Ether and photons In biquaternionic presentation

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Abstract

The paper is related to the construction of solutions of biquaternionic equation of either. It is Electro-Gravimagnetic (EGM) field equation which is generalization of Maxwell equations for EM-field in biquaternions algebra. The special class of solutions of this biquaternionic wave (biwave) equation is mono-chromatic wave solutions that describe periodic oscillations and waves of a fixed frequency. Here, fundamental and generalized solutions of this equation are constructed and studies that describe photons as EGM waves of a fixed frequency, emitted by EGM charges and EGM currents. Solutions of the homogeneous biwave equation are also constructed that describe free photons as free EGM waves of a fixed frequency. The density and motion of photons, their energy-momentum are determined. Solutions of the homogeneous biwave equation are also constructed that describe free photons as free EGM waves of a fixed frequency. Based on them, a biquaternion representation of light and its energy-momentum density are given.

Keywords: *ether, electro-gravimagnetic field, biquaternion, photon, Maxwell equation, stationary oscillations, energy-momentum, light*

I. INTRODUCTION

In [1–5], the author developed a biquaternion model of the electro-gravimagnetic field (EGM field), EGM charges and EGM currents, and EGM interactions based on biquaternion generalizations of the Maxwell and Dirac equations. Note that the biquaternion representation of Maxwell's equations, which describes the relationship of the EM field with electric charges and currents, is based on the representation of the EM field vectors in the form of one biquaternion with a certain restriction on its scalar part, which is zero. The Maxwell equations in the biquaternionic representation are equivalent to one biquaternion wave equation (*biwave equation*), which expresses the charge-current biquaternion through the biquaternionic gradient (*bigradient*) of the EM field. The biquaternion wave equation belongs to the class of hyperbolic and describes solutions of hyperbolic systems of 8 differential equations in partial derivatives of the first order. Note that the quaternionic representation of Maxwell's equations began with Maxwell himself and has a rather extensive bibliography ([6-16] and others).

Removing the restriction on the zero scalar part of the biquaternion of EM field intensity allows us to give a biquaternion representation of the EGM field, as well as a biquaternion representation of mass charges and currents, which contains the EGM charge in the complex scalar part, and the EGM field strength vector in the complex vector part. The latter contains the electric field strength (in the real part) and the gravimagnetic field strength in the complex field. It is the union of the vortex magnetic field with the potential gravitational field into one gravimagnetic field.

We name *ether* the EGM field, which is described by the biquaternion of the strength of the EGM field. Its scalar part is naturally called the density of the ether. And the vector part, by construction, describes the strength of the electric and gravimagnetic fields. The biwave equation in this case expresses EGM charges and currents through the bigradient of the EGM field strength.

The present work is related to the construction of periodic solutions of the EGM field equation. A special class of solutions of the biwave equation is monochromatic wave solutions that describe periodic oscillations and waves of a fixed frequency. The equation for biquaternionic amplitudes of oscillations (*biamplitute*) is the stationary biwave equation, in this case becomes elliptical. Here, fundamental and generalized solutions of this equation are constructed that describe photons as EGM waves of a fixed frequency, emitted by EGM charges and EGM currents. Solutions of the homogeneous biwave equation are also constructed that describe free photons as free EGM waves of a fixed frequency.

II. SCALAR AND VECTOR COMPLEX CHARACTERISTICS OF EGM FIELD

We introduce the notation for the known and new quantities to describe the EGM field, electric and gravimagnetic charges.

- Vectors E, H are the electric and gravimagnetic fields;
- Scalars ρ_E, ρ_H are densities of electric and gravimagnetic charges (*mass charges*);
- Vectors j_E, j_H are densities of electric and gravimagnetic currents.

