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Degree Based Neighborhood Indices of Some Nanostructures

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Abstract: *In Chemical Science, the topological indices are used to analysis of molecular drug structures. These indices are helpful for chemical scientists to find out the chemical characteristics of drugs. This paper introduces the modified first and second neighborhood indices, inverse sum indeg neighborhood index of a graph. Also, we introduce the modified first and second neighborhood polynomials, inverse sum indeg neighborhood polynomial, harmonic neighborhood polynomial, general neighborhood polynomial of a graph. Furthermore, we compute and obtain the comparative analysis of certain neighborhood indices and their polynomials of some important nanostructures which appeared in nanoscience.*

Keywords: ***nanoscience,*** *neighborhood indices, neighborhood polynomials, dendrimer.*

Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*90.

I. INTRODUCTION

Chemical Graph Theory, concerning the definition of the topological index on the molecular graph and concerning chemical properties of drugs, can be studied by the topological index calculation. See [1, 2]. Several degree-based indices of a graph have been appeared in the literature, see [3, 4, 5, 6] and have found some applications, especially in QSPR/QSAR study.

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let $N(u)=\{v:uv\in E(G)\}$. Let $S_G(u)=\sum_{v\in N(u)} d_G(v)$ be the degree sum of neighbor vertices. For undefined terms and notation, we refer to the book [7].

Recently, some neighborhood indices were introduced and studied, such M_1 and M_2 Zagreb indices [8], fifth hyper M_1 and M_2 Zagreb indices [9], general first M-Zagreb indices [9], F-neighborhood index [10], fifth arithmetic-geometric index [11], fifth multiplicative Zagreb indices, fifth multiplicative hyper Zagreb indices, fifth multiplicative sum, and product connectivity indices, general fifth multiplicative Zagreb indices [13], fourth multiplicative atom bond connectivity index [14], multiplicative F_1-neighborhood index, general first multiplicative neighborhood index [15]. Also, some neighborhood indices were studied in [16, 17, 18].

In 2011 [8], Graovac et al. introduced the fifth M-Zagreb indices (now we call the first and second neighborhood indices) defined as

$$NM_1(G)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right],$$

$$NM_2(G)=\sum_{uv\in E(G)}S_G(u)S_G(v).$$

In 2017 [9], Kulli introduced the fifth hyper M-Zagreb indices (now we call the first and second neighborhood indices) defined as

$$HNM_1(G)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right]^2,$$

$$HNM_2(G)=\sum_{uv\in E(G)}\left[S_G(u)S_G(v)\right]^2.$$

In 2017 [9], Kulli introduced the general fifth M-Zagreb indices (now we call the general first and second neighborhood indices) defined as

$$NM_1^a(G)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right]^a,$$

$$NM_2^a(G)=\sum_{uv\in E(G)}\left[S_G(u)S_G(v)\right]^a,$$

Where a is a real number.

The modified first and second neighborhood indices of a graph G are defined as

$${}^mNM_1(G)=\sum_{uv\in E(G)}\frac{1}{S_G(u)+S_G(v)},$$

$${}^mNM_2(G)=\sum_{uv\in E(G)}\frac{1}{S_G(u)S_G(v)}.$$

The inverse sum indeg neighborhood index of a graph G is defined as

$$INM(G)=\sum_{uv\in E(G)}\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}.$$

The harmonic neighborhood and general harmonic neighborhood indices of a graph G are defined as

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$$HNM(G)=\sum_{uv\in E(G)}\frac{2}{S_G(u)+S_G(v)},$$

$$HNM^a(G)=\sum_{uv\in E(G)}\left(\frac{2}{S_G(u)+S_G(v)}\right)^a.$$

The general neighborhood index of a graph G is defined as

$$NM^a(G)=\sum_{uv\in E(G)}\left[S_G(u)^a+S_G(v)^a\right].$$

In [9], Kulli introduced the fifth M_1 and M_2 Zagreb polynomials (we now call the first and second neighborhood polynomials) of a graph G, defined as

$$NM_1(G,x)=\sum_{uv\in E(G)}x^{S_G(u)+S_G(v)},$$

$$NM_2(G,x)=\sum_{uv\in E(G)}x^{S_G(u)S_G(v)}.$$

In [9], Kulli introduced the fifth M_1 and M_2 Zagreb polynomials (we now call the first and the second hyper neighborhood polynomials) of a graph G, defined as

$$HNM_1(G,x)=\sum_{uv\in E(G)}x^{\left[S_G(u)+S_G(v)\right]^2},$$

$$HNM_2(G,x)=\sum_{uv\in E(G)}x^{\left[S_G(u)S_G(v)\right]^2}.$$

In [9], Kulli proposed the fifth general M_1 and M_2 Zagreb polynomials (we now call the general first and second neighborhood polynomials) of a graph G, and they are defined as

$$NM_1^a(G,x)=\sum_{uv\in E(G)}x^{\left[S_G(u)+S_G(v)\right]^a},$$

$$NM_2^a(G,x)=\sum_{uv\in E(G)}x^{\left[S_G(u)S_G(v)\right]^a}.$$

We now introduce the modified first and second neighborhood polynomials of a graph G, defined as

$${}^mNM_1(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{S_G(u)+S_G(v)}},$$

$${}^mNM_2(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{S_G(u)S_G(v)}}.$$

