Photon Density and the Big Bang

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Abstract

A contracting universe, for parameter k = +1 in Robertson Walker metric at very high temperatures close to the singularity is modelled as a highly dense assembly of photons. We use the black body approximation for assembly of photons to show that available photon states in the phase space are completely filled up at temperature $T_S = (5.52).\ 10^{25}$ *K. When the temperature increases beyond* T_S*, the system transforms to a highly unstable, superheated, non-equilibrium state, and leads to the inflation of space-time and the big bang.*

Keywords

Big Bang, Cosmology, Quantum mechanics

I. INTRODUCTION

The contributions of Guth [1] and Linde [2] to the inflation model have led to many studies using different types of potentials and ideas from the standard model of particle physics. Linde [3] has reviewed different models of inflation. We consider the case k = + 1 of Robertson Walker metric for the Einstein's equation for the universe [4]. In this case, initially the universe expands but later on it starts contracting and collapses into a singularity. Mostly photons are present in the singularity region at T < Planck temperature $T_{PL} \sim 10^{32}$ K, and linear dimensions > Planck length $l_{PL} \sim 10^{-35}$ m.

II. PHOTON THERMODYNAMICS

We use quantum statistical mechanics [5] to evaluate properties of an assembly of photons in the singularity region. Mean number of states occupied by photons at temperature T in frequency interval ν and ν+ dν is

$$2f(\nu).\, d\nu \quad = (8\pi / c^3).\, \nu^2\, d\nu \ \ (e^{h\nu/kT} - 1)^{-1} .$$

Hence F, total number of states **per unit volume** in phase space, at temperature T is

$$F = 2\int_0^{\infty} f(\nu)\, .\, d\nu$$

$$= \int_0^{\infty} \left(\frac{8\pi}{c^3}\right).\, \nu^2\, .\, d\nu \ \Big/ \left(e^{\frac{h\nu}{kT}} - 1\right)$$

$$= 8\pi.(2.4).(kT/hc)^3 \quad . \qquad (1)$$

The mean energy density of radiation is thus

$$U_T = \int_0^{\infty} 8\pi h.\, c^{-3}.\, \nu^3\, d\nu \Big/ \left(e^{\frac{h\nu}{kT}} - 1\right)$$

$$= A.\, T^4 \qquad (2)$$

with $A = (8\pi^5/15)\,(k^4/h^3c^3) = (7.56)\, .\, 10^{-15}$ erg. cc^{-1} . deg^{-4} .

Radiation pressure

$$p = U_T/3 = (A/3)\, T^4$$

$$= (2.52).\ 10^{-15}.\ T^4 \ \text{bar}. \qquad (3)$$

The entropy of this highly dense photon state is S = k. ln Ω, where Ω is number of states accessible to system. According to Reif [5] "the total number of cells or states available to the system, Ω, is obtained by dividing accessible volume of the phase space contained between the momentum interval **k** and **k** + d**k** in the phase space by h^3 ". Hence Ω is the product of probability of finding photon in the interval **k** and **k** + d**k**, which is the Planck term $(e^{h\nu/kT} - 1)^{-1}$, and the number of states in this interval integrated over **k** from 0 to ∞ . Thus Ω is same as F, the number of states the photons occupy at temperature T. Hence

$$\Omega = F = 2\int_0^{\infty} f(\nu)\, .\, d\nu$$

$$= 8\pi.\, (2.4).(kT/hc)^3 \qquad (4) \quad \text{and}$$

$$S = k \ln F = k \ln [8\pi.(2.4).(k/hc)^3] + 3k.\ln T$$

$$= 3k\,(1 + \ln T) \qquad (5)$$

using $h = 6.67\ .10^{-27}$ erg.s^{-1} for Planck constant, k = 1.38 . 10^{-16} erg/K for Boltzmann constant, and c = 3. 10^{10} cm/sec. Since each state occupies volume h^3 in the phase space, it follows that total volume these states occupy **per unit volume** is

$$V_{ps}(T) = Fh^3 = 8\pi\,(2.4)\,(kT/hc)^3\, .\, h^3$$

$$= (19.785)\, T^3\, .\, h^3$$

In equilibrium state, volume fraction V_{ps} (T) is always less than or equal to 1. The equilibrium is retained up to singularity temperature $T = T_S$ defined by condition

