Theoretical Resolution of EPR Paradox

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Abstract:

Bell's inequality together with widely used CHSH inequality on joint expectation of spin states between Alice and Bob ends are taken as a decisive theorem to challenge EPR Paradox. All experiments on the polarisation states of entangled particles violate Bell's inequality as well as CHSH inequality and Einstein's version of reality now is considered wrong somehow. Bell's argument clearly shows that quantum mechanics simultaneously can not justify both 'locality' and 'counterfactual definiteness'. 'Locality' means that no causal connection can be made faster than light. 'Counterfactual definiteness', sometimes called 'realism' refers to the assumption on the existence of properties of objects prior to measurement. This paper shows on the contrary that Quantum Mechanics is neither. Any kind of hidden variable or information that was advocated in EPR paradox to make Quantum Mechanics compatible with 'local realism' at once becomes redundant if standard Copenhagen interpretation is upheld assuming Quantum Mechanics 'nonlocal' and 'counterfactually indefinite'. This article shows that Quantum Mechanics is nonlocal as regards conservation laws and does not possess counterfactual definiteness for that particle of the entangled pair whose spin state is detected earlier in the polariser and possess counterfactual definiteness for that particle detected later in the analyser.

Keywords — *Quantum Mechanics; Entangled Pair; EPR Paradox; Bell's inequality; CHSH Inequality; Conservation Laws; Non-locality; Counter Factual indefiniteness*

I. INTRODUCTION

Many literatures and papers on Bell's inequality [2] appeared in the theoretical discussions for resolution of EPR Paradox [1] particularly after 1982 when the experiment of Allen Aspect [3] had shown clear violation of Bell's inequality. Other experiments on polarisation states of entangled particles did the same thing using CHSH inequality [4] and EPR paradox is believed being resolved with Einstein's version of reality collapsed forever. In 'Bell test' experiments there might be problems of experimental design or set up that could affect the validity of experimental findings. Therefore experiments with loophole free violation of Bell's inequality have already been designed and those types of experiments are being designed with much higher accuracy and precision till to date. It is the task of experimental physicists to refine their experiments to such a limit that a comprehensive conclusion can be drawn over the matter of dispute.

However two simple hypotheses can theoretically settle the matter in line with the experimental evidences against EPR paradox –

Hypotheses 1: 'Properties' do not exist prior to n i.e. 'counterfactual definiteness' is meaningless in quantum world.

Hypotheses 2:'Conservation Laws' operate only on measured values and therefore have no difficulties being non-local.

With these two hypotheses one can easily calculate the joint expectation for the outcome at Alice and Bob ends. However this article is intended to proceed without these hypotheses and the resulting contradiction with the prediction of quantum mechanics will be seen being removed if and only if the said two hypotheses are taken as conclusive features of quantum world.

In 'Bell test' experiments the two polariser are never like two independent coins thrown for tossing to show up 'head' i.e. 'spin up' and 'tail' i.e. 'spin down' independently of each other. Difference of orientations of the polarisers does never render entries of both particles of the entangled pair with equal 'easiness' and with 'equal difficulties' if prior spin states were randomly selected during their birth following angular momentum conservation. The polarisers do never act independently for an entangled pair but difference of orientations gives a preference on the spin states of one over the other if a definite spin doublet were produced for the entangled particles at their birth in 'Einstein's world'. We will assume that each entangled pair is produced with prior spin orientations. But by Copenhagen interpretation as there is nothing prior to detection so there is nothing like preferred orientation of the polariser which the particle will select to show up its spin. The particle detected first must behave like an 'unpolarised' one before detection and its 'spin up' and 'spin down' chance is 50:50 after detection. The counter particle then must instantly is endowed with the relevant 'spin' as demanded by conservation law and its 'spin up' or ' spin down' detection probability must then be governed by law of polariser in each trial. But if one wants to challenge Einstein, one cannot take it for granted that 'quantity' does not exist before measurement.

