Physical Significance of Wave Function

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Abstract
Wave function is a mathematical tool used in quantum mechanics to describe any physical system. Currently there is no physical explanation about wave function. This paper describes wave function as function spacetime fluctuation.

Keywords – *Wave function, space time interval, space time curvature*

I. INTRODUCTION

The physical interpretation of the wave function is context dependent shown below.

1) One particle in one spatial dimension
2) One particle in two spatial dimensions
3) One particle in three spatial dimensions
4) One particle in one dimensional momentum space

II. DERIVATION OF WAVE FUNCTION IN TERMS OF MIX SPACE TIME MATRIX COMPONENTS

From paper "Electroweak Field Equations" [1], we know that electromagnetic equations are derived from mix matrix shown below.

$$s^2 = \overset{ri}{\eta}_{\mu\nu}\, x_\mu x_\nu \rightarrow \text{Eq1}$$

Schrödinger equation is a partial differential equation that describes how the quantum state of a physical system changes with time.

Schrödinger equation if considered as similar case of electromagnetic waves we can say that wave function is function of space time interval components.

$$\psi(r,t) = f(\overset{ri}{\eta}_{\mu\nu}\, x_\mu x_\nu) \rightarrow \text{Eq2}$$

III. ONE PARTICLE IN ONE SPATIAL DIMENSION

Let us consider particle in box scenario as shown below in Fig 1.

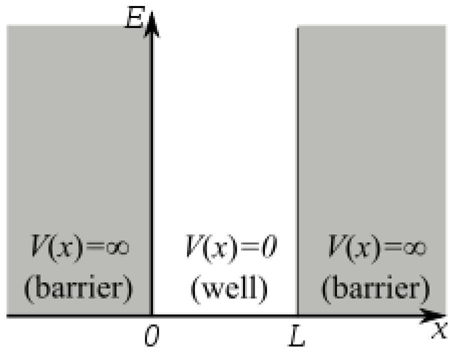

Fig 1: The barriers outside a one-dimensional box have infinitely large potential, while the interior of the box has a constant, zero potential.

Wave equation of particle in one dimension is given as [2].

$$\psi(x,t) = |\Psi_0|\, e^{-i(kx-\omega t)} \rightarrow \text{Eq3}$$

Units [3] of the above wave function is 1/(meter)$^{1/2}$. If we solve Schrödinger equation for particle inside a box we get following equation [4].

$$\psi_n(x,t) = \begin{cases} \sqrt{\dfrac{2}{L}}\sin(\dfrac{n\pi}{L}x) & 0 \le \mathrm{x} \le \mathrm{L} \\ 0 & 0 > \mathrm{x} > \mathrm{L} \end{cases} \rightarrow \text{Eq4}$$

From Eq4 we know that

$$|\Psi_0| = \sqrt{\frac{2}{L}} \rightarrow \text{Eq5}$$

From Eq2 if we consider only one dimension we would get following equation

$$\psi(x,t) = f(x_r x_i) \rightarrow \text{E6}$$

Where $x_r x_i$ is the first component of mix matrix $\overset{ri}{\eta}_{\mu\nu}$. From paper "Electroweak Field Equations" [5], we know that we can expand the matrix by assuming

$$x_r x_i = (\overset{1}{x_r^2}) + (\overset{1}{x_i^2}) \rightarrow \text{Eq7}$$

We can express x_r and x_i as

$$\overset{1}{X} = (\overset{1}{x_r}) + i(\overset{1}{x_i}) \rightarrow \text{Eq8}$$

We know that these dimensions in Eq8 should be waves and should be function of kx-wt. Hence we can write it as.

$$(\overset{1}{x_r}) = |\overset{1}{X}| \sin(kx - \omega t) \rightarrow \text{Eq9}$$

$$(\overset{1}{x_i}) = \left|\overset{1}{X}\right| \cos(kx-\omega t) \rightarrow \text{Eq10}$$

If we put Eq9 and Eq10 in Eq8 we get following expression.

$$\overset{1}{X} = \left|\overset{1}{X}\right| e^{-i(kx-\omega t)} \rightarrow \text{Eq11}$$

We know from Eq8 that

$$\left|\overset{1}{X}\right| = \sqrt{x_r x_i} \rightarrow \text{Eq12}$$

Putting Eq12 in Eq11 we get

$$\boxed{\overset{1}{X} = \sqrt{x_r x_i} e^{-i(kx-\omega t)}} \rightarrow \text{Eq13}$$

Eq 13 is very close to Eq3 but still units do not match, to match units one will have to divide Eq13 by $\lambda^{\frac{3}{2}}$ (where λ is wavelength) we will get

$$\boxed{\frac{\overset{1}{X}}{\lambda^{\frac{3}{2}}} = \frac{\sqrt{x_r x_i}}{\lambda^{\frac{3}{2}}} e^{-i(kx-\omega t)}} \rightarrow \text{Eq14}$$

Eq 14 and Eq 3 are now same and units also match, so now we can say that.