We propose the inverse sum indeg neighborhood polynomial of a graph G, defined as

$$INM(G,x)=\sum_{uv\in E(G)}x^{\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}}.$$

We introduce the harmonic and general harmonic neighborhood polynomials of a graph G, defined as

$$HNM(G,x)=\sum_{uv\in E(G)}x^{\frac{2}{S_G(u)+S_G(v)}},$$

$$HNM^a(G,x)=\sum_{uv\in E(G)}x^{\left[\frac{2}{S_G(u)+S_G(v)}\right]^a}.$$

We also introduce the general neighborhood polynomial of a graph G, defined as

$$NM^a(G,x)=\sum_{uv\in E(G)}x^{\left[S_G(u)^a+S_G(v)^a\right]}.$$

In Chemical Graph Theory, graph polynomials related to molecular graphs were studied in [19, 20, 21, 22, 23, 24, 25, 26].

In this paper, some new and old neighborhood indices of dendrimers are determined. Furthermore, some neighborhood polynomials of dendrimers are computed. For dendrimers, see [27].

II. RESULTS FOR $NS_2[n]$ DENDRIMERS

In this section, we focus on the class of $NS_2[n]$ dendrimers, where n□1. The graph of $NS_2[3]$ is presented in Figure 1.

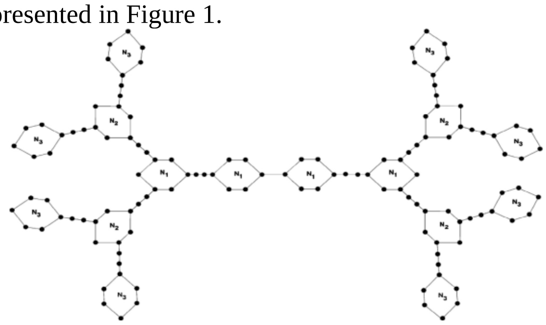

Figure 1. The molecular structure of $NS_2[3]$

Let G be the molecular graph of $NS_2[n]$. By calculation, we obtain that G has $16\times2^n-4$ vertices and $18\times2^n-5$ edges. Also, by calculation, we obtain that G has seven types of edges based on $S_G(u)$ and $S_G(v)$ the degree of end vertices of each edge, as given in Table 1.

$S_G(u), S_G(v) \setminus uv$ □ $E(G)$	Number of edges
(4, 4)	2×2^n
(5, 4)	2×2^n
(5, 5)	$2\times2^n+2$
(5, 6)	6×2^n
(7, 7)	1
(5, 7)	4
(6, 6)	$6\times2^n-12$

Table 1. Edge partition of $NS_2[n]$ based on $S_G(u)$, $S_G(v)$.

We compute the general first neighborhood index of $NS_2[n]$.

Theorem 1. The general first neighborhood index of a dendrimer $NS_2[n]$ is given by

$$NM_1^a\left(NS_2[n]\right)=\left(2\times 2^n\right)\left(8^a+9^a+10^a\right)$$
$$+\left(6\times 2^n\right)\left(11^a+12^a\right)$$
$$+2\times 10^a+14^a-8\times 12^a. \qquad (1)$$

Proof: Let G be the molecular graph of $NS_2[n]$. Using definition and Table 1, we deduce

$$NM_1^a\left(NS_2[n]\right)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right]^a$$
$$=2\times 2^n(4+4)^a+2\times 2^n(5+4)^a$$
$$+\left(2\times 2^n+2\right)(5+5)^a+6\times 2^n(5+6)^a$$
$$+(7+7)^a+4(5+7)^a$$
$$+\left(6\times 2^n-12\right)(6+6)^a$$
$$=\left(2\times 2^n\right)\left(8^a+9^a+10^a\right)+\left(6\times 2^n\right)\left(11^a+12^a\right)$$
$$+2\times 10^a+14^a-8\times 12^a.$$

We obtain the following results by using Theorem 1.

Corollary 1.1. The first neighborhood index of $NS_2[n]$ is

$$NM_1\left(NS_2[n]\right)=192\times 2^n-62.$$

Corollary 1.2. The first hyper neighborhood index of $NS_2[n]$ is

$$HNM_1\left(NS_2[n]\right)=1475\times 2^n-756.$$

Corollary 1.3. The modified first neighborhood index of $NS_2[n]$ is

$$^m NM_1\left(NS_2[n]\right)=\frac{3401}{1980}\times 2^n-\frac{83}{210}.$$

Proof: Put a = 1, 2, –1 in equation (1), we get the desired results.

We determine the general second neighborhood index of $NS_2[n]$.