II. ENTANGLED PAIR'S SAMPLE SPACE AND LAWS OF PHYSICS IN LOCAL REALISM

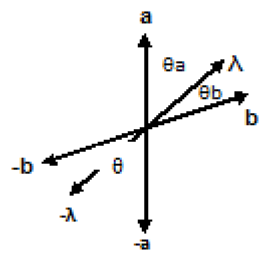
Law of physics is invariant not only in a particular frame but also in all inertial frames. Laws of physics are either laws of 'phenomena' or laws operative on 'boundary conditions'. For example Navier- Stokes equation is the law of fluid flow and 'no slip' condition is the law of boundary. Maxwell's four electrodynamical laws constitute the law for field propagation and continuity equations for normal component of **B** field and tangential component for **E** field are laws at boundary. Similarly conservation of angular momentum is the law by which angular momenta are distributed between entangled particles and law of orientation (e.g. Malus law in classical optics) is the law operative on the orientation of polariser. To the particle it is not only a 'to be or not to be' situation just before the polariser, but also with what 'easiness' or with what 'difficulty' it will do that is also an important question. This 'easiness' or 'difficulty' together with any kind of functional settlement between the entangled particles is expected to have close association with the required hidden variable λ.

In the present article a direct approach without any hidden variable is being outlined to calculate the joint expectation defying the two hypotheses resulting in contradiction with the prediction of quantum mechanics and thereafter calculating the same in accordance with the two hypotheses. The result is just the prediction of quantum mechanics. Clearly the necessity of any hidden variable is found redundant.

Let the 'easiness' or 'difficulty' of the entangled particles with which their polarisation states be determined after detection are two variables θ_a and θ_b .Here θ_a is the angle between the prior spin direction $\boldsymbol{\lambda}$ and field direction **a** for particle in Alice's polariser and θ_b is the angle between prior spin direction $-\boldsymbol{\lambda}$ and field direction **b** for Bob's polariser. Although the output is either +1 or -1 but the decision is taken by the particle being equipped with law of probability on spot and for this no communication with anything is necessary. Decisions are local. Law of physics must produce identical results on identical wave functions if and only if boundary condition i.e., experimental conditions are identical. If experimental condition changes, the law operative on boundary will tell in what way outcome changes. Bell's analysis did not employ the law of polariser directly but put the responsibility on hidden variable λ to count the final spin states.

If a particle has spin axis a priori fixed with the axis of polariser then probability of 'success' (s) i.e., 'spin up' say is f (θ_a). Here θ_a is the angle made by a priori spin axis with polariser's axis of Alice. Obviously probability of failure i.e, probability of 'spin down' is given by P (f) = 1-P (s). For photon, f $(\theta_a) = \cos^2 \theta_a$ and for e^-, f $(\theta_a) = \cos^2 (\theta_a /2)$.Einstein said θ_a exists a priori whether we know it or not and Bohr said it does not exist before we observe it.

Now let's take the test run. Alice's field direction **a** is taken vertical. θ Is the angle of orientation of Bob's polariser with that of Alice. θ is chosen θ_1 and θ_2 in two separate runs with Bob's field directions along **b** and **c**. Let us now configure Bell's sample space of possible outcomes of spin states for a pair of entangled e^- e^+ produced either from π neutral decay or from decay of positronium. The parent particle has zero spin. Therefore spins of e^- or e^+ will be $\pm$ (1/2) $\hbar$ or $\mp$ (1/2) $\hbar$ at the time they are produced to conserve angular momentum. Let is **a** unit vector in the field direction of Alice in Stem Gerlach analyser and **b** is unit vector in the field direction of Bob's analyser where angle between **a** and **b** is θ. Let us assume that e^- produced in the pair production has its spin in direction of $\boldsymbol{\lambda}$ and its counterpart i.e. e^+ has its spin orientation along$-\boldsymbol{\lambda}$ before they enter into the respective analysers. The situation is represented by figure 1.



(Figure-1: A And B are Unit Vectors in the Magnetic Field Directions in the Analysers Of Alice and Bob. Λ Is the Initial Spin Direction of Any (E^- Say) Of The Entangled Particles. Θ is the Angle Between. A And B . Θ_a And Θ_b Are Angles Between Λ And A , Λ And B Respectively.)

