A New Approach on Anti-L-Fuzzy Soft Subhemiring of a Hemiring

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Abstract

The notion of Anti L-Fuzzy soft subhemiring in a Hemiring is introduced and its basic properties are investigated.The purpose of this study is to implement the concept of anti-L-fuzzy soft subhemiring of a hemiring in L-fuzzy soft subhemiring of a hemiring.

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Soft set, Fuzzy soft set, L-fuzzy set,L-fuzzy soft subhemiring, anti-L-fuzzy soft subhemiring, pseudo anti-L-fuzzy soft coset.

I.INTRODUCTION

Hemirings appear in a natural manner in some applications to the automata, the theory of formal languages, graph theory, design theory and combinatorial geometry .Hemirings which are regarded as a generalization of rings have been found useful in solving problems in different areas of applied Mathematics and Computer sciences. So many researchers have studied different aspects of hemirings. Zhan et al gave the concept of h-hemiregularity of hemirings and investigated some properties in terms of fuzzy theory. The concept of fuzzy subset was introduced by L.A Zadeh [16], Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among Mathematicians working in different field of Mathematics. A Semiring R is said to be a hemiring if it is additively commutative with zero. The notion of anti-fuzzy left h-ideals in Hemiringswas introduced by Akram. M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by M.Palaniappan and K.Arjunan [10]. In this paper we introduce the some theorems in anti- L-fuzzy soft subhemirings of a hemiring.

II. PERLIMINARIES

In this section we have some basic definitions which we need for our further studies.

1.1 DEFINITION

pair (F,E) is called a soft set (over U) iff F is a mapping E into the set of all subsets of the set U.

1.2 DEFINITION

Let (U,E) be a soft universe and $A \subseteq E$. Let $f(U)$be the set of all fuzzy subsets in U. A pair ($\tilde{F}$,A) is a called a fuzzy soft set over U,where $\tilde{F}$ is a mapping given by$\tilde{F}$:A→ $f(U)$.

1.3 DEFINITION

Let X be a non-empty set and L=(L,≤) be a lattice with least element 0 and greatest element 1.

1.4 DEFINITION

Let X be a non-empty set A L-fuzzy subset A of X is function A: X → L.

1.5 DEFINITION

Let (R,+,·) be a hemiring. A L-fuzzy subset (F,A) of R is said to be an anti-L-fuzzy soft subhemiring (ALFSSHR) of R if it satisfies the following conditions.

(1) $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)} \vee \mu_{(F,A)}(y_{(F,A)})\}$

(2) $\mu_{(F,A)}(x_{(F,A)} y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)} \vee \mu_{(F,A)}(y_{(F,A)})\}$

1.6 DEFINITION

Let (R,+,·) be a hemiring. An anti- L-fuzzy soft subhemiring (F,A) of R is said to be an anti-L-fuzzy soft normal subhemiring (ALFSNSHR) of R. If $(x_{(F,A)} y_{(F,A)}) = \mu_{(F,A)}(y_{(F,A)} x_{(F,A)})$ for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

1.7 DEFINITION

Let (F,A) and (G,B) be L-fuzzy soft subsets of sets G and H, respectively. The anti-product of (F,A) and (G,B) denoted by (F,A)×(G,B) is defined as(F,A)×(G,B)= $\{(x_{(F,A)} y_{(G,B)}), \mu_{(F,A)\times(G,B)}(x_{(F,A)}, y_{(G,B)}) \,/\,$ for all $x_{(F,A)}$ in G and $y_{(G,A)}$ in $H\}$ Where$\mu_{(F,A)\times(G,B)}(x_{(F,A)}, y_{(G,B)}) =$ max $\{\mu_{(F,A)}(x_{(F,A))}, \mu_{(G,B)}(y_{(G,B)})\}$

1.8 DEFINITION

Let (R,+,·) and $(R',+,\cdot)$ be any two hemirings. Let f:R→ R' be any function and (F,A) be an anti-L-fuzzy soft subhemiring in R . (G,V)be an anti-L-fuzzy soft subhemiring in f(R) = R' defined by $\mu_{(G,V)}(y_{(G,V)}) = \inf_{x\in f^{-1}(y)} \mu_{(F,A)}(x_{(F,A)})$ for all $x_{(F,A)}$ and $y_{(G,V)}$ in R'. Then (F,A) is called pre image of (G,V) under f and is denoted by $f^{-1}(V)$.