Theorem 2. The general second neighborhood index of a dendrimer $NS_2[n]$ is given by

$$NM_2^a\left(NS_2[n]\right)=2\times 2^n\left(16^a+20^a+25^a\right)$$
$$+\left(6\times 2^n\right)\left(30^a+36^a\right)+2\times 25^a$$
$$+49^a+4\times 35^a-12\times 36^a. \qquad (2)$$

Proof: Let G be the molecular graph of $NS_2[n]$. By using definition and Table 1, we derive

$$NM_2^a\left(NS_2[n]\right)=\sum_{uv\in E(G)}\left[S_G(u)S_G(v)\right]^a$$
$$=\left(2\times 2^n\right)(4\times 4)^a+\left(2\times 2^n\right)(5\times 4)^a$$
$$+\left(2\times 2^n+2\right)(5\times 5)^a+6\times 2^n(5\times 6)^a$$
$$+(7\times 7)^a+4(5\times 7)^a+\left(6\times 2^n-12\right)(6\times 6)^a$$
$$=2\times 2^n\left(16^a+20^a+25^a\right)+\left(6\times 2^n\right)\left(30^a+36^a\right)$$
$$+2\times 25^a+49^a+4\times 35^a-12\times 36^a.$$

We establish the following results from Theorem 2.

Corollary 2.1. The second neighborhood index of $NS_2[n]$ is

$$NM_2\left(NS_2[n]\right)=518\times 2^n-193.$$

Corollary 2.2. The second hyper neighborhood index of $NS_2[n]$ is

$$HNM_2\left(NS_2[n]\right)=15738\times 2^n-7001.$$

Corollary 2.3. The modified second neighborhood index of $NS_2[n]$ is

$$^m NM_2\left(NS_2[n]\right)=\frac{403}{120}\times 2^n-\frac{3052}{25725}.$$

Proof: Put a = 1, 2, –1 in equation (2), we obtain the desired results.

Theorem 3. The inverse sum indeg neighborhood index of a dendrimer $NS_2[n]$ is

$$INM\left(NS_2[n]\right)=\frac{4733}{99}\times 2^n-\frac{95}{6}.$$

Proof: By using the definition and Table 1, we obtain

$$INM\left(NS_2[n]\right)=\sum_{uv\in E(G)}\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}$$
$$=2\times 2^n\left(\frac{4\times 4}{4+4}\right)+2\times 2^n\left(\frac{5\times 4}{5+4}\right)$$
$$+\left(2\times 2^n+2\right)\left(\frac{5\times 5}{5+5}\right)+6\times 2^n\left(\frac{5\times 6}{5+6}\right)+\left(\frac{7\times 7}{7+7}\right)$$
$$+4\left(\frac{5\times 7}{5+7}\right)^4\times\left(6\times 2^n-12\right)\left(\frac{6\times 6}{6+6}\right)$$
$$=\frac{4733}{99}\times 2^n-\frac{95}{6}.$$

Theorem 4. The harmonic neighborhood index of a dendrimer $NS_2[n]$ is

$$HNM\left(NS_2[n]\right)=\frac{3401}{990}\times 2^n-\frac{83}{105}.$$

Proof: By using the definition and Table 1, we have

$$HNM\left(NS_2[n]\right)=\sum_{uv\in E(G)}\frac{2}{S_G(u)+S_G(v)}$$
$$=2\times 2^n\left(\frac{2}{4+4}\right)+2\times 2^n\left(\frac{2}{5+4}\right)$$
$$+\left(2\times 2^n+2\right)\left(\frac{2}{5+5}\right)+6\times 2^n\left(\frac{2}{5+6}\right)+\frac{2}{7+7}$$
$$+4\left(\frac{2}{5+7}\right)\times\left(6\times 2^n-12\right)\left(\frac{2}{6+6}\right)$$
$$=\frac{3401}{990}\times 2^n-\frac{83}{105}.$$

Theorem 5. The general neighborhood index of a dendrimer $NS_2[n]$ is

$$NM^a\left(NS_2[n]\right)=\left(6\times 4^a+12\times 5^a+12\times 6^a\right)\times 2^n$$

$$+8\times 5^a - 12\times 6^a + 6\times 7^a.$$

Proof: By using definition and Table 1, we deduce

$$NM^a(NS_2[n]) = \sum_{uv\in E(G)} \left[S_G(u)^a + S_G(v)^a\right]$$
$$= 2\times 2^n(4^a + 4^a) + 2\times 2^n(5^a + 4^a)$$
$$+(2\times 2^n + 2)(5^a + 5^a) + 6\times 2^n(5^a + 6^a)$$
$$+(7+7)^a + 4(5+7)^a + (6\times 2^n - 12)(6+6)^a$$
$$= (6\times 4^a + 12\times 5^a + 12\times 6^a)\times 2^n$$
$$+8\times 5^a - 12\times 6^a + 6\times 7^a.$$

We compute the general first neighborhood polynomial of $NS_2[n]$.

