Neutrino Interaction Cross-sections from MeV to GeV scales of Energy

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Abstract: The neutrino cross-sections on nucleons and nuclei across all energy scales from eV to GeV and beyond are very small and the uncertainties in flux are also large. Moreover, there are many processes which contribute simultaneously to neutrino interaction cross-sections. This makes the analysis of data and theoretical predictions more challenging. Yet it is crucial to determine the cross-sections as an essential ingredient to the growing pursuit of neutrino oscillations and the kind of signals observed there-off. In the present contribution, we have generated the events using NuWro event generator to compute the cross-sections for neutrino interactions with different targets, from MeV to GeV scales of energy, with an emphasis on comparison with experimental findings.

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1. Introduction

Generally cross-sections determine the effective area of an interaction between two or more particles but due to an extremely small size in case of the subatomic domain, targeting of individual particles at each other is out of way. At best, one may shoot a lot of them in the same general area. For instance, the proton-proton to top-antitop cross-section is measured by counting the top-antitop pairs created when a given number of protons are hit at each other. For neutrinos, the problem is even worse; because these did not interact with the other particles in the Universe even only after one second of the the Big Bang [1]. This is because these had even less energy-per-particle than the photons do, as electron/positron pairs are still around at that time. The reaction cross-sections of neutrinos are much smaller than those of other particle species, since these interact only by the weak force. For the elastic scattering of electron-neutrinos with electrons, the charged and neutral weak bosons ($W^{\pm}$ and Z) mediate the interaction, whereas the scattering of muon and tau neutrinos from electrons is mediated only by the neutral boson. Hence the total cross-sections are different.

Neutrino cross-sections are an important component in all neutrino experiments. Interest in the study of neutrino scattering has increased due to the need of such information in the interpretation of neutrino oscillation data. For both charged current (CC) and neutral current (NC) interaction channels, neutrino scattering results have been collected over many decades for a variety of targets and analysis techniques and also for the different detector technologies. New and intense accelerator based neutrino experiments such as NoνA [2], MINOS[3], MiniBOONE[4] etc., have been set up for neutrino oscillation investigations. These experiments are remeasuring the neutrino cross-sections with an incorporation of nuclear effects and of improved neutrino flux calculations. The conventional neutrino scattering measurements have provided an insight into nucleon structure and the Standard Model. While-as electron scattering measurements have uncovered many mysteries of nuclear structure, but not all of those mysteries have been solved. Neutrinos, because these sample the quarks and the nucleons in a nucleus differently from charged leptons, can provide new insight into the nuclear environment.

However, the elusive nature of neutrinos and the inherent difficulty in their detection creates many false signals. The detection of neutrino is inferred indirectly by the particles produced in the process, if at all it undergoes any interaction. As an example, if a ν_μ scatters quasi-elastically off a neutron, we are left with a charged muon and proton

$$\nu_\mu + n \rightarrow \mu^- + p$$

This implies that we need to build low mas detectors with extremely fine tracking capability. However, due to the extremely small size of neutrino cross-sections, the actually constructed detectors have large size and high mass to have enough event rate to perform useful stud-

ies. Therefore, the oscillation experiments such as NovA [2], T2K [5] and ArgoNeut [6] etc. rely on kton-scale far detectors, and use heavier targets such as carbon, iron, heavy water or argon.
Also, the neutrino experiments need to have very intense beams and fine-grained detectors so as to minimize the statistical and systematic errors. The precision detection techniques of the bubble chamber experiments in (70s-80s) measuring neutrino interactions were limited by low statistics and large flux uncertainties. The neutrino scattering measurements were mostly carried out at higher energies to probe nucleon structure and get higher statistics but in the recent past, the neutrino oscillation measurements have necessitated the lower beam energies in oscillation space such as in T2K[5]. There is also a need for these neutrino oscillation experiments to determine more precisely the signal and the background rates in their detectors in the first few GeV energy range. Such measurements had first been taken in bubble and spark chamber experiments but were not updated for decades. Therefore more and more such measurements are sorely needed for present and future neutrino oscillation experiments operating in this energy range.