1.9 DEFINITION

Let (F,A) be an anti L-fuzzy soft subhemiring of a hemiring (R,+,·) and a in R. Then the pseudo anti-L-fuzzy soft coset $(a(F,A)^P$ is defined by $(a\mu_{(F,A)}P)(x_{(F,A)}) = P(a)\mu_{(F,A)}(x_{(F,A)})$ for every $x_{(F,A)}$ in R and for some p in P.

1.10 DEFINITION

Let (R,+,·) and $(R',+,\cdot)$ be any two hemirings then the function f:R→ R' is called a hemiring anti-homomorphism. If it satisfies the following axioms.

(i) $f(x+y) = f(y) + f(x)$
(ii) $f(xy) = f(y)f(x)$ for all x and y in R.

1.11 DEFINITION

Let (R,+,·) and $(R',+,\cdot)$ be any two hemirings. Then the function f: R→ R' be a hemiring homomorphism. If it is one-to-one and on to, then f is called a hemiring isomorphism.

1.12 DEFINITION

Let (R,+,·) and $(R',+,\cdot)$ be any two hemirings. Then the function f:R→ R' be a hemiring anti-homomorphism. If it is one-to-one and onto, then f is called a hemiring anti-isomorphism.

III. PROPERTIES OF ANTI- L-FUZZY SOFT SUBHEMIRINGS OF A HEMIRING R

In this section, we investigate some properties of Anti-L-fuzzy soft subhemiring of a hemiring.

2.1 THEOREM: Union of any two anti-L-fuzzy soft subhemiring of a hemiring R is an anti-L-Fuzzy soft subhemiring of a hemiring R.

PROOF: Let (F,A) and (F,B) be any two anti-L-fuzzy subhemirings of a hemiring R and$x_{(F,A)}$ and $y_{(G,B)}$ in R.Let$(F,A) = \{x_{(F,A)}, \mu_{(F,A)}(x_{(F,A)})/x_{(F,A)} \in R\}$and $(G,B) = \{(x_{(G,B)}), \mu_{(G,B)}(x_{(G,B)})/x_{(G,B)} \in R\}$ and also Let $(H,C) = (F,B) \cup (G,B) = \{(x_{(H,C)}, \mu_{(H,C)}(x_{(H,C)}))/x_{(H,C)} \in R\}$. Where $\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(G,B)}(x_{(G,B)}))\} = \mu_{(H,C)}(x_{(H,C)})$ it is clear that $[(F,A) \cup (G,B)](x) = (F,A)(x)$ when $(x_{(F,A)}) \neq 0 \text{ and } \mu_{(G,B)}(x_{(G,B)}) = 0$.Also $[(F,B) \cup (G,B)](x) = (G,B)(x)$ when $\mu_{(G,B)}(x_{(G,B)}) \neq 0 \text{ and } \mu_{(F,A)}(x_{(F,A)}) = 0$. It is enough to prove that $[(F,A) \cup (G,B)](x) = \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(G,B)}(x_{(G,B)})\}$.

Now $\mu_{(H,C)}(x_{(H,C)} + y_{(H,C)}) = \{\{\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)})\} \vee \{\mu_{(G,B)}(x_{(G,B)} + y_{(G,B)}\}\} \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\} \vee \{\mu_{(G,B)}(x_{(G,B)}) \vee \mu_{(G,B)}(y_{(G,B)})\} = \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(G,B)}, (x_{(G,B)})\} \vee \{\mu_{(F,A)}(y_{(F,A)}) \vee \mu_{(G,B)}(y_{(G,B)})\} = \{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\} \text{ and } \mu_{(H,C)}(x_{(H,C)}y_{(H,C)}) \leq \{\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\} \vee \{\mu_{(G,B)}(x_{(G,B)}) \vee \mu_{(G,B)}(y_{(G,B)})\} = \{\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(G,B)}(x_{(G,B)})\} \vee \{\mu_{(F,A)}(y_{(F,A)}) \vee \mu_{(G,B)}(y_{(G,B)})\}\}$ $=\{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\}$. Therefore $\mu_{(H,C)}(x_{(H,C)} + y_{(H,C)}) \leq \{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\}$, for all $x_{(F,A)}$ and $y_{(G,B)}$ in R. And, $\mu_{(H,C)}(x_{(H,C)}y_{(H,C)}) = \{\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \vee \mu_{(G,B)}(x_{(G,B)}y_{(G,B)})\} \leq \{\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\} \vee \{\mu_{(G,B)}(x_{(G,B)}) \vee \mu_{(G,B)}(y_{(G,B)})\}\} = \{\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(G,B)}(x_{(G,B)})\} \vee \{\mu_{(F,A)}(y_{(F,A)}) \vee \mu_{(G,B)}(y_{(G,B)})\}\} =$