Theorem 6. The general first neighborhood polynomial of a dendrimer $NS_2[n]$ is

$$NM_1^a(NS_2[n],x) = 2\times 2^n x^{8^a} + 2\times 2^n x^{9^a}$$
$$+(2\times 2^n + 2)x^{10^a} + 6\times 2^n x^{11^a}$$
$$+x^{14^a} + (6\times 2^n - 8)x^{12^a}.$$

Proof: By using definition and Table 1, we deduce

$$NM_1^a(NS_2[n],x) = \sum_{uv\in E(G)} x^{[S_G(u)+S_G(v)]^a}$$
$$= 2\times 2^n x^{(4+4)^a} + 2\times 2^n x^{(5+4)^a}$$
$$+(2\times 2^n + 2)x^{(5+5)^a} + 6\times 2^n x^{(5+6)^a}$$
$$+x^{(7+7)^a} + x^{(5+7)^a} + (6\times 2^n - 12)x^{(6+6)^a}$$
$$= 2\times 2^n x^{8^a} + 2\times 2^n x^{9^a} + (2\times 2^n + 2)x^{10^a}$$
$$+6\times 2^n x^{11^a} + x^{14^a} + (6\times 2^n - 8)x^{12^a}.$$

We obtain the following results from Theorem 6.

Corollary 6.1. The first neighborhood polynomial of $NS_2[n]$ is

$$NM_1^a(NS_2[n],x) = 2\times 2^n x^8 + 2\times 2^n x^9$$
$$+(2\times 2^n + 2)x^{10} + 6\times 2^n x^{11}$$
$$+x^{14} + (6\times 2^n - 8)x^{12}.$$

Corollary 6.2. The first hyper neighborhood polynomial of $NS_2[n]$ is

$$HNM_1(NS_2[n],x) = 2\times 2^n x^{64} + 2\times 2^n x^{81}$$
$$+(2\times 2^n + 2)x^{100} + 6\times 2^n x^{121}$$
$$+x^{196} + (6\times 2^n - 8)x^{144}.$$

Corollary 6.3. The modified first neighborhood polynomial of $NS_2[n]$ is

$${}^mNM_1(NS_2[n],x) = 2\times 2^n x^{\frac{1}{8}} + 2\times 2^n x^{\frac{1}{9}}$$
$$+(2\times 2^n + 2)x^{\frac{1}{10}} + 6\times 2^n x^{\frac{1}{11}}$$
$$+x^{\frac{1}{14}} + (6\times 2^n - 8)x^{\frac{1}{12}}.$$

We compute the general second neighborhood polynomial of a dendrimer $NS_2[n]$.

Theorem 7. The general second neighborhood polynomial of a dendrimer $NS_2[n]$ is

$$NM_2^a(NS_2[n],x) = 2\times 2^n x^{16^a} + 2\times 2^n x^{20^a}$$
$$+(2\times 2^n + 2)x^{25^a} + 6\times 2^n x^{30^a}$$
$$+x^{49^a} + 4x^{35^a} + (6\times 2^n - 12)x^{36^a}. \quad (4)$$

Proof: By using definition and Table 1, we derive

$$NM_2^a(NS_2[n],x) = \sum_{uv\in E(G)} x^{[S_G(u)S_G(v)]^a}$$
$$= 2\times 2^n x^{(4\times 4)^a} + 2\times 2^n x^{(5\times 4)^a}$$
$$+(2\times 2^n + 2)x^{(5\times 5)^a} + 6\times 2^n x^{(5\times 6)^a}$$
$$+x^{(7\times 7)^a} + x^{(5\times 7)^a} + (6\times 2^n - 12)x^{(6\times 6)^a}$$
$$= 2\times 2^n x^{16^a} + 2\times 2^n x^{20^a} + (2\times 2^n + 2)x^{25^a}$$
$$+6\times 2^n x^{30^a} + x^{49^a} + 4x^{35^a} + (6\times 2^n - 12)x^{36^a}.$$

We establish the following results by using Theorem 7.

Corollary 7.1. The second neighborhood polynomial of $NS_2[n]$ is

$$NM_1(NS_2[n],x) = 2\times 2^n x^{16} + 2\times 2^n x^{20}$$
$$+(2\times 2^n + 2)x^{25} + 6\times 2^n x^{30}$$
$$+x^{49} + 4x^{35} + (6\times 2^n - 12)x^{36}.$$

Corollary 7.2. The second hyper neighborhood polynomial of $NS_2[n]$ is

$$HNM_1(NS_2[n],x) = 2\times 2^n x^{256} + 2\times 2^n x^{400}$$
$$+(2\times 2^n + 2)x^{625} + 6\times 2^n x^{900}$$
$$+x^{2401} + 4x^{1225} + (6\times 2^n - 12)x^{1296}.$$

Corollary 7.3. The modified second neighborhood polynomial of $NS_2[n]$ is

$${}^mNM_2(NS_2[n],x) = 2\times 2^n x^{\frac{1}{16}} + 2\times 2^n x^{\frac{1}{20}}$$
$$+(2\times 2^n + 2)x^{\frac{1}{25}} + 6\times 2^n x^{\frac{1}{30}}$$
$$+x^{\frac{1}{49}} + 4x^{\frac{1}{35}} + (6\times 2^n - 12)x^{\frac{1}{36}}.$$

Proof: Put a = 1, 2, –1 in equation (4), we obtain the desired results.

