedge B$ and complement A^c are also fuzzy sets with membership functions defined as follows :

(a) $\mu_{A^c}(x) = 1-\mu_A(x), \forall x \in X$,

(b) $\mu_{A\vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$,

(c) $\mu_{A\wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$.

Further,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x), \forall x \in X$,

(b) $A = B$ if and only if $\mu_A(x) = \mu_B(x), \forall x \in X$.

Definition 2.3: [3] A family τ of fuzzy sets is called fuzzy topology (FT) for X if it satisfy the three axioms:

(a) $\bar{0}, \bar{1} \in \tau$

(b) $\forall$ A, B $\in \tau \Rightarrow$ A $\wedge$ B $\in \tau$

(c) $\forall \left(A_j\right)_{j \in J} \in \tau \Rightarrow \vee_{j \in J} A_j \in \tau$

The pair (X, τ) is called a fuzzy topological space (FTS). The elements of τ are called fuzzy open sets in X and their respective complements are called fuzzy closed sets of (X, τ).

Definition 2.4: [6] A fuzzy set A in a FTS (X, τ) is said to be a

(a) fuzzy γ closed set (FγCS) if cl(int(A)) $\wedge$ int(cl(A)) $\leq$ A

(b) fuzzy γ open set (FγOS) if A $\leq$ int(cl(A)) $\vee$ cl(int(A))

Definition 2.5: [6] Let A be a fuzzy set in a FTS X. Then we define the γ interior and γ closure as

γcl(A) = $\wedge$ { B : B $\geq$ A, B is a fuzzy γ closed set in X}

γint(A) = $\vee$ {B : B $\leq$ A, B is a fuzzy γ open set in X}.

Properties 2.6: [6] Let A be a fuzzy set in a FTS X. Then

$\gamma cl(A^c) = (\gamma int(A))^c$

$\gamma\, int(A^c) = (\gamma cl(A))^c$

Definition 2.7: [7] A fuzzy set A is quasi-coincident with a fuzzy set B, denoted by A_qB, if there exists x $\in$ X such that A(x)+B(x) > 1.

Definition 2.8: [7] If A and B are not quasi-coincident then we write $A_{\bar{q}}B$ and A $\leq$ B $\Leftrightarrow$ $A_{\bar{q}}(1-B)$.

Definition 2.9: [9] A fuzzy point p̃ in a set X is also a fuzzy set with membership function:

$$\mu_{\tilde{p}}(x) = \begin{cases} r, & \text{for } x = y \\ 0, & \text{for } x \neq y \end{cases}$$

where x $\in$ X and 0 < r $\leq$ 1, y is called the support of p̃ and r the value of p̃. We denote this fuzzy point by x_r or p̃. A fuzzy point x_r is said to be belonged to a fuzzy subset $\tilde{A}$ in X, denoted by $x_r \in \tilde{A}$ if and only if r $\leq$ $\mu_{\tilde{A}}(x)$.

Definition 2.10:[5] An fuzzy set A of a FTS (X, τ) is said to be a fuzzy γ* generalized closed set (Fγ*GCS) if cl(int(A)) $\wedge$ int(cl(A)) $\leq$ U, whenever A $\leq$ U and U is a fuzzy open set in X.

The complement A^c of a Fγ*GCS A in a FTS (X, τ) is called fuzzy γ* generalized open set (Fγ*GOS) in X.

The family of all Fγ*GOSs of a FTS (X, τ) is denoted by Fγ*GO(X).

Definition 2.11: [8] Let f be a function from a FTS (X, τ_1) into a FTS (Y, τ_2). The map f is said to be fuzzy continuous if every U $\in$ τ_2, f^{-1} (U) $\in \tau_1$.

3. Fuzzy γ* Generalized Continuous Mappings

In this section we have introduced fuzzy $\gamma*$ generalized continuous mappings and investigated some of their properties.

Definition 3.1: A mapping f: (X, τ_1) → (Y, τ_2) is called a fuzzy $\gamma*$ generalized continuous (Fγ*G continuous) mapping if f^{-1}(V) is a Fγ*GCS in (X, τ_1) for every FCS V of (Y, τ_2).