Here we combined a potential gravitational field with a vortex magnetic field, which, along with electric charges and currents, allows us to introduce a gravimagnetic charge and current. Using these values, we introduce the following complex characteristics of the EGM field:

- *ether tension*

$$A = A^E + iA^H = \sqrt{\varepsilon}\ E + i\sqrt{\mu}\ H,$$

- *ether density*

$$\alpha = i\alpha^E / \sqrt{\varepsilon} + \alpha^H / \sqrt{\mu},$$

- *charge density*

$$\rho = -\rho^E / \sqrt{\varepsilon} + i\rho^H / \sqrt{\mu},$$

- *current density*

$$J = J^E + iJ^H = -\sqrt{\mu}\ j^E + i\sqrt{\varepsilon} j^H.$$

Here $\rho^E(x,t), j^E(x,t)$ are densities of electric charge and current; $\rho^H(x,t), j^H(x,t)$ are densities of gravimagnetic charge and current. Constants ε, $\mu$are electrical conductivity and magnetic permeability of vacuum, $c = 1/\sqrt{\varepsilon\mu}$ is the speed of light.

Note that such complex characteristics for the EM-field were introduced by Hamilton, which allows a system of 8 Maxwell equations (two vector for the rotors E and H and two scalar for their divergence) to be reduced to 4 equations (one vector and one scalar for rotor and divergence of the complex vector of EM-field intensity). Generalized and fundamental solutions for such a Hamiltonian system of equations were constructed by the author in [16].

The system of Maxwell equations allows further generalization in biquaternions algebra, which leads it to a single biquaternionic wave equation.

III. EITHER AND CHARGE-CURRENT BIQUATERNIONS

Let us introduce the following biquaternions of electro - gravimagnetic field, mass charges and currents in Murkowski space $\mathrm{M} = \left\{(\tau, x) : x \in R^3\right\}$, $\tau = ct$, t is time, c is light speed:

tension

$$\mathbf{A}(\tau,x) = i\alpha(\tau,x) + A(\tau,x),$$

charge-current

$$\mathbf{\Theta}(\tau,x) = i\rho(\tau,x) + J(\tau,x),$$

Either energy-momentum

$$\mathbf{\Xi}(\tau,x) = 0{,}5\mathrm{A} \circ \mathrm{A}^* =$$

$$= w(\tau,x) + iP(\tau,x)$$

charge-current energy-momentum

$$\mathbf{\Xi}_\Theta = 0{,}5\mathbf{\Theta} \circ \mathbf{\Theta}^* = w_\Theta + iP_\Theta$$

Here we use the *conjugated* biquaternion: $\mathbf{A}^* = \bar{\mathbf{A}}^-$,where $\mathbf{A}^- \triangleq a - A$ is *mutual* biquaternion. Everywhere the line above the symbol means the complex conjugation of the scalar and vector parts of the biquaternion.

When determining the energy-momentum, the operation of quaternion multiplication is used according to the rule:

$$\mathbf{F} \circ \mathbf{B} = (f + F) \circ (b + B) \triangleq$$
$$\triangleq \{fb - (F,B)\} + \{fB + bF + [F,B]\} \tag{1}$$

Hereinafter, we use the Hamiltonian scalar-vector notation of biquaternions:

$$\mathbf{B} = b + B, \quad \mathbf{G} = g + G,$$

denoting scalar and vector parts with the same lowercase and capital letters in italics (with the exception of basic elements e_m , $m = 0,1,2,3$, of biquaternion algebra). In formulas (1)

$$(F,B) = \sum_{j=1}^{3} F_j B_j,$$

$$[F,B] = \sum_{k,l,m=1}^{3} \varepsilon_{klm} F_k B_l\ e_m,$$

the scalar and vector product of these vectors, ε_{klm} is the pseudo-Levi-Civita tensor,.