Theorem 8. The inverse sum indeg neighborhood polynomial of a dendrimer $NS_2[n]$ is

$$INM(NS_2[n],x) = 2\times 2^n x^2 + 2\times 2^n x^{\frac{20}{9}}$$
$$+(2\times 2^n + 2)x^{\frac{5}{2}} + 6\times 2^n x^{\frac{30}{11}}$$
$$+x^{\frac{7}{2}} + 4x^{\frac{35}{12}} + (6\times 2^n - 12)x^3.$$

Proof: By using definition and Table 1, we obtain

$$INM\left(NS_2[n],x\right)=\sum_{uv\in E(G)} x^{\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}}$$

$$=2\times 2^n x^{\frac{4\times4}{4+4}}+2\times 2^n x^{\frac{5\times4}{5+4}}+\left(2\times 2^n+2\right)x^{\frac{5\times5}{5+5}}$$
$$+6\times 2^n x^{\frac{5\times6}{5+6}}+x^{\frac{7\times7}{7+7}}+4x^{\frac{5\times7}{5+7}}\times\left(6\times 2^n-12\right)x^{\frac{6\times6}{6+6}}$$
$$=2\times 2^n x^2+2\times 2^n x^{\frac{20}{9}}+\left(2\times 2^n+2\right)x^{\frac{5}{2}}$$
$$+6\times 2^n x^{\frac{30}{11}}+x^{\frac{7}{2}}+4x^{\frac{35}{12}}+\left(6\times 2^n-12\right)x^3.$$

Theorem 9. The harmonic neighborhood polynomial of a dendrimer $NS_2[n]$ is

$$HNM\left(NS_2[n],x\right)=2\times 2^n x^{\frac{1}{4}}+2\times 2^n x^{\frac{2}{9}}$$
$$+\left(2\times 2^n+2\right)x^{\frac{1}{5}}+6\times 2^n x^{\frac{2}{11}}$$
$$+x^{\frac{1}{7}}+\left(6\times 2^n-8\right)x^{\frac{1}{6}}.$$

Proof: By using the definition and Table 1, we have

$$HNM\left(NS_2[n],x\right)=\sum_{uv\in E(G)} x^{\frac{2}{S_G(u)+S_G(v)}}$$
$$=2\times 2^n x^{\frac{1}{4}}+2\times 2^n x^{\frac{2}{9}}+\left(2\times 2^n+2\right)x^{\frac{1}{5}}$$
$$+6\times 2^n x^{\frac{2}{11}}+x^{\frac{1}{7}}+\left(6\times 2^n-8\right)x^{\frac{1}{6}}.$$

Theorem 10. The general neighborhood polynomial of dendrimer $NS_2[n]$ is

$$NM^a\left(NS_2[n],x\right)=2\times 2^n x^{2\times4^a}+2\times 2^n x^{5^a+4^a}$$
$$+\left(2\times 2^n+2\right)x^{2\times5^a}+6\times 2^n x^{5^a+6^a}$$
$$+x^{2\times7^a}+4x^{5^a+7^a}+\left(6\times 2^n-12\right)x^{2\times6^a}.$$

Proof: By using the definition and Table 1, we deduce

$$NM^a\left(NS_2[n],x\right)=\sum_{uv\in E(G)} x^{\left[S_G(u)^a+S_G(v)^a\right]}$$
$$=2\times 2^n x^{4^a+4^a}+2\times 2^n x^{5^a+4^a}$$
$$+\left(2\times 2^n+2\right)x^{5^a+5^a}+6\times 2^n x^{5^a+6^a}$$
$$+x^{7^a+7^a}+4x^{5^a+7^a}+\left(6\times 2^n-12\right)x^{6^a+6^a}$$
$$=2\times 2^n x^{2\times4^a}+2\times 2^n x^{5^a+4^a}+\left(2\times 2^n+2\right)x^{2\times5^a}$$
$$+6\times 2^n x^{5^a+6^a}+x^{2\times7^a}+4x^{5^a+7^a}+\left(6\times 2^n-12\right)x^{2\times6^a}.$$

III. RESULTS FOR $NS_3[n]$ DENDRIMERS

In this section, we consider another type of dendrimers $NS_3[n]$, where $n\geq 1$. The graph of $NS_3[2]$ dendrimer is shown in Figure 2.