In Bell's argument, outcomes of spin states of Alice and Bob can be written as A(**a,** λ) = $\pm$1 and B(**b,** λ) = $\pm$1 in the unit of (1/2) $\hbar$. Here λ is any possible hidden variable and locality assumption demands that A $\neq$ A (**a, b,** λ) and B $\neq$ B (**a, b,** λ). In other words outcome of an experiment on a system is independent of the actions performed on a different system which has no causal connection with the first. Simultaneously the existence of hidden variable λ determines the complete set of e^- e^+ pair. λ may vary in some way that we neither understand nor control from one pair to another. Einstein wanted to know about the hidden variable which is missing from quantum mechanics. So he wanted a theory whose experiments uncover properties (such as λ) that are pre-existing. Such a theory is called a counterfactual one and sometimes this counterfactual definiteness property is called 'realism'. What Bell did is stunningly simple; he calculated the expectation value E (**a**,**b**) by the following equations-

$$E(\mathbf{a},\mathbf{b}) = \rho(\lambda)\, A(\mathbf{a}, \lambda)\, B(\mathbf{b}, \lambda)\, d\lambda \qquad (1)$$

$$\int \rho(\lambda)\, d\lambda = 1 \qquad (2)$$

$$A(\mathbf{a}, \lambda) = -B(\mathbf{a}, \lambda) \quad (3)$$

Equation (3) simply means that for parallel setting of both analysers, Alice and Bob must get opposite answers in each individual run as demanded by conservation of angular momentum. With these three equations Bell derived the following inequality

$$|E[A(\mathbf{a})B(\mathbf{b})] - E[A(\mathbf{a})B(\mathbf{c})]| \leq 1 + E[A(\mathbf{b})B(\mathbf{c})] \quad (4)$$

For certain specific settings or orientations of **a**, **b**, **c** quantum mechanics obey the inequality and not for any other arbitrary settings. Experiments too violate Bell′s inequality for these settings. So Bell′s theorem simply asserts that quantum mechanics cannot be both' local' and 'counterfactually definite' and Einstein is declared wrong. The nature of probability density given by equation (2) is unknown. One might say that the knowledge of p.d.f is not necessary as the inequality given by relation (4) is of main interest against which quantum mechanics at least can be put to test.

Let us now rewrite equation (1) without any hidden variable in the following way taking **λ**, **-λ** as the a priori directions of e^- or e^+ spins. Whether Alice gets an e^- or e^+, does not matter as Stern Gerlach experiment splits only 'spin up' from 'spin down' by opposite deflections depending upon sign of charge and upon direction of spin either existing a priori as demanded by 'realism' or produced by the experiment itself as demanded by Copenhagen interpretation. When a particle enters Alice's analyser, it enters with either its a priori spin in the direction of **λ** or in the direction of -λ; but the directions are the mutually exclusive if 'properties' exist prior to detection. Now let us modify Bell's expectation value of joint outcome (A B) considering the process being local and counterfactual. The notations to be used are (±**a**) for 'spin up' or 'spin down' for Alice and (±**b**) for Bob after detection and (± **λ**) for 'spin up' or 'spin down' prior to detection i.e. before entry to the analysers. According to classical configuration ('locality') outcomes of Alice and Bob are mutually independent as **a** and **b** can be chosen arbitrarily. Let's denote probability density function by ρ as usual. For example ρ (e^-, **λ**, **a**) represents probability that e^- detected with 'spin up' in Alice's analyser which had initial 'spin up' in the direction of **λ**. So ρ (e^-,**-λ**, **a**) is probability of 'spin up' for e^- detected in Alice's analyser which had initial 'spin down' in the direction of **-λ** before entry to analyser. ρ (e^-,**λ**, - **a**) represents probability of 'spin down' for e^- detected in Alice's analyser which had initial 'spin up' in the direction of **λ** and ρ (e^-,**-λ**, - **a**) represents probability of 'spin down' for e^- detected in Alice's analyser which had initial 'spin down' in the direction of **-λ**. For Bob same notation applies with counter particle e^+ and field direction **b.** Now it is clear that Alice can receive any particle e^- or e^+ and Bob can receive any of the counter particles. Although the nature of detected particles are mutually exclusive but prior to detection this destined feature does not exist by quantum mechanics. With due regards to 'locality' assumption let's assume that prior to detection, destinations of either e^- or e^+ is a 'determined' fact and Alice receives e^- and Bob receive e^+ in a particular trial. Denoting A for Alice and B for Bob we can write the prior event set and detected event set along with the probability set of Alice and Bob in a single trial considering either **λ** or **-λ** exists to Alice and either **-λ** or **λ** exists to Bob by virtue of angular momentum conservation from initial singlet state of zero angular momentum. Here we are assuming three very important things – (1)spin half particle can have only either 'spin up' (say along **λ**) or 'spin down' (say along –**λ**) state even before 'detection' against quantum mechanical prescription of superposition of states; (2) if e^- has a-priori spin in the direction of **λ** then e^+ must have the spin along **-λ** and etc; it is destined a-priori that if A is on the way to receive e^- then sample space of B is to be designed with the counter particle e^+ and (3) the detected spin states ±**a** and ±**b** are mutually independent for any arbitrary angle between **a** and **b** other than parallel or antiparallel setting. The first assumption is nothing but assuming only 'counterfactual properties' and it means (**-λ** ∩ **λ**) =Φ in a single trial for a particl where Φ is a null set. The second one is principle of conservation of angular momentum. The third one is 'locality' assumption. With a view to these assumptions we are in a position to write the states of Alice and Bob prior to detections and after detections.