Example 3.2: Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1} and τ_2 = {$\bar{0}$, $\bar{1}$, G_2} are FTs on X and Y respectively, where G_1 = ⟨x, $(0.5_a, 0.5_b)$⟩ and G_2 = ⟨y, $(0.6_u, 0.6_v)$⟩. Then (X, τ_1) and (Y, τ_2) are FTSs. Define a mapping f: (X, τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. The fuzzy set G_2^c = ⟨y, $(0.4_u, 0.4_v)$⟩ is a FCS in Y. Then $f^{-1}(G_2^c)$ = ⟨x, $(0.4_a, 0.4_b)$⟩ is a Fγ*GCS in (X, τ_1) as $f^{-1}(G_2^c) \leq G_1$ and $cl(int(f^{-1}(G_2^c))) \wedge int(cl(f^{-1}(G_2^c))) = \bar{0} \leq G_1$, where G_1 is a FOS in X. Therefore f is a fuzzy γ*G continuous mapping.

Theorem 3.3: Every fuzzy continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy continuous mapping. Let V be a FCS in Y. Then f^{-1}(V) is a FCS in X. Since every FCS is a Fγ*GCS [5], f^{-1}(V) is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.4: In Example 3.2, $f^{-1}(G_2^c)$ is a fuzzy γ*G continuous mapping but not a fuzzy continuous mapping in X, as G_2^c is a FCS in Y but $f^{-1}(G_2^c)$ is not a FCS in X.

Theorem 3.5: Every fuzzy semi continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy semi continuous mapping [1]. Let V be a FCS in Y. Then f^{-1}(V) is a FSCS in X. Since every FSCS is a Fγ*GCS [5], f^{-1}(V) is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.6: In Example 3.2, $f^{-1}(G_2^c)$ is a fuzzy γ*G continuous mapping but not a fuzzy semi continuous mapping in X, as G_2^c is a FSCS in Y but $f^{-1}(G_2^c)$ is not a FSCS in X.

Theorem 3.7: Every fuzzy pre continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy pre continuous mapping [2]. Let V be a FCS in Y. Then f^{-1}(V) is a FPCS in X. Since every FPCS is a Fγ*GCS [5], f^{-1}(V) is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.8: Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are FTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.5_v)\rangle$. Then (X, τ_1) and (Y, τ_2) are FTSs. Define a mapping f: (X, τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ*G continuous mapping but not a fuzzy pre continuous mapping as $cl(int(f^{-1}(G_3^c))) = G_1^c \nleq f^{-1}(G_3^c)$.

Theorem 3.9: Let f: (X, τ_1) → (Y, τ_2) be a mapping and $f^{-1}(A)$ be a FRCS in X for every FCS A in Y. Then f is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let A be a FCS in Y and $f^{-1}(A)$ is a FRCS in X. Since every FRCS is a Fγ*GCS [5], $f^{-1}(A)$ is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.10: In Example 3.8, $f^{-1}(G_3^c)$ is a fuzzy γ*G continuous mapping but not a mapping as in Theorem 3.9.

Theorem 3.11: Every fuzzy α continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy α continuous mapping [2]. Let V be a FCS in Y. Then $f^{-1}(V)$ is a FαCS in X. Since every FαCS is a Fγ*GCS [5], $f^{-1}(V)$ is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.12: Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are FTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.6_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v)\rangle$. Then (X, τ_1) and (Y, τ_2) are FTSs. Define a mapping f: (X, τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ*G continuous mapping but not a fuzzy α continuous mapping as $cl(int(cl(f^{-1}(G_3^c)))) = G_1^c \nleq f^{-1}(G_3^c)$.

Theorem 3.13: Every fuzzy γ continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy γ continuous mapping [4]. Let V be a FCS in Y. Then $f^{-1}(V)$ is a FγCS in X. Since every FγCS is a Fγ*GCS [5], $f^{-1}(V)$ is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.14: Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are FTs on X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$ and $G_2 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_3 = \langle y, (0.6_u,$