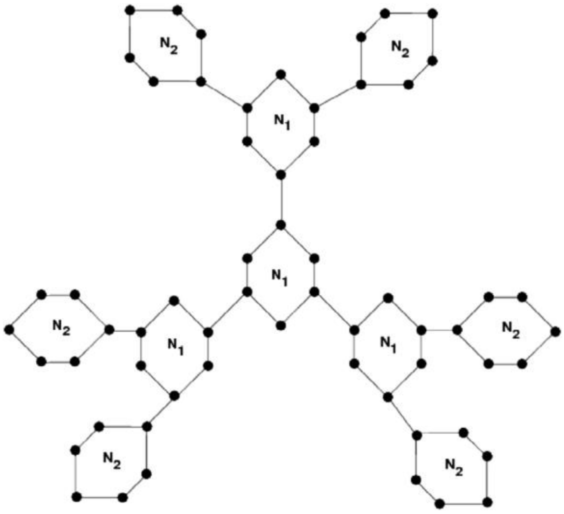

Figure 2. The molecular structure of $NS_3[2]$

Let G be the molecular graph of $NS_3[n]$. By calculation, we obtain that G has $18\times2^n - 12$ vertices and $21\times2^n - 15$ edges. Also, by calculation, we obtain that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degree of end vertices of each edge, as given in Table 2.

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(4, 4)	3×2^n
(4, 5)	3×2^n
(5, 7)	3×2^n
(6, 7)	$9\times2^n - 12$
(7, 7)	$3\times2^n - 3$

Table 2. Edge partition of $NS_3[n]$ based on $S_G(u)$, $S_G(v)$

We compute the general first neighborhood index of $NS_3[n]$.

Theorem 11. The general first neighborhood index of a dendrimer $NS_3[n]$ is given by

$$NM_1^a\left(NS_3[n]\right)=\left(3\times 2^n\right)\left(8^a+9^a+12^a+14^a\right)$$
$$+9\times 2^n\times 13^a-\left(12\times 13^a+3\times 14^a\right). \quad (5)$$

Proof: Let G be the molecular graph of $NS_3[n]$. By using definition and Table 2, we deduce

$$NM_1^a\left(NS_3[n]\right)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right]^a$$
$$=3\times 2^n(4+4)^a+3\times 2^n(4+5)^a+3\times 2^n(5+7)$$
$$+\left(9\times 2^n-12\right)(6+7)^a+\left(3\times 2^n-3\right)(7+7)^a$$
$$=\left(3\times 2^n\right)\left(8^a+9^a+12^a+14^a\right)$$
$$+9\times 2^n\times 13^a-\left(12\times 13^a+3\times 14^a\right).$$

We establish the following results by using Theorem 11.

Corollary 11.1. The first neighborhood index of $NS_3[n]$ is

$$NM_1\left(NS_3[n]\right) = 246\times 2^n - 198.$$

Corollary 11.2. The first hyper neighborhood index of $NS_3[n]$ is

$$HNM_1\left(NS_3[n]\right) = 2976\times 2^n - 2616.$$

Corollary 11.3. The modified first neighborhood index of $NS_3[n]$ is

$${}^{m}NM_1\left(NS_3[n]\right) = \frac{12219}{19656}\times 3\times 2^n - \frac{207}{182}.$$

Proof: Put a = 1, 2, –1 in equation (5), we get the desired results.

We compute the general second neighborhood index of $NS_3[n]$.

Theorem 12. The general second neighborhood index of a dendrimer $NS_3[n]$ is given by

$$NM_2^a\left(NS_3[n]\right) = 3\times 2^n\left(16^a + 20^a + 35^a + 49^a\right) + 9\times 2^n\times 42^a\left(12\times 42^a + 3\times 49^a\right) \quad (6)$$

Proof: By using definition and Table 2, we derive

$$NM_2^a\left(NS_3[n]\right) = \sum_{uv\in E(G)}\left[S_G(u)S_G(v)\right]^a$$

$$= 3\times 2^n(4\times 4)^a + 3\times 2^n(4\times 5)^a + 3\times 2^n(5\times 7)^a + (9\times 2^n - 12)(6\times 7)^a + (3\times 2^n - 3)(7\times 7)^a$$

$$= 3\times 2^n\left(16^a + 20^a + 35^a + 49^a\right) + 9\times 2^n\times 42^a\left(12\times 42^a + 3\times 49^a\right).$$

We obtain the following results by using Theorem 12.

Corollary 12.1. The second neighborhood index of $NS_3[n]$ is

$$NM_2\left(NS_3[n]\right) = 738\times 2^n - 651.$$

Corollary 12.2. The second hyper neighborhood index of $NS_3[n]$ is

$$HNM_2\left(NS_3[n]\right) = 21519\times 2^n - 28367.$$

Corollary 12.3. The modified second neighborhood index of $NS_3[n]$ is

$${}^{m}NM_2\left(NS_3[n]\right) = \frac{134211}{576240}\times 3\times 2^n - \frac{51}{147}.$$

Proof: Put a = 1, 2, –1 in equation (6), we obtain the desired results.

