Relation between Different Time Intervals in Inertial Frame

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Abstract

This paper has been studied that – for the distance between the two points in the rest frame there will be three different time intervals ($\Delta t'_a$, $\Delta t'_b$ & $\Delta t'_f$) in moving frame. And among these three time intervals the difference between the two time interval $\Delta t'_a$ & $\Delta t'_b$ will be twice the other time interval ($\Delta t'_f$). This is to say $\Delta t'_0 = 2\Delta t'_f$. $\Delta t'_0$ = Difference between the two time intervals in moving frame, $\Delta t'_f$ = Time interval in moving frame (If the time interval in rest frame is zero then we will gate this time interval).

Also, in the moving frame there will be another equation of difference between the two time intervals

$$\Delta t'_0 = -\frac{2v\Delta x}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

Δx = The distance between two points in rest frame, v = Velocity of moving frame and c = The velocity of light.

Keywords - *Special relativity · Lawrence transformation · time interval*

I. INTRODUCTION

There is nothing in universe as absolute rest or absolute motion, in the universe rest or motion matter is relative. famous astronomer Galileo first gave the decision is relative which is known as classical relativity .with the respect to rest frame (s) if the moving frame (s’) is movable in constant velocity (v) then between s and s’ frame which transformation will happen that Galileo innovated first. These transformation equations are called Galilean transformation equation. Space and time will be same in moving and rest frame and both are not relative these are known as Galilean transformation equation. Starting of nineteenth century Michelson- Morley proof by it’s experiment that in every medium of space the speed of light is constant. And this constant value is $c = 3 \times 10^8$. The speed of light is constant in space so the scientist Law

rence determined the transformation equations between S and S’ frame thinking in rest and moving frame the speed of light is constant Which is known as Lawrence transformation or Special Relativity. In 1905 Einstein- Lawrence transformed the equation by discovering that the space and time depends on the rest and moving frame. Einstein said space and time are relative. Through Einstein mathematical equations it proved that the time interval in the rest frame will be longer than the time internal of moving frame. That is to say $t > t_0$, t_0 =time interval in rest frame, t =time interval in moving frame.

However, there will be different types of time interval in moving frame at constant velocity with respect to the rest frame and all these time intervals can be interpreted by the definition of Lawrence. There is one type time interval the moving frame for one point in the rest frame and this time interval will be longer than the time interval in the rest frame this big time interval is called the time dilation.

Also, more than three times interval can be interpreted from Lawrence transformation or special relativity to moving frame. And these three time intervals can be found in two ways in a moving frame by Lawrence transformation. For example- if there are two events happen at the same time in the rest frame at two points then a time interval ($\Delta t'_f$) can be found in the moving frame and the rest frame has the same time interval (Δt) between two points there will be two more time interval ($\Delta t'_a$ & $\Delta t'_b$) in the moving frame and these three time intervals ($\Delta t'_a$, $\Delta t'_b$ & $\Delta t'_f$) depends on the distance between the two points in the rest frame and in the moving frame this will be the difference between the $\Delta t'_a$ and $\Delta t'_b$ time interval double the other time interval ($\Delta t'_f$) in the moving frame.

II. BASIC EQUATIONS

If the other frame (s’) is moving in constant motion relative to the rest frame (s) and the rest frame has the same time interval (Δ t) between two points (x_1 & x_2) then, there will be two different time intervals in moving frame. And this time the intervals will be two

$$\Delta t'_a = \frac{\Delta t + v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (1)$$

$$\Delta t'_b = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (2)$$

[$\Delta t'_b = t'_4 - t'_1$, $\Delta t'_a = t'_3 - t'_2$] $\Delta t'_a$, $\Delta t'_b$ = Two time intervals in moving frame,

$\Delta x =$The distance between two points in the rest frame, and $v =$The velocity of moving frame, $c =$ speed of light. Two time intervals in moving frame, $\Delta x =$The distance between two points in rest frame, and $v =$The velocity of moving frame.