The Nuwro event Generator In order to evaluate the feasibility of a proposed experiment, the neutrino event generators are an interface between theory and experiment as these play a vital role from conception of an experiment to the final physics publication, by way of optimizing the detector design, analyzing the collected data samples and evaluating systematic errors.
The present study has been carried out with NuWro [7] which is a relatively new Monte Carlo generator. It handles all important processes in neutrino-nucleus interactions as well as the hadronization due to deep inelastic scattering (DIS) and intra nuclear cascade. It is simple, elaborate and light weight but full featured and serves as a tool to assess the relevance of various theoretical models being investigated currently [8]. NuWro is organized around the event structure which contains three vectors of particles viz; incoming, temporary and outgoing. It also contains a structure with all the parameters used and a set of boolean flags tagging the event as quasi-elastic (QE), resonance excited scattering (RES), deep inelastic scattering (DIS), charged current (CC), neutral current (NC) etc. The input parameters are read at start-up from a text file and the events are stored in the ROOT tree file to simplify further analysis. The main motivation of the authors of NuWro was to have tools to investigate the impact of nuclear effects on directly observable quantities with all the final state interactions included. Now, NuWro simulates all the essential interactions and it is possible to be used in the experiments. As for instance, it has been included in the ICARUS experiment[9] with the task of improving NUX+FLUKA code in the single pion production region.
The basic algorithms of NuWro follow the other known codes such as NEUGEN/GENIE [10], NEUT [11], FLUKA [12], NUANCE[13] etc. In order to facilitate comparisons, NuWro allows running simulations choosing easily the values of parameters, sets of form factors, models of nucleus etc. The important features of NuWro are: fine hadronization model [14], description of resonance region without Rein-Sehgal approach [15] and implementation of spectral function as an improvement with respect to Fermi gas model [16]. NuWro is a generator of interactions only. The neutrino is selected according to information about the beam and the target is selected as (nucleus or free nucleon). This is followed by choosing a model of nucleus such as (Fermi gas, local density approximation, effective potential, spectral function) and the internuclear cascade is switched on.

2. Results and Discussions

To perform the neutrino cross-section measurements, the conventional neutrino beams are produced artificially in much the same way as neutrinos from cosmic ray interactions. A proton synchroton delivers bunches of high energy protons onto a fixed target creating a beam of pions and kaons. Pions decay to muons and muon-neutrinos with a branching ratio of 100% [17]. Similarly the kaons decay to muons and muon-neutrinos with a branching fraction of 63%.

$$M^+ \to \mu^+ + \nu_\mu \qquad (M = \pi, K)$$

A complication is that muons can also decay to generate electron-neutrinos which one may not want in a pure ν_μ beam.

$$\mu^+ \to e^+ + \bar{\nu_\mu} + \nu_e$$

Therefore, a long shield designed to absorb mesons is kept at the end of the decay tunnel to stop the charged muons which haven't decayed in the beam, letting only the neutrinos through.
The neutrino energy spectrum can be determined from the kinematics of the two-body decay of the meson. The

energy E_ν of the neutrino at an angle θ_ν in the laboratory frame can be related to the same quantities labelled with asterisk $(*)$ in the meson rest frame by the following equations

$$E_\nu = \gamma E_\nu^*(1+\beta cos\theta^*) \qquad cos\theta = \frac{cos\theta^* + \beta}{1+\beta cos\theta^*} \tag{1}$$

$$where \ \ \beta = \frac{p_M}{E_M}, \quad \gamma = \frac{E_M}{m_M}, \quad E_\nu^* = \frac{m_M^2 - m_\mu^2}{2m_M} \tag{2}$$

and where $p_M, \ E_M \ and \ m_M$ respectively refer to the momentum, energy and mass of the meson in consideration and the other quantities have their usual meanings. The minimum and maximum neutrino energies come from the cases where the neutrino is emitted forwards and backwards, respectively in the pion rest frame. That is when $cos\theta^* = \pm 1$. In this case

$$E_\nu^{min} = \frac{E_M}{m_M}\frac{m_M^2 - m_\mu^2}{2m_M}(1 - \frac{p_M}{E_M}) \tag{3}$$