Theorem 13. The inverse sum indeg index neighborhood index of a dendrimer $NS_3[n]$ is

$$INM\left(NS_3[n]\right) = \frac{9515}{468}\times 3\times 2^n - \frac{1281}{26}.$$

Proof: By using definition and Table 2, we derive

$$INM\left(NS_3[n]\right) = \sum_{uv\in E(G)}\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}$$

$$= 3\times 2^n\left(\frac{4\times 4}{4+4}\right) + 3\times 2^n\left(\frac{4\times 5}{4+5}\right) + 3\times 2^n\left(\frac{5\times 7}{5+7}\right) + (9\times 2^n - 12)\left(\frac{6\times 7}{6+7}\right) + (3\times 2^n - 3)\left(\frac{7\times 7}{7+7}\right)$$

$$= \frac{9515}{468}\times 3\times 2^n \times - \frac{1281}{26}.$$

Theorem 14. The harmonic neighborhood index of a dendrimer $NS_3[n]$ is

$$HNM\left(NS_3[n]\right) = \frac{4073}{468}\times 3\times 2^n - \frac{207}{91}.$$

Proof: By using definition and Table 2, we deduce

$$HNM\left(NS_3[n]\right) = \sum_{uv\in E(G)}\frac{2}{S_G(u)+S_G(v)}$$

$$= 3\times 2^n\left(\frac{2}{4+4}\right) + 3\times 2^n\left(\frac{2}{4+5}\right) + 3\times 2^n\left(\frac{2}{5+7}\right) + (9\times 2^n - 12)\left(\frac{2}{6+7}\right) + (3\times 2^n - 3)\left(\frac{2}{7+7}\right)$$

$$= \frac{4073}{3276}\times 3\times 2^n - \times\frac{207}{91}.$$

Theorem 15. The general neighborhood index of a dendrimer $NS_3[n]$ is

$$NM^a\left(NS_3[n]\right) = \left(3\times 4^a + 2\times 5^a + 3\times 6^a + 6\times 7^a\right)\times 3\times 2^n - \left(12\times 6^a + 18\times 7^a\right).$$

Proof: Using the definition and Table 2, we obtain

$$NM^a\left(NS_3[n]\right) = \sum_{uv\in E(G)}\left[S_G(u)^a + S_G(v)^a\right]$$

$$= 3\times 2^n\left(4^a + 4^a\right) + 3\times 2^n\left(4^a + 5^a\right) + 3\times 2^n\left(5^a + 7^a\right) + (9\times 2^n - 12)\left(6^a + 7^a\right) + (3\times 2^n - 3)\left(7^a + 7^a\right)$$

$$= \left(3\times 4^a + 2\times 5^a + 3\times 6^a + 6\times 7^a\right)\times 3\times 2^n - \left(12\times 6^a + 18\times 7^a\right).$$

We compute the general first neighborhood polynomial of $NS_3[n]$.

Theorem 16. The general first neighborhood polynomial of a dendrimer $NS_3[n]$ is

$$NM_1^a\left(NS_3[n], x\right) = 3\times 2^n x^{8^a} + 3\times 2^n x^{9^a} + 3\times 2^n x^{12^a} + (9\times 2^n - 12)x^{13^a} + (3\times 2^n - 3)x^{11^a}. \quad (7)$$

Proof: By using definition and Table 2, we deduce

$$NM_1^a\left(NS_3[n], x\right) = \sum_{uv\in E(G)} x^{\left[S_G(u)+S_G(v)\right]^a}$$

$$= 3\times 2^n x^{(4+4)^a} + 3\times 2^n x^{(4+5)^a} + 3\times 2^n x^{(5+7)^a} + (9\times 2^n - 12)x^{(6+7)^a} + (3\times 2^n - 3)x^{(7+7)^a}$$

$$= 3\times 2^n x^{8^a} + 3\times 2^n x^{9^a} + 3\times 2^n x^{12^a} + (9\times 2^n - 12)x^{13^a} + (3\times 2^n - 3)x^{14^a}.$$

The following results are obtained by using Theorem 16.

Corollary 16.1. The first neighborhood polynomial of $NS_3[n]$ is

$$NM_1(NS_3[n],x)=3\times 2^n x^8+3\times 2^n x^9+3\times 2^n x^{12}+(9\times 2^n-12)x^{13}+(3\times 2^n-3)x^{14}.$$

Corollary 16.2. The first hyper neighborhood polynomial of $NS_3[n]$ is

$$NM_1(NS_3[n],x)=3\times 2^n x^{64}+3\times 2^n x^{81}+3\times 2^n x^{144}+(9\times 2^n-12)x^{169}+(3\times 2^n-3)x^{196}.$$

Corollary 16.3. The modified first neighborhood polynomial of $NS_3[n]$ is

$${}^mNM_1(NS_3[n],x)=3\times 2^n x^{\frac{1}{8}}+3\times 2^n x^{\frac{1}{9}}+3\times 2^n x^{\frac{1}{12}}+(9\times 2^n-12)x^{\frac{1}{13}}+(3\times 2^n-3)x^{\frac{1}{14}}.$$

Proof: Put a = 1, 2, –1 in equation (7), we get the desired results.

We compute the general second neighborhood polynomial of a dendrimer $NS_3[n]$.