$$A(e^-)\,[\text{either } \boldsymbol{\lambda} \text{ or} {-\boldsymbol{\lambda}}] \equiv A(e^-)\,[(\boldsymbol{\lambda} \cup {-\boldsymbol{\lambda}}) - (\boldsymbol{\lambda} \cap {-\boldsymbol{\lambda}})]$$

$$\equiv A(e^-)\,[(\boldsymbol{\lambda} \cup {-\boldsymbol{\lambda}})]$$

$$\equiv A(e^-)\,[\{(\boldsymbol{\lambda}, \mathbf{a}) \cup (\boldsymbol{\lambda}, -\mathbf{a}) - (\boldsymbol{\lambda}, \mathbf{a}) \cap (\boldsymbol{\lambda}, -\mathbf{a})\} \cup \{(-\boldsymbol{\lambda}, \mathbf{a}) \cup (-\boldsymbol{\lambda}, -\mathbf{a}) - (-\boldsymbol{\lambda}, \mathbf{a}) \cap (-\boldsymbol{\lambda}, -\mathbf{a})\}]$$

$$\equiv A(e-)\,[\{(\boldsymbol{\lambda}, \mathbf{a}) \cup (\boldsymbol{\lambda}, -\mathbf{a})\} \cup \{(-\boldsymbol{\lambda}, \mathbf{a}) \cup (-\boldsymbol{\lambda}, -\mathbf{a})\}] \quad (5)$$

Here we have taken $(\boldsymbol{\lambda}, \mathbf{a}) \cap (\boldsymbol{\lambda}, -\mathbf{a}) = \Phi$ and $(-\boldsymbol{\lambda}, \mathbf{a}) \cap (-\boldsymbol{\lambda}, -\mathbf{a}) = \Phi$

Similarly we can write

$$B(\text{counter particle})\,[\text{either } {-\boldsymbol{\lambda}} \text{ or } \boldsymbol{\lambda}] \equiv B(e^+)\,[-\boldsymbol{\lambda} \cup \boldsymbol{\lambda}]$$

$$\equiv B(e^+)\,[\{(-\boldsymbol{\lambda}, \mathbf{b}) \cup (-\boldsymbol{\lambda}, -\mathbf{b})\} \cup \{(\boldsymbol{\lambda}, \mathbf{b}) \cup (\boldsymbol{\lambda}, -\mathbf{b})\}] \quad (6)$$

Now we can write joint outcome using the entire three –'localities', 'counterfactual properties' and 'principle of conservation of angular momentum' as

$$A(e^-) \cap B(e^+) = \{(\boldsymbol{\lambda}, \mathbf{a}) \cap (-\boldsymbol{\lambda}, \mathbf{b})\} \cup \{(\boldsymbol{\lambda}, \mathbf{a}) \cap (-\boldsymbol{\lambda}, -\mathbf{b})\}$$

$$\cup \{(\boldsymbol{\lambda}, -\mathbf{a}) \cap (-\boldsymbol{\lambda}, \mathbf{b})\} \cup \{(\boldsymbol{\lambda}, -\mathbf{a}) \cap (-\boldsymbol{\lambda}, -\mathbf{b})\} \cup \{(-\boldsymbol{\lambda}, \mathbf{a}) \cap$$

$$(\boldsymbol{\lambda}, \mathbf{b})\} \cup \{(-\boldsymbol{\lambda}, \mathbf{a}) \cap (\boldsymbol{\lambda}, -\mathbf{b})\} \cup \{(-\boldsymbol{\lambda}, -\mathbf{a}) \cap (\boldsymbol{\lambda}, \mathbf{b})\} \cup$$

$\{(-\boldsymbol{\lambda},-\mathbf{a}) \cap (\boldsymbol{\lambda},-\mathbf{b})\}$ (7)