$\{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\}$. Therefore, $\mu_{(H,C)}(x_{(H,C)}y_{(H,C)}) \leq \{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\}$, for all $x_{(F,A)}$ and $y_{(G,B)}$ in R. Therefore $(x_{(H,C)}y_{(H,C)}) \leq \{\mu_{(H,C)}(x_{(H,C)}) \vee \mu_{(H,C)}(y_{(H,C)})\}$ for all $x_{(F,A)}$ and $y_{(G,B)}$ in R..

2.2 THEOREM: The union of a family of anti-L-fuzzy soft subhemirings of hemiring R is an anti-L-fuzzy soft subhemiring of R.

PROOF: Let $\{(F,V_i): i \in I\}$ be a family of anti-L-fuzzy soft subhemirings of a hemiring R and let (F,A) $= \cup_{i\in I} Vi$. Let $x_{(F,A)}$ and $y_{(F,A)}$ in R. Then $\mu_{(F,A)}(x_{(F,A)}) + y_{(F,A)}) = \sup_{i\in I} \mu_{(F,V_i)}(x_{(F,A)} + y_{(F,A)}) \leq \sup_{i\in I} \{\mu_{(F,V_i)}(x_{(F,A)}) \vee \mu_{(F,V_i)}(x_{(F,A)})\} = \{\sup_{i\in I} \mu_{(F,V_i)}(x_{(F,A)}) \vee \sup_{i\in I} \mu_{(F,V_i)}(x_{(F,A)})\}$

$=\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(x_{(F,A)})$. So $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$ for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

And $\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) = \sup_{i\in I} \mu_{(F,V_i)}(x_{(F,A)}) \leq \sup_{i\in I} \{\mu_{(F,V_i)}(x_{(F,A)}) \vee \mu_{(F,V_i)}(y_{(F,A)})\} = \{\sup_{i\in I} \mu_{(F,V_i)}(x_{(F,A)}) \vee \sup_{i\in I} \mu_{(F,V_i)}(y_{(F,A)} = \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$. Therefore $\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$ for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Hence the union of a family of anti-L-fuzzy soft subhemirings of R is an anti-L-fuzzy soft subhemirings of R.

2.3 THEOREM: If (F,A) and (G,B) are any two anti-L-fuzzy soft subhemirings of the hemirings $R_1$and R_2 respectively then anti-product $(F,A) \times (G,B)$ is an anti-L-fuzzy soft subhemirings of $R_1 \times R_2$.

PROOF: Let (F,A) and (G,B) be two anti-L-fuzzy soft subhemirings of the hemiring R_1 and R_2 respectively. Let $x_{(F,A)_1}$ and $x_{(F,A)_2}$ be in R_1, $y_{(G,B)_1}$ and $y_{(G,B)_2}$ be in R_2. Then $(x_{(F,A)_1}, y_{(G,B)_1})$ and $(x_{(F,A)_2}, y_{(G,B)_2})$ are in $R_1 \times R_2$. Now

$$\mu_{(F,A)\times(G,B)}[(x_{(F,A)_1}, y_{(G,B)_1}) + (x_{(F,A)_2}, y_{(G,B)_2})]$$
$$\leq \{\mu_{(F,A)}(x_{(F,A)_1}) \vee \mu_{(F,A)}(x_{(F,A)_2})\} \vee$$
$$\{\mu_{(G,B)} y_{(G,B)_1} \vee \mu_{(G,B)} y_{(G,B)_2}\} =$$
$$[\mu_{(F,A)\times(G,B)}(x_{(F,A)_1}, y_{(G,B)_1} \vee \mu_{(F,A)\times(G,B)}(x_{(F,A)_2}, y_{(G,B)_2})]$$