Theorem 17. The general second neighborhood polynomial of dendrimer $NS_3[n]$ is

$$NM_2^a(NS_3[n],x)=3\times 2^n x^{16^a}+3\times 2^n x^{20^a}+3\times 2^n x^{35^a}+(9\times 2^n-12)x^{42^a}+(3\times 2^n-3)x^{49^a}. \quad (8)$$

Proof: By using definition and Table 2, we derive

$$NM_2^a(NS_3[n],x)=\sum_{uv\in E(G)} x^{[S_G(u)S_G(v)]^a}$$
$$=3\times 2^n x^{(4\times4)^a}+3\times 2^n x^{(4\times5)^a}+3\times 2^n x^{(5+7)^a}+(9\times 2^n-12)x^{(6\times7)^a}+(9\times 2^n-12)x^{(7\times7)^a}$$
$$=3\times 2^n x^{16^a}+3\times 2^n x^{20^a}+3\times 2^n x^{35^a}+(9\times 2^n-12)x^{42^a}+(3\times 2^n-3)x^{49^a}.$$

We establish the following results from Theorem 17.

Corollary 17.1. The second neighborhood polynomial of $NS_3[n]$ is

$$NM_3(NS_3[n],x)=3\times 2^n x^{16}+3\times 2^n x^{20}+3\times 2^n x^{35}+(9\times 2^n-12)x^{42}+(3\times 2^n-3)x^{49}.$$

Corollary 17.2. The second hyper neighborhood polynomial of $NS_3[n]$ is

$$HNM_3(NS_3[n],x)=3\times 2^n x^{256}+3\times 2^n x^{400}+3\times 2^n x^{1225}+(9\times 2^n-12)x^{1764}+(3\times 2^n-3)x^{2401}.$$

Corollary 17.3. The modified second neighborhood polynomial of $NS_3[n]$ is

$${}^mNM_2(NS_3[n],x)=3\times 2^n x^{\frac{1}{16}}+3\times 2^n x^{\frac{1}{20}}+3\times 2^n x^{\frac{1}{35}}+(9\times 2^n-12)x^{\frac{1}{42}}+(3\times 2^n-3)x^{\frac{1}{49}}.$$

Proof: Put a = 1, 2, –1 in equation (8), we obtain the desired results.

Theorem 18. The inverse sum indeg neighborhood polynomial of a dendrimer $NS_3[n]$ is

$$INM(NS_3[n],x)=3\times 2^n x^2+3\times 2^n x^{\frac{20}{9}}+3\times 2^n x^{\frac{35}{12}}+(9\times 2^n-12)x^{\frac{42}{13}}+(3\times 2^n-3)x^{\frac{7}{2}}.$$

Proof: Using definition and Table 2, we deduce

$$INM(NS_3[n],x)=\sum_{uv\in E(G)} x^{\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)}}$$
$$=3\times 2^n x^{\frac{4\times4}{4+4}}+3\times 2^n x^{\frac{4\times5}{4+5}}+3\times 2^n x^{\frac{5\times7}{5+7}}+(9\times 2^n-12)x^{\frac{6\times7}{6+7}}+(3\times 2^n-3)x^{\frac{7\times7}{7+7}}$$
$$=3\times 2^n x^2+3\times 2^n x^{\frac{20}{9}}+3\times 2^n x^{\frac{35}{12}}+(9\times 2^n-12)x^{\frac{42}{13}}+(3\times 2^n-3)x^{\frac{7}{2}}.$$

Theorem 19. The harmonic neighborhood polynomial of a dendrimer $NS_3[n]$ is

$$HNM(NS_3[n],x)=3\times 2^n x^{\frac{1}{4}}+3\times 2^n x^{\frac{2}{9}}+3\times 2^n x^{\frac{1}{6}}+(9\times 2^n-12)x^{\frac{2}{13}}+(3\times 2^n-3)x^{\frac{1}{7}}.$$

Proof: By using definition and Table 2, we have

$$HNM(NS_3[n],x)=\sum_{uv\in E(G)} x^{\frac{2}{S_G(u)+S_G(v)}}$$
$$=3\times 2^n x^{\frac{1}{4}}+3\times 2^n x^{\frac{2}{9}}+3\times 2^n x^{\frac{1}{6}}+(9\times 2^n-12)x^{\frac{2}{13}}+(3\times 2^n-3)x^{\frac{1}{7}}.$$

Theorem 20. The general neighborhood polynomial of a dendrimer $NS_3[n]$ is

$$NM^a(NS_3[n],x)=3\times 2^n x^{2\times4^a}+3\times 2^n x^{4^a+5^a}+3\times 2^n x^{5^a+7^a}+(9\times 2^n-12)x^{6^a+7^a}+(3\times 2^n-3)x^{2\times7^a}$$

Proof: By using definition and Table 2, we deduce

$$NM^a(NS_3[n],x)=\sum_{uv\in E(G)} x^{[S_G(u)^a+S_G(v)^a]}$$
$$=3\times 2^n x^{2\times4^a}+3\times 2^n x^{4^a+5^a}+3\times 2^n x^{5^a+7^a}+(9\times 2^n-12)x^{6^a+7^a}+(3\times 2^n-3)x^{2\times7^a}.$$

IV. CONCLUSION

In this study, we have proposed some neighborhood indices and their polynomials. We have also computed some new and old neighborhood indices and their polynomials for two types of dendrimers.

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