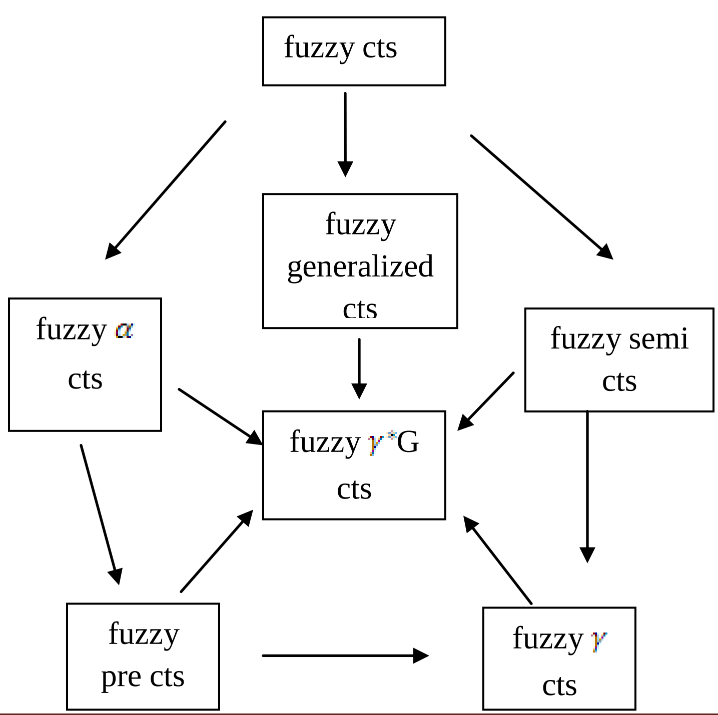
$0.6_v)\rangle$. Then (X, τ_1) and (Y, τ_2) are FTSs. Define a mapping f: (X, τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ*G continuous mapping but not a fuzzy γ continuous mapping as $cl(int(f^{-1}(G_3^c))) \wedge int(cl(f^{-1}(G_3^c))) = G_2 \nleq f^{-1}(G_3^c)$.

Theorem 3.15: Every fuzzy g continuous mapping is a fuzzy γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ_1) → (Y, τ_2) be a fuzzy g continuous mapping [3]. Let V be a FCS in Y. Then $f^{-1}(V)$ is a FGCS in X. Since every FGCS is a Fγ*GCS [5], $f^{-1}(V)$ is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Example 3.16: Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are FTs on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.5_v)\rangle$. Then (X, τ_1) and (Y, τ_2) are FTSs. Define a mapping f: (X, τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ*G continuous mapping but not a fuzzy g continuous mapping as $cl(f^{-1}(G_3^c)) = G_2^c \nleq G_1$.

The relation between various types of fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous



Theorem 3.17: A mapping f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ*G continuous mapping if and only if the inverse image of each FOS in Y is a Fγ*GOS in X.

Proof: Necessity: Let A be a FOS in Y. Then A^c is a FCS in Y. Since f is a fuzzy γ*G continuous mapping, $f^{-1}(A^c)$ is a Fγ*GCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a Fγ*GOS in X.

Sufficiency: Let A be a FCS in Y. Then A^c is a FOS in Y. By hypothesis $f^{-1}(A^c)$ is a Fγ*GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a Fγ*GCS in X. Hence f is a fuzzy γ*G continuous mapping.

Theorem 3.18: If f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ*G continuous mapping then for each FP $\mu_{\tilde{p}}(x)$ of X and each $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x)) \in A$, there exists a F$\gamma$*GOS B of X such that $\mu_{\tilde{p}}(x) \in B$ and $f(B) \leq A$.

Proof: Let $\mu_{\tilde{p}}(x)$ be a FP of X and $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is a Fγ*GOS in X such that $\mu_{\tilde{p}}(x) \in B$ and $f(B) = f(f^{-1}(A)) \leq A$.

Theorem 3.19: If f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ*G continuous mapping then for each FP $\mu_{\tilde{p}}(x)$ of X and each $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x))\ _q\ A$, there exists a F$\gamma$*GOS B of X such that $\mu_{\tilde{p}}(x))\ _q\ B$ and $f(B) \leq A$.

Proof: Let $\mu_{\tilde{p}}(x)$ be a FP of X and $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x))\ _q\ A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is a Fγ*GOS in X such that $\mu_{\tilde{p}}(x)\ _q\ B$ and $f(B) = f(f^{-1}(A)) \leq A$.

Definition 3.20: If every Fγ*GCS in (X, τ) is a FγCS in (X, τ), then the space can be called as a fuzzy γ* $T_{1/2}$ (F γ* $T_{1/2}$)space.

Example 3.21: Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a FT on X, where $G_1 = \langle x, (0.6_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.6_b)\rangle$. Then $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ is a FT on X and the space (X, τ) is a fuzzy γ*$T_{1/2}$ space.

Definition 3.22: A FTS (X, τ) is a fuzzy γ*$_cT_{1/2}$ (Fγ*$_cT_{1/2}$) space if every Fγ*GCS is a FCS in X.