Therefore
$\mu_{(F,A)\times(G,B)}[(x_{(F,A)_1}, y_{(G,B)_1} + (x_{(F,A)_2}, y_{(G,B)_2})] \leq [\mu_{(F,A)\times(G,B)}(x_{(F,A)_1}, y_{(G,B)_1}) \vee \mu_{(F,A)\times(G,B)}(x_{(F,A)_2}, y_{(G,B)_2})]$
also
$\mu_{(F,A)\times(G,B)}[(x_{(F,A)_1}, y_{(G,B)_1})(x_{(F,A)_2}, y_{(G,B)_2})]$
$\leq \{\mu_{(F,A)}(x_{(F,A)_1}) \vee \mu_{(F,A)}(x_{(F,A)_2})\} \vee \{\mu_{(G,B)} y_{(G,B)_1} \vee \mu_{(G,B)} y_{(G,B)_2}\}[\mu_{(F,A)\times(G,B)}$

$$(x_{(F,A)_1}, y_{(G,B)_1}) \vee \mu_{(F,A)\times(G,B)}(x_{(F,A)_2}, y_{(G,B)_2})]$$

Therefore $\mu_{(F,A)\times(G,B)}[(x_{(F,A)_1}, y_{(G,B)_1}(x_{(F,A)_2}, y_{(G,B)_2})] \leq \mu_{(F,A)\times(G,B)}(x_{(F,A)_1}, y_{(G,B)_1}) \vee \mu_{(F,A)\times(G,B)}(x_{(F,A)_2}, y_{(G,B)_2})$Hence $(F, A) \times (G, B)$ is an anti-L-fuzzy soft subhemirings of $R_1 \times R_2$.

2.4 THEOREM :Let (F,A) is an anti-L-fuzzy soft subhemirings of a hemiring (R,+,·) if and only if $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)} \vee \mu_{(F,A)} y_{(F,A)})\}$, $\mu_{(F,A)}(x_{(F,A)} y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$ for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

PROOF: It is trivial.

2.5 THEOREM: If (F,A) is an anti-L-fuzzy soft subhemirings of a hemiring (R,+,·) then $H = \{x_{(F,A)}/x_{(F,A)} \in$R:$\mu_{(F,A)}(x_{(F,A)})$=0} is either empty (or) is a subhemiring of R.

PROOF : If no element satisfies this condition, then H is empty. If $x_{(F,A)}$ and $y_{(F,A)}$ in H then $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\} = \{0 \vee 0\} = 0$. Therefore, $\mu_{(F,A)}(x_{(F,A)} y_{(F,A)}) = 0$. we get $x_{(F,A)}(x_{(F,A)} y_{(F,A)}) = 0$ we get $(x_{(F,A)} + y_{(F,A)}),(x_{(F,A)} y_{(F,A)})$ in H. Therefore, H is a subhemiring of R. Hence H is either empty or is a subhemiring of R.

2.6 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemirings of a hemiring (R,+,·). If $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)})$=1, then either $\mu_{(F,A)}(x_{(F,A)}) = 1$ (or) $\mu_{(F,A)}(y_{(F,A)}) = 1$for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

PROOF: Let $x_{(F,A)}$ and $y_{(F,A)}$ in R. By the definition $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)} \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$, which implies that $1 \leq \{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$. Therefore, either $\mu_{(F,A)}(x_{(F,A)}) = 1$ (or) $\mu_{(F,A)}(y_{(F,A)}) = 1$.

2.7 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring (R,+,·), then the pseudo anti-L-fuzzy soft coset $(a(F,A))^P$ is an anti-L-fuzzy soft subhemiring of a hemiring R. For every a in R.