As the instruments can take only local decisions by locality assumption so Alice's detected state (**a**) or (-**a**) is independent of Bob's arbitrarily chosen state (**b**) or (-**b**). Now considering the event terms within { } as independent and any two { } pairs as mutually exclusive on ground of 'locality' and 'counter factualness' we get the joint probability distribution as

$$\rho[(\pm\boldsymbol{\lambda}, \pm\mathbf{a})\cap(\mp\boldsymbol{\lambda}, \pm \mathbf{b})]$$

$$= (C)\ [\rho(\boldsymbol{\lambda}, \mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b})+\rho(-\boldsymbol{\lambda}, -\mathbf{b})\}+\rho(\boldsymbol{\lambda}, -\mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b}) + \rho(-\boldsymbol{\lambda}, -\mathbf{b})\} + \rho(-\boldsymbol{\lambda}, \mathbf{a})\ \{\rho(\boldsymbol{\lambda}, \mathbf{b})+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}+\rho(-\boldsymbol{\lambda},-\mathbf{a})\{\rho(\boldsymbol{\lambda}, \mathbf{b})]+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}] \quad (8)$$

As $\rho(-\boldsymbol{\lambda}, -\mathbf{a}) = \rho(\boldsymbol{\lambda}, \mathbf{a})$, $\rho(-\boldsymbol{\lambda}, -\mathbf{b}) = \rho(\boldsymbol{\lambda}, \mathbf{b})$, $\rho(\boldsymbol{\lambda}, -\mathbf{a}) = \rho(-\boldsymbol{\lambda}, \mathbf{a})$ and $\rho(\boldsymbol{\lambda}, -\mathbf{b})] = \rho(-\boldsymbol{\lambda}, \mathbf{b})$, so by symmetry of the respective situations (figure 1), so we can find the normalisation constant (C) in the following way
.

$$(C)\ \rho\ [(\pm\boldsymbol{\lambda}, \pm\mathbf{a}) \cap (\mp\boldsymbol{\lambda}, \pm \mathbf{b})] = 1$$

$$\text{or,}(C)\ [\rho(\boldsymbol{\lambda}, \mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b})+\rho(\boldsymbol{\lambda}, \mathbf{b})\}+\rho(\boldsymbol{\lambda},-\mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b}) +\rho(\boldsymbol{\lambda}, \mathbf{b})\}+\rho(-\boldsymbol{\lambda}, \mathbf{a})\{\rho(\boldsymbol{\lambda}, \mathbf{b})+\rho(\boldsymbol{\lambda},-\mathbf{b})\}+\rho(\boldsymbol{\lambda}, \mathbf{a}) \{\rho(\boldsymbol{\lambda}, \mathbf{b})]+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}] = 1$$

$$\text{or, } (C)\ \{\rho(\boldsymbol{\lambda}, \mathbf{b})+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}\ 2\ \{\rho(\boldsymbol{\lambda}, \mathbf{a})+\rho(\boldsymbol{\lambda}, -\mathbf{a})\} = 1$$

As the states $\rho(\boldsymbol{\lambda}, \mathbf{a})$ and $\rho(-\boldsymbol{\lambda}, -\mathbf{a})$ are equivalent in terms of classical picture (figure 1) and as quantum mechanics says that the probability of occurrence of the single state **a** from the single state **λ** is given by $\cos^2(\theta_a/2)$, so in the given situation each of $\rho(\boldsymbol{\lambda}, \mathbf{a})$ and $\rho(-\boldsymbol{\lambda}, -\mathbf{a})$ should have the value $(½)\cos^2(\theta_a/2)$ and by similar argument each of $\rho(\boldsymbol{\lambda}, -\mathbf{a})$ and $\rho(-\boldsymbol{\lambda}, \mathbf{a})$ should have the value $(½)\sin^2(\theta_a/2)$. Same thing must hold also for $\rho(\boldsymbol{\lambda}, \mathbf{b})$ and $\rho(\boldsymbol{\lambda}, -\mathbf{b})$ with angle θ_b. If separate probabilities $\rho(\pm\boldsymbol{\lambda}, \pm\mathbf{a})$ and $\rho(\mp\boldsymbol{\lambda}, \pm \mathbf{b})$ are calculated from equations (5) and (6), it is found that each probability density function is separately normalised. We see that the joint probability distribution given by equation (8) contains total of 8 terms following angular momentum conservation; but considering the decisions taken by each polariser as 'local decisions' or 'independent decisions', the joint probability computed multiplying equations (5) and (6) might contain 16 terms and the density function is itself normalised. But 8 of them defy angular momentum conservation. $\rho(\boldsymbol{\lambda}, \mathbf{a})\ \{\rho(\boldsymbol{\lambda}, \mathbf{b})+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}$, $\rho(\boldsymbol{\lambda}, -\mathbf{a})\ \{\rho(\boldsymbol{\lambda}, \mathbf{b})]+\rho(\boldsymbol{\lambda}, -\mathbf{b})\}$, $\rho(-\boldsymbol{\lambda}, \mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b})+\rho(-\boldsymbol{\lambda}, -\mathbf{b})\}$, $\rho(-\boldsymbol{\lambda}, -\mathbf{a})\{\rho(-\boldsymbol{\lambda}, \mathbf{b})]+\rho(-\boldsymbol{\lambda}, -\mathbf{b})\}$ are 8 such terms those go against angular momentum conservation in the joint probability distribution. Therefore it is clear that 'conservation law' can greatly reduce size of the sample space of joint outcome to half of the simple product space. With a view to this and considering all elementary brackets equally likely, we see that the value of normalising constant (C) in equation (8) cumes out to 2 instead of 1.