$$\boxed{\psi(x,t) = \frac{\sqrt{x_r x_i}}{\lambda^{\frac{3}{2}}} e^{-i(kx-\omega t)}} \rightarrow \text{Eq15}$$

Where

$$|\Psi_0| = \frac{\sqrt{x_r x_i}}{\lambda^{\frac{3}{2}}} \rightarrow \text{Eq16}$$

The figure below shows wave functions of first 2 energy levels

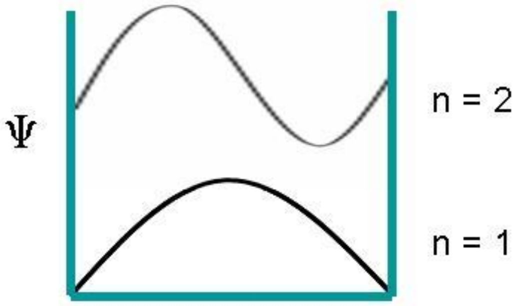

Fig2: The wave function for a particle in a box at the n=1 and n=2 energy levels look like this

The probability of finding a particle a certain spot in the box is determined by squaring Psi[6]. The probability distribution for a particle in a box at the n=1 and n=2 energy levels looks like this

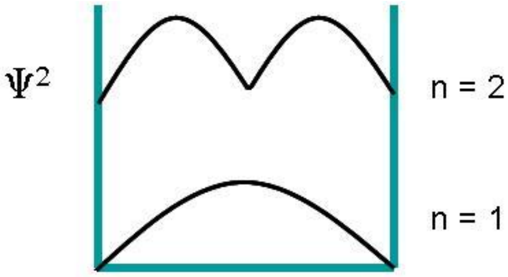

Fig 3: The probability distribution for a particle in a box at the n=1 and n=2

Multiplying Eq4 by its conjugate and equating it with Eq14 (multiplying E14 with its conjugate) we get probability density function

$$P_{df} = \left|\frac{\overset{1}{X}_n}{\lambda^{\frac{3}{2}}}\right|^2 = \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)\right)^2 \rightarrow \text{Eq17}$$

$$P_{df} = \frac{(x_r x_i)_n}{\lambda^{\frac{3}{2}}} = \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)\right)^2$$

From Eq 4 we know that wave number is

$$k = \frac{n\pi}{L} \rightarrow \text{Eq18}$$

We know that wave number is given by

$$k = \frac{2\pi}{\lambda} \rightarrow \text{Eq19}$$

Equating Eq18 and Eq19 we get

$$\lambda = \frac{2L}{n} \rightarrow \text{Eq20}$$

Putting Eq20 in Eq17 we get

$$\frac{(x_r x_i)_n}{\left(\frac{2L}{n}\right)^3} = \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)\right)^2 \rightarrow \text{Eq21}$$

$$\frac{(x_r x_i)_n}{\left(\frac{8L^3}{n^3}\right)} = \frac{2}{L}\left(\sin\left(\frac{n\pi}{L}x\right)\right)^2 \rightarrow \text{Eq22}$$

$$\frac{(x_r x_i)_n}{\left(\frac{8L^2}{n^3}\right)} = 2\left(\sin\left(\frac{n\pi}{L}x\right)\right)^2 \rightarrow \text{Eq23}$$

$$\boxed{(x_r x_i)_n = \frac{16L^2}{n^3}\left(\sin\left(\frac{n\pi}{L}x\right)\right)^2} \rightarrow \text{Eq24}$$

If we plot the graph of $x_r x_i$ for n = 1 , 2 we get following output see Fig 4 and Fig 5.

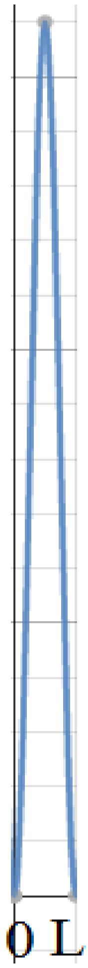

Fig 4: plot of $(x_r x_i)_n$ for n=1

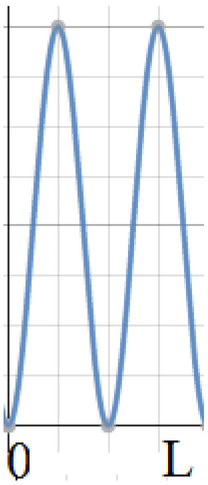

Fig 5: plot of $(x_r x_i)_n$ for n=2

From Fig 3, 4 and 5 we can say that Probability of finding a particle is correlated to $(x_r x_i)_n$ in space, which is evident from Eq17 as well.

Quantum mechanics successfully predicted the probability without knowing the reason behind it; the main reason is fluctuation in mix matrix space time components.