PROOF: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. For every $x_{(F,A)}$ and$y_{(F,A)}$ in R. We have $((a\mu_{(F,A)})^P)(x_{(F,A)} + y_{(F,A)}) \leq P(a)\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\}$ $=P(a)\mu_{(F,A)}(x_{(F,A)}) \vee P(a)\mu_{(F,A)}(y_{(F,A)})$= $(a\mu_{(F,A)})^P)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^P)(y_{(F,A)})$. Therfore $((a\mu_{(F,A)})^P)(x_{(F,A)} + y_{(F,A)}) \leq ((a\mu_{(F,A)})^P)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^P)(y_{(F,A)})$. Now $((a\mu_{(F,A)})^P)(x_{(F,A)} y_{(F,A)}) \leq P(a)\{\mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})\} =$ $P(a)\mu_{(F,A)}(x_{(F,A)}) \vee P(a)\mu_{(F,A)}(y_{(F,A)})$= $(a\mu_{(F,A)})^P)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^P)(y_{(F,A)})$.Therefore $(a\mu_{(F,A)})^P)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^P)(y_{(F,A)})$. Hence $(a\mu_{(F,A)})^P)$ is an anti-L- fuzzy soft subhemiring R.

2.8 THEOREM: Let (R,+,·) and $(R^{'}$,+,·) be any two hemirings. The homomorphic image of an anti-L-fuzzy soft subhemiring of a hemiring of R is an anti-L-fuzzy soft subhemiringof $R^{'}$ **PROOF** :Let $f: R \rightarrow R^{'}$ be a homomorphism. Then $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all x and y in R. Let $(G, V) = f(F, A), Where\ (F, A)$ is an anti-L-fuzzy soft subhemiring of R. Now for $f(x)_{(G,V)} f(y)_{(G,V)}$in $R^{'}$. $\mu_{(G,V)}(f(x)_{(G,V)} + f(y)_{(G,V)}) \leq \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})$. Which implies that $\mu_{(G,V)}(f(x)_{(F.A)} + f(y)_{(F,A)}) \leq \mu_{(G,V)}(f(x)_{(F.A)} \vee \mu_{(G,V)} f(y)_{(F,A)})$. again $\mu_{(G,V)}(f(x)_{(G,V)} + f(y)_{(G,V)}) \leq \mu_{(G,V)}(x_{(F,A)} y_{(F,A)}) \leq \mu_{(G,V)}(x_{(F,A)}) \vee \mu_{(G,V)}(y_{(F,A)})$ which implies that $\mu_{(G,V)}(f(x)_{(F.A)} + f(y)_{(F,A)}) \leq \mu_{(G,V)}(f(x)_{(F.A)} \vee \mu_{(G,V)} f(y)_{(F,A)})$. hence (G,V) is an anti-L-fuzzy soft subhemirings of $R^{'}$.

2.9 THEOREM : Let (R,+,·) and $(R^{'}$,+,·) be any two hemirings. The homomorphic preimage of an anti-L-fuzzy soft subhemiring of $R^{'}$ is an anti-L-fuzzy soft subhemiring of R.

PROOF :Let (G,V) = f((F,A)),where (G,V) is an anti-L-fuzzy soft subhemiring of $R^{'}$. Let $x_{(F,A)}$ and $y_{(F,A)}$ in R. Then $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) = \mu_{(G,V)}(f(x)_{(F.A)} + f(y)_{(F,A)}) \leq \mu_{(G,V)}(f(x)_{(F.A)} \vee \mu_{(G,V)} f(y)_{(F,A)}) = \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})$ which implies that $\mu_{(F,A)}(x_{(F,A)}) + (y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})$again $\mu_{(F,A)}(x_{(F,A)} y_{(F,A)}) = \mu_{(G,V)}(f(x_{(F,A)} y_{(F,A)}) \leq \mu_{(G,V)}(f(x_{(F,A)})) \vee \mu_{(G,V)}(f(y_{(F,A)}) = \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})$which implies that

$\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)})$.
Hence (F,A) is an anti-L-fuzzy soft subhemiring of R.

2.10 THEOREM : Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two hemirings. The anti-homomorphic image of an anti-L-fuzzy soft subhemiring of R is an anti-L-fuzzy soft subhemiring of R'.