$V_{ps} = F_S .h^3 = 1$ or

$F_S = h^{-3} = 8\pi\,(2.4)\,(kT_S /hc)^3$

$= (19.785)\, T_S^{\,3}$. (6) and

$T_S = (60.288)^{-1/3} . (c/k) = (5.52)\, 10^{25}$ K . (7)

III. NON-EQUILIBRIUM STATE

The assembly of photons transforms to a non-equilibrium state above T_S . U_T increases with increasing T. However, since V_{ps} cannot exceed 1 at $T > T_S$, F and S remain constant at temperatures $T > T_S$ and take values $F_S = (19.785)\, T_S^{\,3}$ and

$S_S = k \ln F_S = k. (2.985) + 3k .\ln T_S$

Using Eq. (7), this gives

$S_S = 3k\,(1 + \ln T_S) = k.(180.8)$ (8)

S_S is the entropy of photons confined to a volume of the order of $(l_X)^3$ where $l_X \sim$ but $> l_{PL}$. The non-equilibrium state is retained by the inward force of the collapsing universe which opposes the radiation pressure U_T . We have proposed two models for distribution of photons in non-equilibrium state. These models are discussed in Appendix.

IV. DISCUSSION

Let us suppose that the force of contraction, which drives the system to singularity, takes it beyond T_S to temperature $T_X = 10^3\, T_S = (5.52).10^{28}$ K which is $< T_{PL} \sim 10^{32}$ K. At temperature $(5.52).10^{28}$ K, the $U \sim 10^{12}$ times the value U has at T_S . The superheated state of photons has energy $U_X \sim 7.\, 10^{100}$ erg.cm $^{-3}$ and pressure $p \sim 10^{100}$ bar. When the confining pressure is dissipated, the release of energy of this superheated state causes a sudden and rapid inflation of the phase space (or space time) to return system to the equilibrium state. During expansion, entropy remains constant until the temperature decreases below T_S . This spontaneous, adiabatic expansion of the space time is inflation. The scale of expansion of the universe for Robertson Walker metric for the Einstein equation for photons, following Weinberg [4], is

$R_t = R_0 . e^{t/\tau}$. (9)

Here $(4\pi/3).\, R_0^{\,3}$ is the initial volume of the universe and

$\tau = (3c^2 /8\pi\, GU)^{1/2} = (4.61) . 10^{20} / T^2$ sec (10)

is the time constant of the expansion.

For $U_x = 10^{100}$ erg.cm $^{-3}$ considered here, above equation gives $\tau \sim 4.\, 10^{-37}$ sec. We use for present discussion $R_0 = 10^{-33}$ m which is $< l_{PL} \sim 10^{-35}$ m . It follows from Eqs. (9) and (10) that after $t = 4.10^{-36}$ sec, R_0 increases 10^5 times to 10^{-28} m, and after 10^{-35} sec, 10^{11} times to 10^{-22} m, which is the inflation scenario. Guth has considered $U_x = 10^{16}$ GeV.cm^{-3} (= $1.6\, .10^{29}$ K) which gives $\tau = 10^{-37}$ sec. Guth has discussed how the inflation resolves the flatness problem and horizon problem in the expanding universe. An important property of the inflation is that the following inflation the space time grows at a speed greater than the speed of light, as is seen from the example given here. The expansion of the universe slows down when the temperature falls below T_S or when the photon dominated universe becomes matter dominated universe. The inflation models mentioned earlier [1, 2] also relate the singularity - a state of low entropy, non-equilibrium state of linear dimension $> l_{PL} \sim 10^{-35}$ m and energy of the order of Planck energy $\sim 10^{18}$ GeV. Since there can only be photons under these conditions, in these models also the cause of inflation and big bang has to be the instability of assembly of photons discussed in this article.