In the rest frame, if there are two events happen at the same time in both the points (x_1 and x_2) there will be another time interval in the moving frame. And this time interval will be

$$\Delta t'_f = -\frac{v\Delta x}{c^2\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

Where, $\Delta t'_f$ $=$Time interval in moving frame (if the time interval in rest frame is zero then will get this interval).

There for, we can say from (1) no, (2) no and (3) no equation these three different time intervals ($\Delta t'_a$,$\Delta t'_b$ and $\Delta t'_f$) in the moving frame can be found for the distance between the two points in the rest frame.

Again, there will be a difference between two time intervals in moving frame.

$$\Delta t'_0 = 2\Delta t'_f \tag{4}$$

Where, $\Delta t'_0 = \Delta t'_b - \Delta t'_a$, $\Delta t'_b - \Delta t'_a =$ Difference between the two times interval in moving frame.

Also, in the moving frame there will be another equation of difference between the two time intervals.

$$\Delta t'_0 = -\frac{2v\Delta x}{c^2\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

Where, $\Delta x =$ The distance between two points in rest frame

III. SOLUTIONS

Thinking, the other frame (s') with respect to the rest frame (s) is moving at the constant velocity v with the positive (X) axis direction. Convicted the first two incidents occur at the same time t_1 in two points (x_1 and x_2) in the rest frame.

So, can we tell according to the Lawrence transformation equation for the two events at the same time t_1 in the rest frame at the points two (x_1& x_2).

$$t'_1 = \frac{t_1 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{6}$$

$$t'_2 = \frac{t_1 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

In this case, the time interval in the rest frame will be zero ($\Delta t_1 = 0$). Because x_1 and x_2 points in time is t_1. Here, t_1& t_2 are two times in moving frame.

Now, $$t'_2 - t'_1 = \frac{t_1 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Or, $$t'_2 - t'_1 = -\frac{v(x_2-x_1)/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\therefore$ $$\Delta t'_f = -\frac{v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

Where, $\Delta t'_f = t'_2 - t'_1 =$ time interval in moving frame, $\Delta x = x_2 - x_1 =$ The distance between two points in rest frame.

Noteworthy, this time interval $\Delta t'_f$ can be found in moving frame if the rest frame does not have time interval.

Again, in the rest frame the second two incidents occurred in same two points x_1 and x_2 at the time t_2. So, according to the Lawrence transformation equation for the second time in two events

$$t'_3 = \frac{t_2 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

$$t'_4 = \frac{t_2 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

Hear, t'_3 and t'_4 are two time in moving frame.

Now, $$t'_4 - t'_3 = \frac{t_2 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_2 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{v(x_2-x_1)/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\therefore$ $$\Delta t'_i = -\frac{v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{11}$$

$\Delta t'_i = t'_4 - t'_3 =$ Time interval in moving frame, $\Delta x = x_2 - x_1 =$ the distance between two points in rest frame.

Now, (8) and (11) get the numbers from the equation.

$$\Delta t'_i = \Delta t'_f \;\therefore t'_4 - t'_3 = t'_2 - t'_1 \tag{12}$$

So, the rest frame is in two points (x_1& x_2) when the time is t_1 while the moving frame time interval is $\Delta t'_f$.

Again, when the time is increased to t_2 in those two points (x_1& x_2) in the rest frame then the time interval in moving frame is $\Delta t'_i$.

Significant, this two time intervals will be the same.

Again, the rest frame is the first time t_1 and the second time t_2 in two specific points x_1 and x_2 then we can say the rest frame will have the same time interval (Δt) in these two points.

Now, from the equation (10) we get subtracted from the number (6).

$$t'_4 - t'_1 = \frac{t_2 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{t_2 - t_1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{v(x_2 - x_1)/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \quad , \quad \Delta t'_b = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (13)$$

Where, $\Delta t'_b = t'_4 - t'_1 =$ a time interval in moving frame, $\Delta t =$time interval in rest frame

Again, from the equation (9) we get subtracted from the number (7).

$$t'_3 - t'_2 = \frac{t_2 - vx_1/c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1 - vx_2/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_2 - t_1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v(x_2 - x_1)/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

There for, $\Delta t'_a = \frac{\Delta t + v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ (14)

$\Delta t'_a = t'_3 - t'_2 =$ It's one more time interval in the moving frame.