IV. ONE PARTICLE IN TWO SPATIAL DIMENSION

If a particle is trapped in a two-dimensional box, it may freely move in the x and y-directions, between barriers separated by lengths L_x and L_y respectively. Using a similar approach to that of the one-dimensional box, it can be shown that the wave functions is

$$\psi_{n_x,n_y} = \sqrt{\frac{4}{L_x L_y}} \sin(k_{n_x} x) \sin(k_{n_y} y) \rightarrow \text{Eq24}$$

Where the two-dimensional wave vector is given by

$$k_{n_x,n_y} = k_{n_x} \hat{x} + k_{n_y} \hat{y} \rightarrow \text{Eq25}$$

Equivalent of wave function of Eq24 in terms of mix matrix can be written as

$$\psi_{n_x,n_y} = \frac{X^{1} Y^{1}}{\lambda_x^{\frac{3}{2}} \lambda_y^{\frac{3}{2}}} = \frac{\sqrt{(x_r x_i)(y_r y_i)}}{\lambda_x^{\frac{3}{2}} \lambda_y^{\frac{3}{2}}} \sin(k_{n_x} x) \sin(k_{n_y} y) \rightarrow \text{Eq26}$$

$$\psi_{n_x,n_y} = \frac{X^{1} Y^{1}}{\lambda_x^{\frac{3}{2}} \lambda_y^{\frac{3}{2}}} = \sqrt{\frac{4}{L_x L_y}} \sin(k_{n_x} x) \sin(k_{n_y} y) \rightarrow \text{Eq27}$$

Because we know that

$$\frac{\sqrt{(x_r x_i)(y_r y_i)}}{\lambda_x^{\frac{3}{2}} \lambda_y^{\frac{3}{2}}} = \sqrt{\frac{4}{L_x L_y}} \rightarrow \text{Eq28}$$

If we plot Eq6 we get following output (see Fig 6) in terms of wave function.

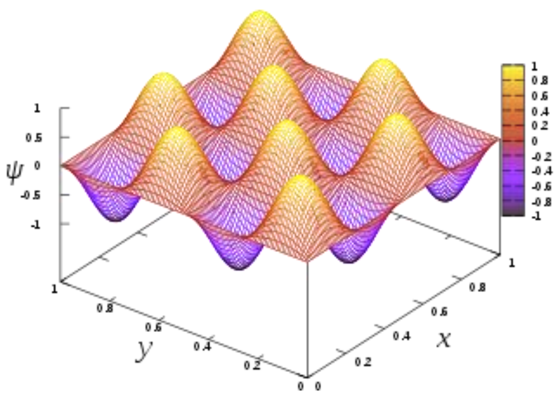

Fig 6: The wave function of a 2D well with nx=4 and ny=4

From Eq26 and Eq27 we can see the wave function is function of $(x_r x_i)$ and $(y_r y_i)$.

V. ONE PARTICLE IN 3 SPATIAL DIMENSION

If particle is trapped in 3 dimensional box, its wave function is give as.

$$\psi_{n_x,n_y,n_z} = \sqrt{\frac{8}{L_x L_y L_z}} \sin(k_{n_x} x)\sin(k_{n_y} y)\sin(k_{n_z} y)$$

→Eq29

Where the 3-dimensional wave vector is given by

$$k_{n_x,n_y,n_z} = k_{n_x}\hat{x} + k_{n_y}\hat{y} + k_{n_z}\hat{z}$$ →Eq30

Similarly if we follow approach done for particle in one or two dimension we can write wave function in terms of mix matrix in Eq31

$$\psi_{n_x,n_y} = \frac{\overset{1}{X}\overset{1}{Y}\overset{1}{Z}}{(\lambda_x\lambda_y\lambda_z)^{\frac{3}{2}}} = \frac{\sqrt{(x_r x_i)(y_r y_i)}}{(\lambda_x\lambda_y\lambda_z)^{\frac{3}{2}}} \sin(k_{n_x} x)\sin(k_{n_y} y)\sin(k_{n_z} z)$$

→Eq30

VI. ONE PARTICLE IN ONE DIMENSIONAL MOMENTUM SPACE

Momentum-space wave functions frequently are most easily obtained by the Fourier transform of the already available position-space wav e function. For the particle in a One-dimensional box the Fourier transforms is given by the following equation [7]:

$$\phi(n,p,L) = \frac{1}{\sqrt{2\pi}} \int_0^L e^{-ipx} \sqrt{\frac{2}{L}} \sin(\frac{n\pi}{L} x)dx$$

→Eq31

If we put Eq16 value in Eq31 we get following equation

$$\boxed{\phi(n,p,L) = \frac{1}{\sqrt{2\pi}} \int_0^L e^{-ipx} \frac{\sqrt{x_r x_i}}{\lambda^{\frac{3}{2}}} \sin(\frac{n\pi}{L} x)dx}$$

→Eq32

From above we can say that momentum space function is also function of $x_r x_i$ (mix matrix component's).

$$\phi(n,p,L) = f(x_r x_i, n, p, L)$$ →Eq33

VII. CONCLUSION

Wave function and Momentum wave function are due to fluctuations in mix space time matrix components, as we move towards the particle these disturbance will be higher compared to when away from the particle. These disturbances give rise to probability in Quantum mechanics.

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