PROOF : Let $f: R \rightarrow R'$ be an anti-homomorphism. Then $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$ for all x and y in R. Let (G,V)=f((F.A)) where (G,V) is an anti-L-fuzzy soft subhemiring of R. Now, for $f(x)_{(G,V)}, f(y)_{(G,V)}$ are in R' $\mu_{(G,V)}(f(x)_{(G,V)} + f(y)_{(G,V)}) = \mu_{(G,V)}(f(y)_{(G,V)} + f(x)_{(G,V)})$ as f is an anti – homomorphism. $\leq \mu_{(F,A)}(y_{(F,A)} + x_{(F,A)}) \leq \mu_{(F,A)}(y)_{(F,A)} \vee \mu_{(F,A)}(x)_{(F,A)}\} = \{\mu_{(F,A)}(x) \vee \mu_{(F,A)}(y)\}$which implies that $\mu_{(G,V)}(f(x_{(G,V)}) + f(y_{(G,V)})) \leq \{ \mu_{(G,V)}(f(x_{(G,V)})) \vee \mu_{(G,V)}(f(y_{(G,V)}))\}$.

Again, $\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) = \mu_{(G,V)}(f(y_{(G,V)}x_{(G,V)}))$, as f is an anti-homomorphism $\leq \mu_{(F,A)}(y_{(F,A)}x_{(F,A)}) \leq \{ \mu_{(F,A)}(y_{(F,A)}) \vee \mu_{(F,A)}(x_{(F,A)}) \} = \{ \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)}) \}$which implies that $\mu_{(G,V)}(f(x_{(G,V)})f(y_{(G,V)})) \leq \{ \mu_{(G,V)}(f(x_{(G,V)})) \vee \mu_{(G,V)}(f(y_{(G,V)}))\}$.

Hence (G,V) is an anti-L-fuzzy soft subhemiring of R^{I}.

We denote the composition of operations by ∘ for the following:

2.11 THEOREM : Let (F, A) be an anti-L-fuzzy soft subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then (F,A)∘f is an anti-L-fuzzy soft subhemiring of R.

PROOF: Let $x_{(F,A)}$ and $y_{(F,A)}$ in R and (F, A) be an anti-L-fuzzy soft subhemiring of a hemiring H. we have, $(\mu_{(F,A)}\circ f)(x_{(F,A)}+y_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}+y_{(F,A)})) = \mu_{(F,A)}(f(x_{(F,A)}) + f(y_{(F,A)}))$, as f is an isomorphism $\leq \{ \mu_{(F,A)}(f(x_{(F,A)})) \vee \mu_{(F,A)}(f(y_{(F,A)})) \}, \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$,which implies that $(\mu_{(F,A)}\circ f)(x_{(F,A)}+y_{(F,A)}) \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$.And $(\mu_{(F,A)}\circ f)(x_{(F,A)}y_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}y_{(F,A)})) = \mu_{(F,A)}(f(x_{(F,A)})f(y_{(F,A)}))$, as f is an isomorphism $\leq \{ \mu_{(F,A)}(f(x_{(F,A)})) \vee \mu_{(F,A)}(f(y_{(F,A)})) \}, \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$,which implies that $(\mu_{(F,A)}\circ f)(x_{(F,A)}y_{(F,A)}) \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$.Therefore (F,A)∘f is an anti-L-fuzzy soft subhemiring of a hemiring R.

2.12 THEOREM: Let (F, A) be an anti-L-fuzzy soft subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then (F,A)∘f is an anti-L-fuzzy soft subhemiring of R.