Now representing $A(\boldsymbol{\lambda},\mathbf{a})=+1$, $A(\boldsymbol{\lambda},-\mathbf{a})= -1$ as spin eigen values detected in Alice's analyser and $B(-\boldsymbol{\lambda}, \mathbf{b})= +1$, $B(-\boldsymbol{\lambda},-\mathbf{b})= -1$ as spin eigen values in Bob's analyser we get the following expectation for joint outcome (A . B) from equation (8).

$$E\ [A\ (\lambda, \pm\mathbf{a}).B\ (-\boldsymbol{\lambda}, \pm\mathbf{b})] =2\ [\rho\ (\boldsymbol{\lambda}, \mathbf{a})\ \{\rho\ (-\boldsymbol{\lambda}, \mathbf{b}) -\rho\ (-\boldsymbol{\lambda}, -\mathbf{b})\}-\rho\ (\boldsymbol{\lambda}, -\mathbf{a})\ \{\rho\ (-\boldsymbol{\lambda}, \mathbf{b})-\rho\ (-\boldsymbol{\lambda}, -\mathbf{b})\} +\rho\ (-\boldsymbol{\lambda}, \mathbf{a}) \{\rho\ (\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}-\rho\ (-\boldsymbol{\lambda}, -\mathbf{a})\{\rho\ (\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}] \quad (9)$$

$$=2\ [\rho\ (\boldsymbol{\lambda}, \mathbf{a})\ \{\rho\ (-\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, \mathbf{b})\}-\rho\ (\boldsymbol{\lambda}, -\mathbf{a})\ \{\rho\ (-\boldsymbol{\lambda}, \mathbf{b})- \rho\ (\boldsymbol{\lambda}, \mathbf{b})\} +\rho\ (-\boldsymbol{\lambda}, \mathbf{a})\{\rho(\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda},-\mathbf{b})\}-\rho\ (\boldsymbol{\lambda}, \mathbf{a})\{\rho\ (\boldsymbol{\lambda}, \mathbf{b})- \rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}]$$

$$= 2\ \{\rho\ (\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}\ \{-\rho\ (\boldsymbol{\lambda}, \mathbf{a}) + \rho\ (\boldsymbol{\lambda}, -\mathbf{a}) +\rho(-\boldsymbol{\lambda}, \mathbf{a})- \rho\ (\boldsymbol{\lambda}, \mathbf{a})\}$$

$$= 2\{\rho\ (\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}\ (-2)\ \{\rho\ (\boldsymbol{\lambda}, \mathbf{a}) - \rho\ (\boldsymbol{\lambda}, -\mathbf{a})\ \} \quad (10)$$

$$= -\ 4\ \{\rho\ (\boldsymbol{\lambda}, \mathbf{b})-\rho\ (\boldsymbol{\lambda}, -\mathbf{b})\}\ \{\rho\ (\boldsymbol{\lambda}, \mathbf{a}) - \rho\ (\boldsymbol{\lambda}, -\mathbf{a})\ \}$$

$$= -\ (4/4)[\cos^2(\theta_b/2) -\sin^2(\theta_b/2)][\cos^2(\theta_a/2)-\sin^2(\theta_a/2)]$$