$$= \frac{m_M^2 - m_\mu^2}{2m_M^2}(E_M - p_M) \tag{4}$$

$$= \frac{m_M^2 - m_\mu^2}{2(E_M^2 - p_M^2)}(E_M - p_M) \tag{5}$$

$$= \frac{m_M^2 - m_\mu^2}{2(E_M + p_M)} \tag{6}$$

$$= \frac{m_M^2 - m_\mu^2}{4E_M} \quad \sim 0 \tag{7}$$

and

$$E_\nu^{max} = \frac{E_M}{m_M}\frac{m_M^2 - m_\mu^2}{2m_M}(1 + \frac{p_M}{E_M}) \tag{8}$$

$$\sim \frac{m_M^2 - m_\mu^2}{m_M^2}E_M \tag{9}$$

$$= 0.427E_\pi \qquad for\ pions \tag{10}$$

$$= 0.954E_K \qquad for\ kaons \tag{11}$$

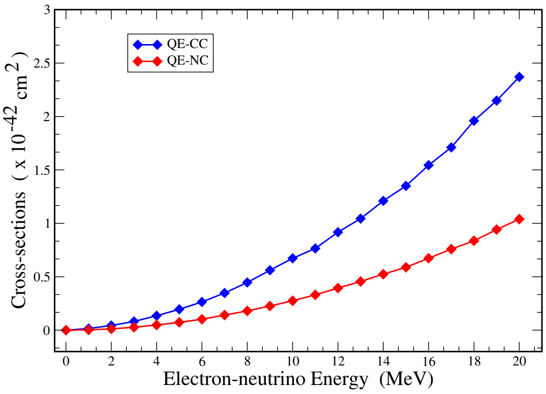

FIG. 1: electron-neutrino cross-sections with Oxygen using NuWro, in few MeV

where we assume that $| E | \sim | p |$. Therefore the low energy parts of the conventional neutrino beams arise from pion decay whereas the higher energy parts arise from kaon decay. Neutrino beams can be built to generate neutrinos in the energy range of 10 MeV to hundreds of GeV. The scale ultimately depends on the energy of the proton beam. As the proton beam energy increases, so does the meson energy and hence the higher neutrino energy. We have begun plotting our results from NuWro with the threshold energy values of a few MeV to look for electron-neutrino cross-sections for their interactions with oxygen as shown in FIG. 1. The cross-sections so obtained are very feeble. The choice of oxygen as target is because it is an abundant gas in the atmosphere. The Homestake experiment in USA [18] which was the earliest to detect electron-neutrinos involved the following reaction

$$\nu_e + \mathrm{cl} \rightarrow \mathrm{Ar} + \mathrm{e}$$

wherein the number of electron-neutrinos detected per day was determined by the number of Argon atoms produced. The average cross-section for $E_\nu > 0.82$ MeV was found out to be of the order of $10^{-45}cm^2/cl$ atom. With this small cross-section it was not a surprise to see that only 0.17 such conversions took place per day. The subsequent experiments such as SNO [19], SAGE [20] and GALLEX [21] which have independently measured the solar neutrino flux with greater precision, have essentially concluded that the solar neutrinos, which are all of electron type, do not oscillate as far as the flight length is less than the diameter of earth. Moreover, it has been found that whereas the matter effects are quite large for $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ oscillations but in vacuum the oscillation probability of electron-neutrinos to other flavours is reduced to 50% [22, 23]. Thus, not much of the reason for the lower value of electron-neutrino cross-sections in the first few MeVs of energy could be attributed to their oscillations from sun to earth. In vacuum, the neutrino oscillation probabilities are a function of the inverse of the neutrino energy. Therefore the current and next generation accelerator-based neutrino experiments such as MINOS [3], MiniBooNE [4], DUNE [24] etc. are focussing on neutrino energies of a few hundred MeV to a handful of GeV. The plot drawn in FIG. 2 gives our