Obviously, this time interval ($\Delta t^{'}_a$) of moving frame depends on the distances between the two points (x_1& x_2) in the rest frame and the time interval (Δt) in the rest of the frame.

Hear, the two times interval of $\Delta t'_b$ and $\Delta t'_a$ will not be the same. $\Delta t'_b$ time interval will be longer than $\Delta t'_a$ time interval. ($\Delta t^{'}_b > \Delta t^{'}_a$)

So, by analyzing the equations (8), (13), (14) we can say there will be three time interval ($\Delta t^{'}_a$, $\Delta t^{'}_b$ and $\Delta t'_f$) in the moving frame.

Significant, these three different time interval can be found for distance between the two points in the rest frame.

When, $\Delta x = 0$ then (8), (13) and (14) no of equations can be found

$$\Delta t'_b = \Delta t'_a \quad \text{And} \quad \Delta t'_f = 0$$

There for, there will never be different time interval available in a moving frame for one point in the rest frame.

Now, from the equation (13) we get subtracted from the number (14)

$$\Delta t'_b - \Delta t'_a = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\Delta t + v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= -\frac{2v\Delta x}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \quad \Delta t'_0 = -\frac{2v\Delta x}{c^2\sqrt{1-\frac{v^2}{c^2}}} \qquad (15)$$

Here, $\Delta t'_0 = \Delta t'_b - \Delta t'_a =$ Difference between the two times interval in moving frame.

Again, from the number of equation (15) we can say in the moving frame difference between the two times interval $\Delta t'_a$ and $\Delta t'_b$ depends on the distance between the two points in the rest frame.

Now, we get two from equation (15) and (8).

$$\Delta t'_0 = 2\Delta t'_f \qquad (16)$$

$$\text{Or,} \quad \Delta t'_f = \frac{\Delta t^{'}_0}{2} \qquad (17)$$

Where, $\Delta t'_f =$ Time interval in moving frame (if the time interval in rest frame is zero then will get this interval).

$\Delta t'_0 =$ Difference between the two time intervals in moving frame

So, we can say from the equation (16) difference between the two time intervals in moving frame double the other time interval in the moving frame.

Again, in other ways if there is no time interval in the rest frame there will be a time interval in the moving frame and this time interval will be half of the difference between the two other time interval in the moving frame.

IV. CONCLUDING REMARKS

Basically this theory is established from special relativity. If there is no time interval in the rest frame there will be a time interval in the moving frame, and this time interval will be half of the difference between the two other time interval in the moving frame.

From this paper we can to know that, there will be three time interval in moving frame. In content this three time intervals ($\Delta t'_a$, $\Delta t'_b$ & $\Delta t'_f$) is available for distance between the two points in the rest frame. also this theory analyzed that it if the rest frame is a time interval there is a difference between the two time interval in the moving frame, for which we can say that differences between the two time interval $(\Delta t'_0)$ in moving frame are relative.

Again, the differences between these two time intervals $(\Delta t'_0)$ in moving frame do not depends on the time interval in the rest frame. This difference time interval depends on the distance between the two points in the rest frame and the moving frame velocity.

Also, the rest frame is in two points $(x_1 \& x_2)$ when the time is t_1 while the moving frame time interval is $\Delta t'_f$. again, when the time is increased to t_2 in those two points $(x_1 \& x_2)$ in the rest frame then the time interval in moving frame is $\Delta t'_i$

Significant, two-time interval will be the same.

If the velocity of moving frame is $\frac{v}{c} \to 0$ then there will be no difference in the time interval in moving frame.

There for, $\Delta t'_0 = \Delta t'_b - \Delta t'_a = 0$ and $\Delta t'_b = \Delta t'_a = \Delta t$ [$\Delta t =$ Time interval in rest frame]

In this case, the time interval in the rest frame will be equal to the time interval in the moving frame.

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