$$= -\ (½)\ [2\cos\theta_b \cos\theta_a]$$

$$= -\ (½)\ [\cos\ (\theta_b + \theta_a) + \cos\ (\theta_b - \theta_a)]$$

$$= -(½)\ [\cos\theta + \cos\ (\theta\text{-}2\ \theta_a)]$$

$$=-(½)\ [\cos\theta + \cos\ (2\ \theta_b - \theta)] \quad (11)$$

If we sum up the expectation value for all possible **λ** i.e. for all possible θ_a for a large number (N) of trials and divide the sum by N we get overall expectation given by

$$E\ [A(\pm\mathbf{a})\ B(\pm\mathbf{b})\ ]=(1/N)\Sigma\ \ E\ [A(\pm\mathbf{a}, \boldsymbol{\lambda})\ B(\pm\mathbf{b}\ ,-\boldsymbol{\lambda}), \theta_a\ ]$$

$$= -(½)\ (N\cos\theta)\ /\ N= -\ (½)\ (\cos\theta)$$

$$= -\ (½)\ (\mathbf{a}.\ \mathbf{b}) \quad (12)$$

III. RESULTS AND DISCUSSION

The expectation value for joint outcome of Alice-Bob pair appears to be just 'half' the prediction of quantum mechanics in local and counterfactual nature of events for all angles except for $\theta_a = 0^0$ or 180^0 or for $\theta_b = 0^0$ or 180^0 in each run. The question is where did something go wrong? We used two laws - principle of conservation of angular momentum to split the initial apriori spin states ('counterfactual definiteness' of the entangled pair) in the directions **λ**, - **λ** and the probability law of quantum mechanics itself that gives probability of detecting spin state (up or down) in the field directions **a** or - **a** and **b** or - **b** if the particles had initial spin states in the directions **λ**, - **λ**. It appears that there might be something wrong to assume counterfactual property of the pair as a whole. The answer can be found in equation (11).Here we have to start from equation (11) instead of equation (12). The reason is that equation (12) represents expectation over large number of runs with θ_a and θ_b chosen randomly ; but equation (11) represents expectation over a single run with particular θ_a and θ_b. If we could choose perfect alignment or perfect opposition either at Alice's end (i.e. either $\theta_a = 0^0$ or 180^0 i.e. **λ** and **a** parallel or antiparallel with $\theta_b = \theta - \theta_a = \theta$ or $\theta - 180^0$) or at Bob's end (i.e. $\theta_b = 0^0$ or 180^0 i.e. **λ** and **b** parallel or antiparallel with $\theta_a = \theta - \theta_b = \theta$ or $\theta - 180^0$) then only E [A (±**a**) B (±**b**)] = - **a. b** for all orientations as per the prescription of quantum mechanics. It means a very serious thing – the entangled particles had no initial spin states before detection; either of the particles behaves in such a way that it is born with its spin state either being parallel (**λ**= either **a** or **b**) or being antiparallel (**λ**= either - **a** or - **b**) to the field direction of either Alice or Bob ! In fact both of 'parallel' and 'antiparallel spin' are equally probable and equation (9) clearly shows the superposition of parallel and antiparallel states . It means spin states are ' created ' by either of the polarisers and the other analyser placed in any arbitrary orientation detects 'spin up 'or 'spin down' by law of probability imposed by the angle θ between them. This is in fact half of the famous 'Copenhagen view'. Let's discuss the thing in short detail in the following paragraphs.

Let's suppose that Alice's polariser produces the spin states of the entangled particles. In this situation spin states in the direction of **a** and – **a** are equally probable. $\theta_a = 0^0$ means that Alice's polariser produces spin states of the entangled particles in such a way that it is parallel to **a** for Alice's particle and antiparallel to **a** for Bob's particle by virtue of angular momentum conservation. Obviously when ρ (**λ**= **a, a**) = (½) for Alice, then ρ (**λ**= **a, -a**) =0 along with all of ρ (- **λ**) =0 for Alice. In that case angular momentum conservation determines spin states in Bob's analyser in the direction –**λ**= - **a** and then for Bob

ρ (-**λ**=-**a**, **b**)=(½)sin^2(θ/2), ρ(-**λ**=-**a**,-**b**)=(½)cos^2(θ/2).