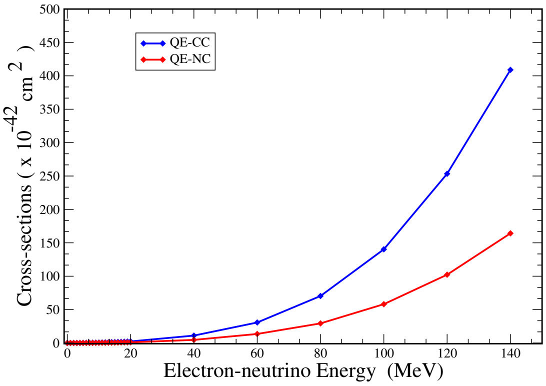

FIG. 2: electron-neutrino cross-sections with Oxygen using NuWro, in hundreds of MeV

cross-section measurements for electron-neutrino interactions with water in the first 100 MeVs of energy. It is needless to mention that water is the choice of target in many experiments [25] for being cheap and abundant. The trend of simulated cross-sections so obtained is much in conformity with the experimental results obtained with different targets [26].

There are a host of nuclei that are under experimental study e.g; the low-energy neutrino cross-sections on ^{127}I have been studied using the proton beam stops at the Los Alamos Meson Physics Facility [27]. Similarly the cross-sections on iron targets have also been explored with low-energy beams at the KARMEN experiment [28] where also the neutrino beam is provided from proton beam stops. When high-energy protons collide on a fixed target, these produce a large π^+ flux which is subsequently stopped and allowed to decay. The majority of low-energy neutrinos are produced from the decay at rest from stopped μ^+ and π^+, providing a well-characterized neutrino beam with energies below 50 MeV. However, the main uncertainty affecting these cross-section measurements arises primarily from the knowledge of the pion flux produced in the proton-target interactions.

The study of low-energy neutrino cross-sections features prominently in a variety of model-building scenarios such as supernova neutrinos, burst and relic neutrinos, solar neutrinos and low energy atmospheric neutrinos. A precise knowledge of the inclusive and differential cross-sections also serves as an essential input to neutrino oscillation tests. However, the number of direct experimental tests of these cross-sections is remarkably few. In our cross-section measurements with NuWro, we see that the neutrino-nucleus interactions at threshold energies proceed through quasi-elastic scattering only. However at 140 MeV, some more interaction channels start appearing on the scene and become significant towards first few Gev. The understanding of neutrino-nucleus interactions around 1 to 2 GeV energy is of great importance in analysing neutrino oscillation experiments [5, 29, 30]. This energy range is also of particular interest for being a cross over region, where the different important interaction channels turn on and off, ranging from elastic and quasi-elastic scattering (QE) [31], through single pion production via resonance excited scattering (RES) [32] and runs into deep inelastic scattering (DIS) [33].

