Edge-magic total labeling of some graphs

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Abstract- Let G be a finite simple graph with v vertices and e edges.An edge-magic total labelling on a graph with v vertices and e edges will be defined as a one-to-one map taking the vertices and edges onto the integers 1,2,…,v+e with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge.And we study edge-magic total labeling of wheel graph.

Keywords- Magic labelling, Edge-magic total labelling, Wheel graph.

1.INTRODUCTION

In this paper we consider only finite, simple,undirected graphs.The set of vertices and edges of a graph G will be denoted by V(G) and E(G),respectively, v=│V(G)│ and e = │E(G)│.

A labelling of a graph G=(V,E) is a one-to-one mapping λ of the vertex set V(G) into the set of non-negative integers.An edge-magic total labelling on G is a one-to-one map λ from V(G) U E(G) onto the integers 1,2,…,v+e,where , v=│V(G)│ and e = │E(G)│, with the property that , given any edge (x,y),

$$\lambda(x) + \lambda(x,y) + \lambda(y) = k$$

for some constant k. It will be convenient to call λ(x) + λ(x,y) + λ(y) = k the edge sum of (x,y) , and k the (constant) magic sum of G. A graph is called edge-magic total if it admits any edge-magic total labelling.

2.WHEEL GRAPH

The wheel graph W_n is defined to be the join of K_1+C_n (i.e), the wheel graph consists of edges which joint a vertex of K_1 to every vertex of C_n.

Thus W_n contains n+1 vertices (say)

$c, v_1, v_2, v_3, \ldots, v_n$ and 2n edges (say)

$cv_1, cv_2, cv_3, \ldots, cv_n, v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1$.

THEOREM 2.1

For n ≡ 6(mod 8) , every wheel W_n has an edge-magic total labelling with the magic constant k = 5n + 2

Proof:

Label vertices and edges of the wheel W_n in the following way.

λ(c) = 2n.

$$\lambda(v_i)=\begin{cases} 3n+2-\frac{i+1}{2} & \text{for } i= 1,3,\ldots,\frac{n}{2} \\ 3n+1-\frac{i+1}{2} & \text{for } i= \frac{n+4}{2}, \frac{n+8}{2},\ldots,\frac{3n-6}{4} \\ \frac{19n+14}{8} & \text{for } i =\frac{3n+2}{4} \\ 3n-\frac{i+1}{2} & \text{for } i= \frac{3n+10}{4}, \frac{3n+18}{2},\ldots, n-3 \\ \frac{11n+2}{4} & \text{for } i=n-1 \\ n+1-\frac{i}{2} & \text{for } i=2,4,\ldots,\frac{3n-2}{4} \\ n-\frac{i}{2} & \text{for } i=\frac{3n+6}{4}, \frac{3n+14}{4},\ldots, n \end{cases}$$

$$\lambda(cv_i)=\begin{cases} \frac{i+1}{2} & \text{for } i= 1,3,\ldots,\frac{n}{2} \\ \frac{i+3}{2} & \text{for } i= \frac{n+4}{2}, \frac{n+8}{2},\ldots,\frac{3n-6}{4} \\ \frac{5n+2}{8} & \text{for } i =\frac{3n+2}{4} \\ \frac{i+5}{2} & \text{for } i= \frac{3n+10}{4}, \frac{3n+18}{2},\ldots, n-3 \\ \frac{n+6}{4} & \text{for } i=n-1 \\ 2n+1+\frac{i}{2} & \text{for } i=2,4,\ldots,\frac{3n-2}{4} \\ 2n+2+\frac{i}{2} & \text{for } i=\frac{3n+6}{4}, \frac{3n+14}{4},\ldots, n \end{cases}$$

$$\lambda(v_iv_{i+1}) = \begin{cases} n+i \text{ for } i = 1,2,3,\dots,\frac{n}{2} \\ n+1+i \text{ for } i = \frac{n+2}{2}, \frac{n+4}{2},\dots,\frac{3n-6}{4} \\ 2n-1 \text{ for } i = \frac{3n-2}{4} \\ 2n+1 \text{ for } i = \frac{3n+2}{4} \\ n+1+i \text{ for } i = \frac{3n+6}{4}, \frac{3n+10}{2},\dots,n-3 \\ \frac{7n+2}{4} \text{ for } i = n-2 \\ \frac{7n+6}{4} \text{ for } i = n-1 \end{cases}$$

$\lambda(v_n,v_1) = \frac{3n+2}{2}$

Hence, λ is an edge-magic total labelling of W_n for $n \equiv 6 \pmod 8$ with the magic constant $k = 5n+2$

Hence proved.