The central region of some supermassive black holes can have highly compressed assembly of photons in non-equilibrium state above T_S under very high pressures and high temperatures. A supermassive black hole can use the mechanism described in this article as well to expel radiation and energy which can transform to the dark matter, known matter in the space beyond it. This is possible because space-time at the core of the black hole, on inflation, can grow at a speed greater than the speed of light just as in the big bang.

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[6] http://www.wolframalpha.com/ , insert integrate x^2/(e^x -1) from x = .. to ∞'

Table I.

Results of calculations for Model 1

$T_S = 5.52 \times 10^{25}$ K, $U_S = 7 \times 10^{88}$ erg/cc

F - number of photon states / vol

T / T_S	5	10	100	1000
F_n / h^{-3} (available in non-eq. state)	1	1	1	1
F_e / h^{-3} (required for hypothetical equilibrium state)	125	10^3	10^6	10^9
U / erg/cc	$(4.3).10^{91}$	$7.\ 10^{92}$	$7.\ 10^{96}$	$7.\ 10^{100}$
ν_n / sec	$(1.93).10^{39}$	$(3.09).10^{40}$	$(3.09).10^{44}$	$(3.09).10^{48}$
$\nu_n / \Delta\nu_n$	5.10^{-11}	$(1.12).10^{-13}$	$(1.32).10^{-25}$	$(1.32).\ 10^{-37}$

APPENDIX

Model 1

In the first model, we assume that since number of states available is very small, the Planck term can be ignored because the photons are forced to occupy all available states. The density of states in non-equilibrium state $F_n = F_S = h^{-3}$ for all temperatures $T > T_S$. Here suffix n stands for the non-equilibrium state. The mean energy per mode in non-equilibrium state is therefore

$$U / F_n = \{(\pi^4/15)(8\pi)(kT)^4/(hc)^3\} / \{8\pi . (2.4)(kT_S/hc)^3\}$$

$$= (2.70) . 10^{-2} (T/T_S)^3 . kT$$

$$= (3.726).10^{-16} . (T/T_S)^3 .T \qquad (11)$$

The average frequency ν_n is

$$\nu_n = (U/F_n)/h$$

$$= (5.586) .10^{10} . (T/T_S)^3 . T . \qquad (12)$$

The value of mean frequency in the equilibrium state ν_e can be calculated for equilibrium state using Eqs. (1) and (2). The calculation gives

$$h\nu_e / kT = 2.67 \quad \text{for } T \leq T_S . \qquad (13)$$

It follows from Eqs, (1) and (6) that the ratio of number of states available at $T > T_S$ to the number of states required if the system were to be in the equilibrium state, $F_n / F_e = (T_S/T)^3$. Hence at temperatures $T = 10^2 .T_S$ and $T = 10^3 .T_S$, available number of states is only 10^{-6} and 10^{-9} times those available in the (hypothetical) equilibrium state, respectively. Under these conditions we can assume that all the occupied states cluster around the average frequency ν_n. For $T >> T_S$ the expression for F_n can be derived from Eq. (1). We omit the Planck term on the right hand side of Eq. (1) under the assumption that the photons are forced to occupy all the available states. This gives

$$F_n = h^{-3} = (8\pi / c^3). \nu_n^{\,2} . \Delta\nu_n \qquad (14)$$

where $\Delta \nu_n$ is the width of narrow frequency band around ν_n. By rearranging the terms we get

$$\Delta\nu_n / \nu_n = (3.62).10^{108} / (\nu_n)^3 . \quad (15)$$

Table I shows the calculated values of F, total energy density U, ν_n and $(\Delta\nu_n / \nu_n)$ for some representative values of (T / T_S). The variation of u_ν (energy at frequency ν) with ν is shown in Fig. 1. Curve A represents the variation in the equilibrium state and it is valid up to temperature T_S and the curve marked B the variation in non-equilibrium state $(T > T_S)$ for Model 1. Very rapid increase in values of U and average frequency ν_n with increasing T is seen from the Table I.