Therefore we have simulated the neutrino interactions

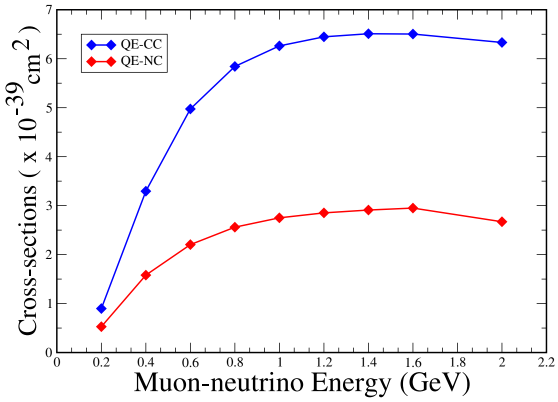

FIG. 3: ν_μ QE interactions with carbon using NuWro

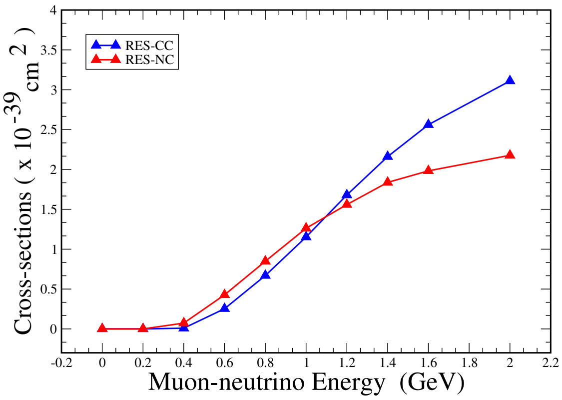

FIG. 4: ν_μ RES interactions with carbon using NuWro

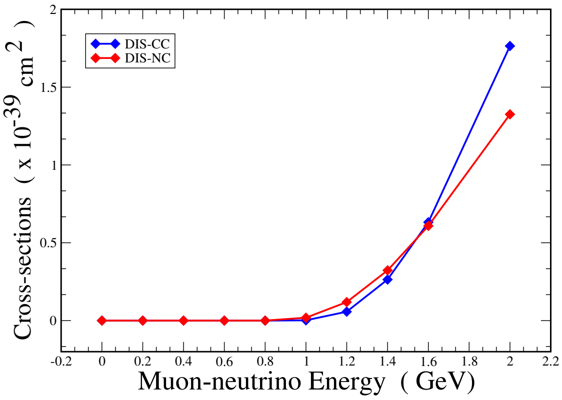

FIG. 5: ν_μ DIS interactions with carbon using NuWro

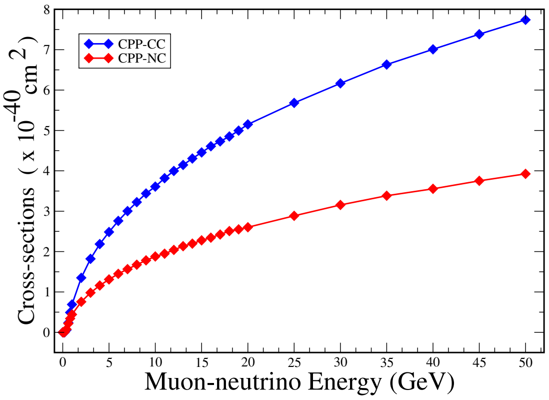

FIG. 6: ν_μ COH interactions with Iron using NuWro

through these three key processes for energies less than 2 GeV and have plotted the cross-sections for muon-neutrino interactions with Carbon Nucleus. Both the charged current and neutral current reaction values have been looked for in each process. The trends so obtained corresponding to these process viz; QE, RES and DIS have been shown in FIG 3, 4 and 5 respectively. At 1 GeV of neutrino energy, we are also in a transition region where QE and RES processes dominate but where there is also a significant DIS component being switched on as we increase the neutrino beam energy. A fourth channel of interaction for coherent pion production (COH) as shown in FIG.6 has also been plotted in our results although its cross-sections are more feeble than the the other three. Therefore, for the measurement of COH cross-sections, we have used a heavier target i.e iron and increased the energy range upto 50 GeV but the process does not show any significant improvement in the cross-sections. Charged current coherent pion production is a rare and poorly understood neutrino interaction. In this process a neutrino scatters off an entire nucleus coherently and produces a very forward-going pion and transfers little or no energy to the nucleus. The neutral current analog is a background with large uncertainties for electron appearance oscillation measurements [4, 34]. On the other hand, the charged current analog has only been seen until recently at high energy neutrino experiments but not at the 1 GeV experiments like K2K [35] and MiniBooNE [4].

It is also pertinent to mention that the bulk of our discussion has centered around measurements of ν_μ-nucleon scattering although many of these arguments also carry over to ν_τ scattering, except for one key difference that the energy threshold for the reaction is severely altered because of the large ν_τ lepton mass [26]. Also, the muon-neutrinos are preferred over electron-neutrinos in experimental studies because of their greater abundance than the later. However, the cross-section formulae for the electron and tau-neutrinos are the same as for muon-neutrinos.

There are adequate theoretical descriptions of quasi-elastic, resonance mediated, and deep inelastic scattering that have been formulated during all these years, however, there is no uniform description which globally describes the transition between these processes or how these should be combined. Moreover, the full extent to which nuclear effects impact this region is a topic that has only recently been appreciated.