In the case of a zero density of the EGM field (α = 0), the energy-momentum biquaternion contains the known density of the EM field, and in the vector part, the Pointing vector:

$$W = 0{,}5\|A\|^2 = 0{,}5(A, A^*) = 0{,}5\left(\varepsilon\|E\|^2 + \mu\|H\|^2\right),$$

$$P = A \times A^* = c^{-1} E \times H$$

The biquaternion of the energy-momentum of an EGM charge-current has the form:

$$\mathbf{\Xi}_\Theta = W_\Theta + iP_\Theta = 0{,}5\mathbf{\Theta} \circ \mathbf{\Theta}^* =$$
$$= 0{,}5\left(|\rho|^2 + \|J\|^2\right) - i\left\{\mathrm{Re}\left(\rho\bar{J}\right) + 0{,}5\,\mathrm{Im}\left[J, \bar{J}\right]\right\} \tag{2}$$

IV. THE RELATIONSHIP BETWEEN EITHER AND CHARGE– CURRENT. GENERALIZED MAXWELL EQUATION

The relationship between the intensity of the EGM field and the density of the EGM of charge-currents has the form of a generalized Maxwell equation in the biquaternion form. For this, differential operators are introduced - mutual bigradients ∇^{+}, ∇^{-}, whose action is determined by the rule of quaternion multiplication:

$$\nabla^{\pm}\mathbf{B} \triangleq \left(\partial_\tau \pm i\nabla\right)\circ\left(b(\tau,x)+B(\tau,x)\right)=$$
$$=\left(\partial_\tau b \mp i(\nabla,B)\right)+\partial_\tau B \pm i\left(\nabla b+[\nabla,B]\right)=$$
$$=\left(\partial_\tau b \mp i\,\mathrm{div}B\right)+\partial_\tau B \pm i\,\mathrm{grad}b \pm i\,\mathrm{rot}\,\mathrm{B}$$

Using them, we represent this relationship in the form of the following postulate [1-5].

Postulate 1. *EGM charges-current are a bigradient of e EGM field tension:*

$$\nabla^{+}\mathbf{A} \triangleq (\partial_\tau + \nabla)\mathbf{A}(\tau,\mathbf{x}) = \mathbf{\Theta}(\tau,\mathbf{x}) \qquad (3)$$

This biwave equation is equivalent to a system of two scalar and vector equations:

$$\begin{aligned}\rho(\tau,x) &= -\partial_\tau\alpha - \mathrm{div}A,\\ J(\tau,x) &= i\,\mathrm{grad}\alpha + \partial_\tau A + i\,\mathrm{rot}\,A\end{aligned} \qquad (4)$$

Hence, for α = 0, the Hamiltonian form [18] of the Maxwell equations follows:

$$\begin{aligned}\mathrm{div}\,A &= -\rho(\tau,x),\\ \partial_\tau A + i\,\mathrm{rot}\,A &= J(\tau,x)\end{aligned} \qquad (5)$$

from which, writing out the real and imaginary parts, we get the classic Maxwell system.

Therefore, we call equation (1) the generalized Maxwell equation (GMEq). And postulate 1 should be called *Maxwell's law* for the EGM field. It has a deep physical meaning, namely:

EGM charges-currents (substance) are derivatives of the EGM field (ether)

Those of the two states of matter (*matter* and *field*), the field is primary, and the substance is secondary. Substance it is a physical manifestation of the heterogeneity and motion of the ether.

Next, we will build monochromatic OAM solutions for describing photons - EGM waves on air with an oscillation frequency

V. BIQUATERNIONS OF PHOTONS

We construct and study the properties of the monochromatic solutions of GMEq that we use to describe the photons emitted by the EGM charge-currents which are on the right side of this equation. In particular, we consider the case of harmonic oscillations with a frequency when the right-hand side of (1) has the form:

$$\mathbf{\Theta}(\tau,x) = \mathbf{\Theta}(x,\omega)\exp(-i\omega t)\,. \qquad (5)$$