Definition 3.23: A FTS (X, τ) is a fuzzy $\gamma^*{}_pT_{1/2}$(Fγ*$_pT_{1/2}$ in short) space if every Fγ*GCS is a FPCS in X.

Theorem 3.24: Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ*G continuous mapping, then

(i) f is a fuzzy γ continuous mapping if X is a Fγ*$T_{1/2}$ space

(ii) f is a fuzzy continuous mapping if X is a Fγ*$_cT_{1/2}$ space

(iii) f is a fuzzy pre continuous mapping if X is a Fγ*$_pT_{1/2}$ space

Proof: (i) Let V be a FCS in Y. Then $f^{-1}(V)$ is a Fγ*GCS in X, by hypothesis. Since X is a Fγ*$T_{1/2}$ space, $f^{-1}(V)$ is a FγCS in X. Hence f is a fuzzy γ continuous mapping.

(ii) Let V be a FCS in Y. Then $f^{-1}(V)$ is a Fγ*GCS in X, by hypothesis. Since X is a Fγ*$_c T_{1/2}$ space, $f^{-1}(V)$ is a FCS in X. Hence f is a fuzzy continuous mapping.

(iii) Let V be a FCS in Y. Then $f^{-1}(V)$ is a Fγ*GCS in X, by hypothesis. Since X is a Fγ*$_p T_{1/2}$ space, $f^{-1}(V)$ is a FPCS in X. Hence f is a fuzzy pre continuous mapping.

Theorem 3.25: Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ*G continuous mapping and g: $(Y, \tau_2) \rightarrow (Z, \tau_3)$ be a fuzzy continuous mapping then $g \circ f : (X, \tau_1) \rightarrow (Z, \tau_3)$ is a fuzzy γ*G continuous mapping.

Proof: Let V be a FCS in Z. Then $g^{-1}(V)$ is a FCS in Y, by hypothesis. Since f is a fuzzy γ*G continuous mapping, $f^{-1}(g^{-1}(V))$ is a Fγ*GCS in X. Hence $g\circ f$ is a fuzzy γ*G continuous mapping.

Theorem 3.26: The composition of two fuzzy γ*G continuous mapping is a fuzzy γ*G continuous mapping if Y is a Fγ*$_c T_{1/2}$ space.

Proof: Let V be a FCS in Z. Then $g^{-1}(V)$ is a Fγ*GCS in Y, by hypothesis. Since Y is a Fγ*$_c T_{1/2}$ space, $g^{-1}(V)$ is a FCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is a Fγ*GCS in X, by hypothesis. Hence $g\circ f$ is a fuzzy γ*G continuous mapping.

Theorem 3.27: Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then the following conditions are equivalent if X and Y are Fγ*$T_{1/2}$ spaces:

(i) f is a fuzzy γ*G continuous mapping

(ii) $f^{-1}(B)$ is a Fγ*GOS in X for each FOS B in Y

(iii) for each FP $\mu_{\tilde{p}}(x)$ in X and for every FOS B in Y such that $f(\mu_{\tilde{p}}(x)) \in B$, there exists a F$\gamma$*GOS A in X such that $\mu_{\tilde{p}}(x) \in A$ and $f(A) \leq B$.

Proof: (i) $\Leftrightarrow$ (ii) is obvious from the Theorem 3.17.

(ii) $\Rightarrow$ (iii) Let B be any FOS in Y and let $\mu_{\tilde{p}}(x) \in X$. Given $f(\mu_{\tilde{p}}(x)) \in B$. By hypothesis $f^{-1}(B)$ is a Fγ*GOS in X. Take A = $f^{-1}(B)$. Then $\mu_{\tilde{p}}(x) \in f^{-1}(B) = A$. This implies $\mu_{\tilde{p}}(x) \in A$ and $f(A) = f(f^{-1}(B)) \leq B$.

(iii) $\Rightarrow$ (ii) Let A be a FCS in Y. Then its complement, say B is a FOS in Y. Let $\mu_{\tilde{p}}(x) \in X$ and $f(\mu_{\tilde{p}}(x)) \in B$. Then there exists a F$\gamma$*GOS, say C in X such that $\mu_{\tilde{p}}(x) \in C$ and $f(C) \leq B$. Therefore $\mu_{\tilde{p}}(x) \in C \leq f^{-1}(B)$ and hence $f^{-1}(B)$ is a Fγ*GOS in X, by

Theorem 3.18. That is $f^{-1}(A^c)$ is a Fγ*GOS in X and hence $f^{-1}(A)$ is a Fγ*GCS in X. Thus f is a fuzzy γ*G continuous mapping.