At the energies of T2K [5] and NOvA [2], the quasi-elastic processes constitute a large fraction of the signal population. These are characterised by the appearance of a lepton and a nucleon in the final state.

$$\nu + n \to l^- + p \quad \text{or} \quad \bar{\nu} + p \to l^+ + n$$

Therefore, the measurement and modelling of QE scattering on nuclear targets, has gained more interest. In this process, the nucleus is essentially described to be composed of individual quasi-free nucleons (Impulse Approximation Scheme), like in the Fermi Gas (FG) model. The typical values of momentum transfer are large enough in the $\sim$ 1 GeV energy region and hence the Impulse Approximation can be used as a reliable approximation. However, in inclusive neutrino measurements there is always a significant fraction of low momentum transfer for events appearing to be CCQE. A remedy for this problem can be to impose suitable cuts in momentum transfer. Thus for an experimentalist it becomes quite obvious to look for QE-like events specified by a condition that there are no mesons in the final state. Most of the neutrino experiments use a relativistic Fermi-gas model [36] when simulating their QE scattering events, although many other independent particle approaches have been developed in recent years that incorporate more sophisticated treatments. Neutrino experiments have therefore begun to remeasure the absolute QE scattering cross-sections by making use of more reliable incoming neutrino fluxes made available in modern experimental setups.

The next most important process is the charged pion production. For a Cerenkov detector experiment or a time projection chamber (TPC), this process can pose as a background since the pion goes undetected as it can be absorbed in the nucleus before it ever reaches the active detector material and can therefore cause ambiguities in neutrino energy measurements. Charged pions are either absorbed or get converted into neutral pions via:

$$n + \pi^+ \longrightarrow p + \pi^0$$

The neutral pion production, while although less probable than charged pion production can still contribute a background in electron-neutrino appearance searches and must be well-simulated. Finally, deep inelastic scattering (DIS) events can also contribute a copious source of neutral pions which may contaminate an electron-neutrino appearance measurement and hence these channels must be well understood. As a result of these competing processes, the products of neutrino interactions include a variety of final states ranging from the emission of nucleons to more complex final states including pions, kaons, and collections of mesons. Moreover the cross-sections for neutrino-nucleon scatterings are not so precisely known as for leptonic reactions. This is due to the poor theoretical knowledge of the nucleon form factors.

Out of the three important interaction channels viz; quasi-elastic scattering (QE), single pion production (SPP) through resonance excited scattering and deep inelastic scattering (DIS), the later dominates the other two completely at intermediate and higher energies [37]. This is because up to a few GeVs, the neutrino scattering takes place from composite entities such as nucleons or nuclei but given enough energy, the neutrinos can actually begin to resolve the internal structure of the target. Whileas the description of the low energy regime exploits approaches like the elementary particle theory, effective field theories or microscopic models such as the Shell Model [38], the fermi gas is the basis for theoretical description in the high energy regime. The most common high energy interactions proceed through deep inelastic scattering (DIS) where the neutrino scatters off a quark in the nucleon via the exchange of a virtual $W^{\pm}$ or Z_0 boson producing a lepton and a hadronic system in the final state. Quarks cannot be individually detected as these recombine quickly and thus appear as a hadronic shower denoted by X.

$$\nu_\mu + N \rightarrow \mu^- + X \qquad \bar{\nu_\mu} + N \rightarrow \mu^+ + X$$
$$\nu_\mu + N \rightarrow \nu_\mu + X \qquad \bar{\nu_\mu} + N \rightarrow \bar{\nu_\mu} + X$$

The FIG. 7 shows our cross-section measurements for DIS process with iron and protons as targets. A lin-

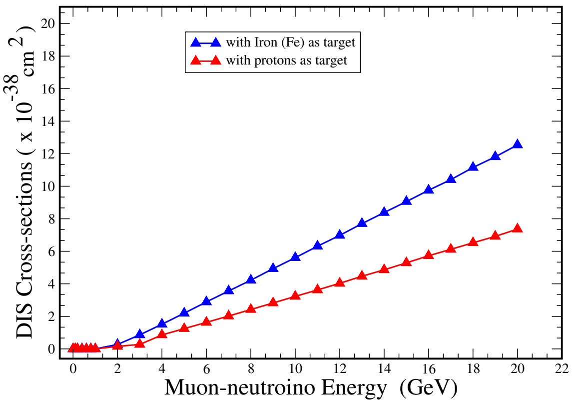

FIG. 7: ν_μ interactions with Iron and protons for charged current DIS process

ear dependence of the cross-sections on neutrino energy which is exhibited in the higher ranges is a confirmation of the quark parton model predictions [39]. This is because as the energy increases, the neutrino begins to probe the nucleus upto nucleon degrees of freedom. There is a point like scattering (one to one interaction) by the quarks which leads to a linear dependence of neutrino cross-sections. This linearity breaks down at low energies because of sensing of nuclear effects [40] by the projectile.