PROOF: Let $x_{(F,A)}$ and $y_{(F,A)}$ in R and (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring H. We have, $(\mu_{(F,A)}\circ f)(x_{(F,A)}+y_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}+y_{(F,A)})) = \mu_{(F,A)}(f(y_{(F,A)})+f(x_{(F,A)}))$, as f is an anti-isomorphism$\leq \{ \mu_{(F,A)}(f(x_{(F,A)})) \vee \mu_{(F,A)}(f(y_{(F,A)})) \}, \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$,which implies that $(\mu_{(F,A)}\circ f)(x_{(F,A)}+y_{(F,A)}) \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$.$(\mu_{(F,A)}\circ f)(x_{(F,A)}y_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}y_{(F,A)})) = \mu_{(F,A)}(f(y_{(F,A)})f(x_{(F,A)}))$, as f is an anti-isomorphism $\leq \{ \mu_{(F,A)}(f(x_{(F,A)})) \vee \mu_{(F,A)}(f(y_{(F,A)})) \}, \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$,which implies that $(\mu_{(F,A)}\circ f)(x_{(F,A)}y_{(F,A)}) \leq \{ (\mu_{(F,A)}\circ f)(x_{(F,A)}) \vee (\mu_{(F,A)}\circ f)(y_{(F,A)}) \}$. Therefore (F,A)∘f is an anti-L-fuzzy soft subhemiring of a hemiring R.

2.13 THEOREM: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring (R, +, .), then the pseudo anti-L-fuzzy soft coset $(a(F,A))^p$ is an anti-L-fuzzy soft subhemiring of a hemiring R, for every a in R.

PROOF: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. For every $x_{(F,A)}$ and $y_{(F,A)}$ in R, we have, $((a\mu_{(F,A)})^p)(x_{(F,A)}+y_{(F,A)}) = p(a)\mu_{(F,A)}(x_{(F,A)}+y_{(F,A)}) \leq p(a) \{ \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)}) \} = \{ p(a)\mu_{(F,A)}(x_{(F,A)}) \vee p(a)\mu_{(F,A)}(y_{(F,A)}) \} = \max \{ ((a\mu_{(F,A)})^p)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^p)(y_{(F,A)}) \}$.Therefore, $((a\mu_{(F,A)})^p)(x_{(F,A)}+ y_{(F,A)}) \leq \{ ((a\mu_{(F,A)})^p)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^p)(y_{(F,A)}) \}$.Now, $((a\mu_{(F,A)})^p)(x_{(F,A)}y_{(F,A)}) = p(a)\mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq p(a) \{ \mu_{(F,A)}(x_{(F,A)}) \vee \mu_{(F,A)}(y_{(F,A)}) \} = \{ p(a)\mu_{(F,A)}(x_{(F,A)}) \vee p(a)\mu_{(F,A)}(y_{(F,A)}) \} = \{ ((a\mu_{(F,A)})^p)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^p)(y_{(F,A)}) \}$.Therefore, $((a\mu_{(F,A)})^p)(x_{(F,A)}y_{(F,A)}) \leq \{ ((a\mu_{(F,A)})^p)(x_{(F,A)}) \vee ((a\mu_{(F,A)})^p)(y_{(F,A)}) \}$.

2.14 THEOREM: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. (F,A^+)be a L-fuzzy soft set in R defined by $[(F,A^+)x_{(F,A)}] = (F,A)(x_{(F,A)}) +1 - (F,A)(0)$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in R . Then (F,A^+) is an anti-L-fuzzy soft subhemiring of a hemiring R.

PROOF: Let $x_{(F,A)},y_{(F,A)} \in R$. We have,$(F,A^+)(x_{(F,A)}+y_{(F,A)}) = (F,A)(x_{(F,A)}+y_{(F,A)}) +(F,A)(0) \leq \{ (F,A)(x_{(F,A)}) \vee (F,A)(y_{(F,A)}) \}+1- (F,A)(0)= \{ \{(F,A)(x_{(F,A)}) +1- (F,A)(0)\} \vee \{ (F,A)(y_{(F,A)}) +1- (F,A)(0)\}= \{ (F,A^+)(x_{(F,A)}) \vee (F,A^+)(y_{(F,A)}) \}$.Therefore $(F,A^+)(x_{(F,A)}+y_{(F,A)}) \leq$

$\{ (F,A^+)(x_{(F,A)}) \vee (F,A^+)(y_{(F,A)}) \}$. Similarly, $(F,A^+)(x_{(F,A)}y_{(F,A)}) = (F,A)(x_{(F,A)}y_{(F,A)}) +1- (F,A)(0) \leq \{ (F,A)(x_{(F,A)}) \vee (F,A)(y_{(F,A)}) \}+1- (F,A)(0)= \{ \{(F,A)(x_{(F,A)}) +1- (F,A)(0)\} \vee \{ (F,A)(y_{(F,A)}) +1- (F,A)(0)\}= \{ (F,A^+)(x_{(F,A)}) \vee (F,A^+)(y_{(F,A)}) \}$ Therefore $(F,A^+)(x_{(F,A)}y_{(F,A)}) \leq$

$\{ (F,A^+)(x_{(F,A)}) \vee (F,A^+)(y_{(F,A)}) \}$.