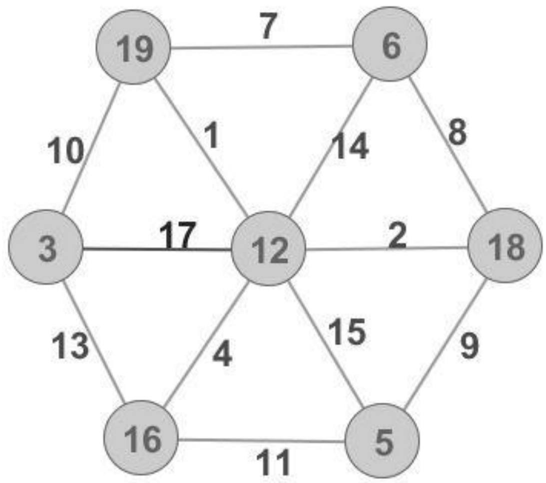
EXAMPLE 2.2

From the above theorem 2.1 we have,

$n \equiv 6 \pmod 8$.

1. $6 \equiv 6 \pmod 8$.
2. $14 \equiv 6 \pmod 8$.
3. $22 \equiv 6 \pmod 8$.
4. $30 \equiv 6 \pmod 8$ and so on.

If n = 6.Then,theorem 2.1 is satisfied.let check with this example.



W_6 with k = 32

$\lambda(c) = 2n$

$= 2(6)$

$\lambda(c) = 12$

$\lambda(v_i) = 3n+2-\frac{i+1}{2}$ for $i = 1,3,\dots,\frac{n}{2}$

Here n=6, for v_1 and v_3 exist.

$\lambda(v_1) = 19$

$\lambda(v_3) = 18$

$\lambda(v_i) = 3n+1-\frac{i+1}{2}$ for $i = \frac{n+4}{2}, \frac{n+8}{2},\dots,\frac{3n-6}{4}$

$\lambda(v_5) = 16$

$\lambda(v_i) = n+1-\frac{i}{2}$ for $i=2,4,\dots,\frac{3n-2}{4}$

$\lambda(v_2) = 6$

$\lambda(v_4) = 5$

$\lambda(v_i) = n-\frac{i}{2}$ for $i=\frac{3n+6}{4}, \frac{3n+14}{4}, \dots, n$

$\lambda(v_6) = 3$

$\lambda(cv_i) = \frac{i+1}{2}$ for $i = 1,3,\dots,\frac{n}{2}$

$\lambda(cv_1) = 1$

$\lambda(cv_3) = 2$

$\lambda(cv_i) = \frac{i+3}{2}$ for $i = \frac{n+4}{2}, \frac{n+8}{2},\dots,\frac{3n-6}{4}$

$\lambda(cv_5) = 4$

$\lambda(cv_i) = 2n+1+\frac{i}{2}$ for $i=2,4,\dots,\frac{3n-2}{4}$

$\lambda(cv_2) = 14$

$\lambda(cv_4) = 15$

$\lambda(cv_i) = 2n+2+\frac{i}{2}$ for $i=\frac{3n+6}{4}, \frac{3n+14}{4}, \dots, n$

$\lambda(cv_6) = 17$

$\lambda(v_iv_{i+1}) = n+i$ for $i = 1,2,3,\dots,\frac{n}{2}$

$\lambda(v_1v_2) = 7$

$\lambda(v_2v_3) = 8$

$\lambda(v_3v_4) = 9$

$\lambda(v_iv_{i+1}) = 2n-1$

$\lambda(v_4v_5) = 11$

$\lambda(v_iv_{i+1}) = 2n+1$

$\lambda(v_5v_6) = 13$

$\lambda(v_n v_1) = \frac{3n+2}{2}$

$\lambda(v_6 v_1) = 10$

Magic constant k = 5n + 2

k = 32

In this example we verify that their exist an edge-magic total labelling for $n \equiv 6 (\text{mod } 8)$ if n = 6.By definition of edge-magic total labelling.

We have,

λ: V(G) U E(G) → { 1,2,……,v+e } and there exist a one-to-one mapping.

Here v = 7 and e = 12

v+e = 19

Hence λ: V(G) U E(G) → { 1,2,…,19}.

EXAMPLE 2.2

If n = 14.Then the theorem 2.1 is satisfied.

W_{14} with k = 5n + 2

k = 72

Similarly an edge-magic total labelling exist for n = {6,14,22,30,38,…..}

CONCLUSION

In this paper we have studied the edge-magic total labelling of graphs mainly wheel graphs.

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