Model 2

In this model we assume that only the modes above a certain frequency ν_0 are occupied in the non-equilibrium state. This gives preference only to the high frequency modes. For this model, Eq. (1) for total number of modes h^{-3} can be written as

$$F = h^{-3} = 8\pi . (kT/hc)^3 . \int_{\eta_0}^{\infty} \frac{\eta^2 . d\eta}{\exp \eta - 1}$$

$$= 8\pi . (kT/hc)^3 . I_1 . \qquad (16)$$

Here $\eta_0 = h\nu_0 / kT = (8.10^{-38}) \nu_0 / T \quad (17)$ and

$$I_1 = \int_{\eta_0}^{\infty} \frac{\eta^2 . d\eta}{\exp \eta - 1}$$

$$= (8\pi)^{-1} .(c/k)^3 / T^3$$

$$= 4.09 \times 10^{77} / T^3 \qquad (18)$$

This integral can be evaluated [6] for $\eta_0 > 0$. We use Eq. (18) to calculate η_0 at required at temperature T to satisfy Eq. (16). Total energy density U is then obtained by substituting the calculated value of η_0 in the following equation :

$$U = 8\pi . (k^4 / h^3c^3) . I_2$$

$$= 1.137 . (10^{-15}).T^4.I_2 \qquad (19)$$

with $I_2 = \int_{\eta 0}^{\infty} \frac{\eta 3.\ d\eta}{\exp \eta - 1}$. (20)

The values of η_0, ν_0 and U calculated for selected temperatures are given in the Table II. The variation of u_ν with ν is shown in Fig. 1, curve C. Analysis based on this model shows that ν_0 and U increase rather slowly with increase in temperature. This model does not yield large increase in U with increasing temperature that is expected and therefore it is unsatisfactory.

Table II.
Results of calculations for Model 2

$T_S = 5.52 \times 10^{25}$ K, $U_S = 7 \times 10^{88}$ erg/cc .
F - number of photon states / vol

T / T$_S$	10	10^2	10^3	10^4
F_n / h^{-3} (available in non-eq. state)	1	1	1	1
F_e / h^{-3} (required for hypothetical equilibrium state)	10^3	10^6	10^9	10^{12}
η_0	10.995	18.91	26.46	33.84
I_1	$(2.43).10^{-3}$	$(2.43).10^{-6}$	$(2.43).10^{-9}$	$(2.43).10^{-12}$
I_2	$(2.96).10^{-2}$	$(4.87).10^{-5}$	$(6.71).10^{-8}$	$(7.07).10^{-11}$
ν_0 /sec	$(2.28).10^{11}$	$(3.91).10^{11}$	$(5.48).10^{11}$	$(7.01).10^{11}$
U/erg/cc	$(3.12).10^{90}$	$(5.14).10^{91}$	$(7.07).10^{92}$	$(8.94).10^{93}$

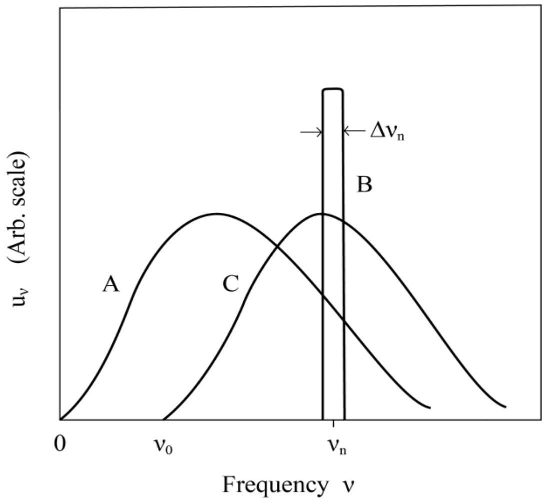

Fig. 1. Schematic variation of uv as a function of frequency ν for a dense assembly of photons in (A) equilibrium state, (B) non-equilibrium state Model 1, and (C) non-equilibrium state Model 2.