On the other hand, as expected for QE and RES processes, we observe that a linearly rising cross-section is damped by the form factors at higher neutrino energies and shows a saturation. Such results have been obtained in our measurements as shown in FIG. 8 for a heavier target (Iron) and in FIG. 9 for a lighter target (oxygen).

Moreover, as we transit from low-energy neutrino interactions to higher energies, our approach is primarily focussed on the scattering off a particular target in any state and whether that be a nucleus, a nucleon, or a parton. This approach is not accidental, as it is theoretically a well-defined problem to have the target constituents treated individually. Therefore it is to be acknowledged

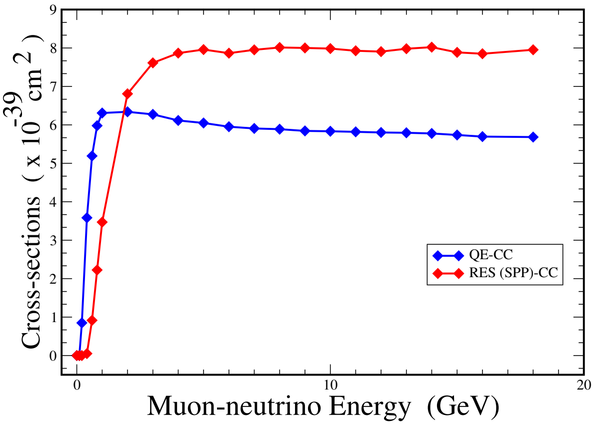

FIG. 8: cross-section saturation for ν_μ interactions with Iron using NuWro

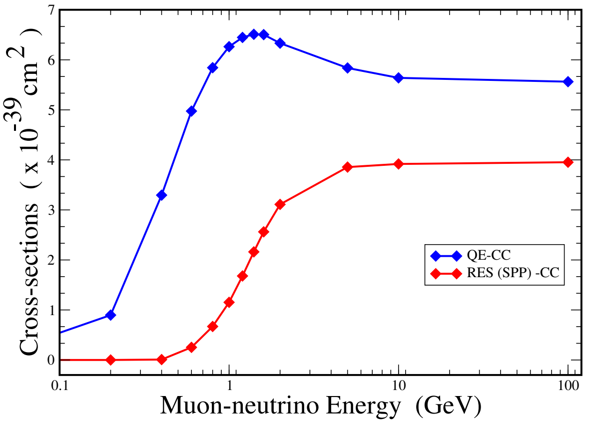

FIG. 9: cross-section saturation for ν_μ interactions with oxygen using NuWro

that the approach is also limited, as it is not able to incorporate the nucleus as a whole. This is the reason that the domains of low-energy and high-energy physics appear so disjointed in both approach and terminology. Until a full, comprehensive model of the entire neutrino-target interaction is formulated, we are constrained to follow this approach.
In the past years, a number of new results have been released on each of the above mentioned interaction channels by using several different target nuclei. These results are often not in agreement with theoretical predictions even from those anchored to deuterium or hydrogen measurements. The new measurements have started to give new hints to the theorists about the need to improve the theoretical description of neutrino interactions, which will ultimately lead to improve on precision oscillation measurements.

3. Conclusions

This work is an attempt to present a comprehensive study on neutrino-nucleus interaction cross-sections. Our discussion ranged from MeV to GeV scale of energy which is important since the naturally occuring solar and atmospheric neutrinos fit in this energy range. Whereas atmospheric neutrinos carry MeVs of energy, the atmospheric neutrinos fall in the GeV range. We generated a single flavour neutrino beam at different energies to measure the cross-sections for various neutrino interaction processes with different targets. The results serve to test the predictions of the Rein Sehgal model and verify old experimental measurements. While our results do not improve on precision, these serve as a useful cross check in a region with few measurements. Therefore more interaction channels we can measure and the more different nuclei we can test, the better we can understand both neutrinos and the nuclei with which these interact.

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The authors declare that there is no conflict of interest regarding the publication of this paper.

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