2.15 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R ,

(F,A^+) be a L-fuzzy soft set in R defined by $(F,A^+)(x_{(F,A)}) = (F,A)(x_{(F,A)}) + 1 - (F,A)(0)$

for all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Then there exists $0 \in R$ such that $(F,A)(0) = 1$ if and only

if $(F,A^+)(x_{(F,A)}) = (F,A)\, x_{(F,A)}$

PROOF: It is trivial.

2.16 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R ,

(F,A^+) be a L-fuzzy soft set in R defined by $(F,A^+)(x_{(F,A)}) = (F,A)(x_{(F,A)}) + 1 - (F,A)(0)$

for all x and y in R. Then there exists $x \in R$ such that $(F,A^+)(x_{(F,A)}) = 1$ if and only

if $x_{(F,A)} = 0$.

PROOF :It is trivial.

2.17 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. ,

(F,A^+) be a L- fuzzy soft set in R defined by $(F,A^+)(x_{(F,A)}) = A(x_{(F,A)}) + 1 - A(0)$, for

all $x_{(F,A)}$ and $y_{(F,A)}$ in R. Then $((F,A^+))^+ = (F,A^+)$.

PROOF: Let $x_{(F,A)}, y_{(F,A)} \in$ R. We have, $((F,A^+))^+(x_{(F,A)}) = (F,A^+)(x_{(F,A)}) + 1 - (F,A^+)(0)$

$= \{(F,A)(x_{(F,A)}) + 1 - (F,A)(0)\} + 1 - \{(F,A)(0) + 1 - (F,A)(0)\} = \{(F,A)(x_{(F,A)}) + 1 - (F,A)(0) = (F,A^+)(x_{(F,A)})$ Hence $((F,A^+))^+ = (F,A)$.

2.18 THEOREM: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. Then (F,A^0) is an anti-L-fuzzy soft subhemiring of a hemiring R , for all $x_{(F,A)}$ and $y_{(F,A)}$ in R .

PROOF :For any $x_{(F,A)} \in$ R, we have $(F,A^0)(x_{(F,A)}+y_{(F,A)}) = (F,A)(x_{(F,A)}+y_{(F,A)}) / (F,A)(0)$

$\leq [1 / (F,A)(0)]\{(F,A)(x_{(F,A)}) \vee (F,A)(y_{(F,A)})\}, = \{[(F,A)(x_{(F,A)}) / (F,A)(0)] \vee [(F,A)(y_{(F,A)}) / (F,A)(0)]\}, = \{(F,A^0)(x_{(F,A)}) \vee (F,A^0)(y_{(F,A)})\}$. That is $(F,A^0)(x_{(F,A)}+y_{(F,A)}) \leq \{(F,A^0)(x_{(F,A)}) \vee (F,A^0)(y_{(F,A)})\}$.Similarly, $(F,A^0)(x_{(F,A)}y_{(F,A)}) = (F,A)(x_{(F,A)}y_{(F,A)}) / A(0) \leq [1 / (F,A)(0)]\{(F,A)(x_{(F,A)}) \vee (F,A)(y_{(F,A)})\}, = \{[(F,A)(x_{(F,A)}) / (F,A)(0)] \vee [(F,A)(y_{(F,A)}) / (F,A)(0)]\}, = \{(F,A^0)(x_{(F,A)}) \vee (F,A^0)(y_{(F,A)})\}$.That is $(F,A^0)(x_{(F,A)}y_{(F,A)}) \leq \{(F,A^0)(x_{(F,A)}) \vee (F,A^0)(y_{(F,A)})\}$. Hence (F,A^0) is an anti-L-fuzzy soft subhemiring of a hemiring R , for all $x_{(F,A)}$ and $y_{(F,A)}$ in R .

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