Theorem 3.28: Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then the following conditions are equivalent if X and Y are Fγ*$T_{1/2}$ spaces:

(i) f is a fuzzy γ*G continuous mapping

(ii) $cl(int(f^{-1}(B))) \wedge int(cl(f^{-1}(B))) \leq f^{-1}(cl(B))$ for each FCS B in Y

(iii) $f^{-1}(int(B)) \leq cl(int(f^{-1}(B))) \vee int(cl(f^{-1}(B)))$ for each FOS B in Y

(iv) $f(int(cl(A)) \wedge cl(int(A))) \leq cl(f(A))$ for each FS A of X.

Proof: (i) $\Rightarrow$ (ii) Let B be a FCS in Y. Then $f^{-1}(B)$ is a Fγ*GCS in X. Since X is a Fγ*$T_{1/2}$ space, $f^{-1}(B)$ is a FγCS in X. Therefore $cl(int(f^{-1}(B))) \wedge int(cl(f^{-1}(B))) \leq f^{-1}(B) = f^{-1}(cl(B))$.

(ii) $\Rightarrow$ (iii) can be easily proved by taking complement in(ii).

(iii) $\Rightarrow$ (iv) Let A $\in$ X. Then B = f(A) in Y and therefore $A \leq f^{-1}(f(A)) \leq f^{-1}(B)$. Here int(f(A)) = int(B) is a FOS in Y. Then (iii) implies that $f^{-1}(int(B)) \leq cl(int(f^{-1}(int(B)))) \vee int(cl(f^{-1}(int(B)))) \leq cl(int(f^{-1}(B))) \vee int(cl(f^{-1}(B)))$. Now $(cl(int(A^c)) \vee int(cl(A^c)))^c \leq (cl(int(f^{-1}(B^c)) \vee int(cl(f^{-1}(B^c)))^c \leq (f^{-1}(int(B^c)))^c$. Therefore $int(cl(A)) \wedge cl(int(A)) \leq f^{-1}(cl(B))$. Now $f(int(cl(A)) \wedge cl(int(A))) \leq f(f^{-1}(cl(B))) \leq cl(f(A))$.

(iv) $\Rightarrow$ (i) Let B be any FCS in Y, then $f^{-1}(B)$ is a FS in X. By hypothesis $f(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B)))) \leq cl(f(f^{-1}(B))) \leq cl(B) = B$. Now $(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B)))) \leq f^{-1}(f(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B))))) \leq f^{-1}(B)$. This implies $f^{-1}(B)$ is a FγCS and hence it is a Fγ*GCS in X. Thus f is a fuzzy γ*G continuous mapping.

Theorem 3.29: A mapping f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ is a fuzzy γ*G continuous mapping if $cl(int(cl(f^{-1}(A)))) \leq f^{-1}(cl(A))$ for every FS A in Y.

Proof: Let A be a FCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A)))) \leq f^{-1}(cl(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is a FαCS and hence it is a Fγ*GCS. Thus f is a fuzzy γ*G continuous mapping.

Theorem 3.30: Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping from a FTS X into a FTS Y. Then the following conditions are equivalent if X is a Fγ*$T_{1/2}$ space:

i. f is a fuzzy γ*G continuous mapping

ii. $cl(int(f^{-1}(A))) \wedge int(cl(f^{-1}(A))) \leq f^{-1}(cl(A))$ for every FS A in Y

Proof: (i) ⇒ (ii) Let A be a FS in Y. Then cl(A) is a FCS in Y. By hypothesis, $f^{-1}(cl(A))$ is a Fγ*GCS in X. Since X is a Fγ*$T_{1/2}$ space, $f^{-1}(cl(A))$ is a FγCS in X. Therefore $cl(int(f^{-1}(cl(A)))) \wedge int(cl(f^{-1}(cl(A)))) \leq f^{-1}(cl(A))$. Now $cl(int(f^{-1}(A))) \wedge int(cl(f^{-1}(A))) \leq cl(int(f^{-1}(cl(A)))) \wedge int(cl(f^{-1}(cl(A)))) \leq f^{-1}(cl(A))$.

(ii) ⇒ (i) Let A be a FCS in Y. By hypothesis $cl(int(f^{-1}(A))) \wedge int(cl(f^{-1}(A))) \leq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is a FγCS in X and hence it is a Fγ*GCS. Thus f is a fuzzy γ*G continuous mapping.

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