Similarly $\theta_a = 180^0$ means that Alice's polariser produces spin states of the entangled particles in such a way that it is antiparallel to **a** for Alice' particle and parallel to **a** for Bob's particle by virtue of law of angular momentum conservation. Obviously when ρ(-**λ**= -**a**,- **a**)= (½) for Alice then it is clear that ρ (-**λ**=- **a, a**) =0 along with all of ρ (**λ**) =0 for Alice. In that case spin states in Bob's analyser is 'counterfactually definite' and is determined along **λ**=**a** and then for Bob

ρ(**λ** =**a**,**b**)=(½) (cos^2(θ/2) , ρ(**λ** =**a**,-**b**)=

(½)sin^2(θ/2). So from equation (8) we have

ρ[(±**λ**=±**a**, ±**a**) ∩ (∓**λ** =∓**b**, ∓**b**)]

=2[ρ(**a**,**a**){ρ(-**a**,**b**)+ρ(-**a**,-**b**)}+ρ(-**a**,-**a**){ρ(**a**,**b**)+ρ(**a**,-**b**)}]

=1

And from equation (9) we have

E [(±**λ**=±**a**, ±**a**)∩(∓**λ**=∓**b**, ∓**b**)]

=2[ρ(**a**,**a**){ρ(-**a**, **b**)-ρ(-**a**,-**b**)}-ρ(-**a**,-**a**){ρ(**a**,**b**)-ρ(**a**,-**b**)}]

= -**a . b**.

Same thing would happen with equal chance to Bob's polariser had Bob first detected the particle. It were detected then either with 'spin up' along **b** or with 'spin down' along -**b** with equal chance and then Alice's particle must have its spin opposite to that of Bob by virtue of angular momentum conservation and Alice's detection of 'spin up 'or 'spin down' will occur with probability density function (½) cos^2(θ/2) or (½) sin^2(θ/2) as the case may be.

Quantum world is therefore neither local with respect to the applicability of conservation laws nor is 100 percent 'counterfactually definite ' for the whole entangled pair. Quantum mechanics possesses 50 percent 'counterfactual definiteness' for entangled pair i.e. it possesses 'counterfactual definiteness' only for that particle whose spin state is detected later. Until one of the particle of the pair is detected, 'properties' cannot be attributed to any of them; but once it is detected for one of them in the pair, it becomes 'counterfactually definite' to the counter particle by virtue of nonlocal nature of conservation laws. Conservation law by itself can not associate a definite value for a particular property to any 'quantum object' until actual measurements take place. It can operate only when detection is done.

IV. CONCLUSION

EPR paradox appeared in 1935 to challenge 'Copenhagen view' of quantum mechanics was a bolt from blue to Niles Bohr.EPR upheld two things – 'properties' like 'position' or 'momentum' possess 'counterfactual definiteness' i.e. they exist even without observation and one can bypass Heisenberg's indeterminacy principle by observing momentum of one of the entangled particle together with position measurement on the counterpart. Obviously one could of course know position and linear momentum together of any from the pair by virtue of linear momentum conservation. But this elegant argument fails if 'properties' are not 'counterfactually definite'. It has already been shown that 'spin state' of any one of the entangled pair is bound to be produced in one polariser when and only when it is observed and that of its counterpart instantly acquires opposite spin by virtue of angular momentum conservation even if it is not observed. In this sense the counter particle's 'property' is 'counterfactually definite' and the corresponding analyser only ascertains its spin state by law of probability. To be sure whether this happens or not, we can take different path length till to the detectors from the site of 'pair production'. It is obvious that spin states of the pair will be produced by the nearer detector (say Alice's polariser) and so out of total count 50 percent will show 'spin up' and 50 percent will show 'spin down' as 'spin' never existed prior to measurement. Therefore Bob's detector acts not as 'polariser' but as 'analyser' as it will detect 'spin up' and 'spin down' count imposed by probability factor $\cos^2(\theta/2)$ or$\sin^2(\theta/2)$ on already detected spin state. But there remains another great twist and this is the nonlocality of the conservation laws. Laws of physics operate on measured values. If a pair of particles is produced from single one then conservation laws are applied only between initial and observed final states even if the particles are widely separated from each other. Definite properties are never produced initially to individual . As soon as 'property' of any one is determined or rather produced by experiment, 'property' of the other is instantly produced following conservation rule and this of course we call 'entanglement'. It will be not out of place to conclude that entanglement is intrinsic character of any 'event' because events occur obeying certain conservation laws; and whenever there is a conservation law, there must be entanglement between the events produced. Einstein's formalism of four dimensional world takes no such account of entanglement.

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