Here is the complex *biamplitude* from the class of generalized biquaternions, whose components are generalized functions of slow growth: $\mathbf{\Theta}(x) \in \mathbf{B}'(R^3)$. Similarly, a solution of Eq. (1) can be represented in a similar form:

$$\mathbf{\Phi}(\tau,x) = \mathbf{\Phi}(x,\omega)\exp(-i\omega\tau) \qquad (6)$$

In this case, from equation (3) it follows

$$\left(-i\omega + i\nabla\right)\circ\mathbf{\Phi} \triangleq -i\nabla_\omega^{-}\mathbf{\Phi} = \mathbf{\Theta}(x,\omega)$$

From here we get the stationary equation for biamplitudes–

photon equation:

$$\nabla_\omega^{-}\mathbf{\Phi}(x) = i\mathbf{\Theta}(x) \qquad (7)$$

Here, the operators $\nabla_\omega^{\pm} = \omega \pm \nabla$ are called ω - *gradients*. Their composition is commutative and equal to the scalar Helmholtz operator:

$$\nabla_\omega^{+}\circ\nabla_\omega^{-} = \nabla_\omega^{-}\nabla_\omega^{+} = \left(\omega+\nabla\right)\circ\left(\omega-\nabla\right) = \omega^2 + \Delta \qquad (8)$$

We use this property to construct solutions (7). Taking from it a mutual ω - gradient we obtain the inhomogeneous Helmholtz equation

$$\left(\omega^2+\Delta\right)\mathbf{\Phi}(x,\omega) = i\nabla_\omega^{+}\mathbf{\Theta}(x,\omega) \qquad (9)$$

to which each component of the biquaternion satisfies. The solution of this equation is easy to construct if to use the fundamental solution of the Helmholtz equation, which satisfies the radiation conditions of Somerfield. It has the form [18]:

$$\psi(x,\omega) = -\frac{e^{i\omega\|x\|}}{4\pi\|x\|} \qquad (10)$$

Taking into account the time factor, it describes diverging harmonic spherical waves that satisfy the Somerfield radiation conditions [18].

From (9), using the properties of the fundamental solution and the properties of convolution, we obtain a solution, that has the form of convolution. That is

photon biquaternion

$$\mathbf{\Phi}(x,\omega)=i\nabla_{\omega}^{+}\mathbf{\Theta}*\psi=$$
$$=-\frac{i}{4\pi}\nabla_{\omega}^{+}\left\{\mathbf{\Theta}(x.\omega)*\frac{e^{i\omega\|x\|}}{\|x\|}\right\} \quad (11)$$

Definition. Let's call a photon *elementary*, generated by a concentrated EGM charge of the form:

$$i\mathbf{\Theta}(x,\omega)e^{-i\omega\tau}=\delta(x)e^{-i\omega\tau}.$$

We calculate its biquaternionic representation using (11):

$$\mathbf{\Phi}_0(x,\omega)=-\frac{1}{4\pi}\nabla_{\omega}^{+}\left\{\delta(x)*\frac{e^{i\omega r}}{r}\right\}=$$
$$=-\frac{1}{4\pi}\nabla_{\omega}^{+}\frac{e^{i\omega r}}{r}=-\frac{1}{4\pi}\left\{\omega\frac{e^{i\omega r}}{r}+\operatorname{grad}\frac{e^{i\omega r}}{r}\right\},$$

$r=\|x\|$. As a result, we obtain the biquaternion of an elementary photon, whose biamplitude is

$$\mathbf{\Phi}_0(x,\omega)=\varphi_0+\Phi_0=-\frac{e^{i\omega r}}{4\pi r}\left\{\omega+e_x\left(i\omega+\frac{1}{r}\right)\right\}\Rightarrow$$
$$\varphi_0=-\frac{\omega e^{i\omega r}}{4\pi r},\quad \Phi_0=-\frac{e^{i\omega r}}{4\pi r}\left(i\omega+\frac{1}{r}\right)e_x,\quad e_x=\frac{x}{r} \quad (12)$$

It describes a spherical EGM wave in ether, emitted by a concentrated electric charge. It moves at a speed of 1 (in the original space-time at the speed of light c) and decays at infinity as $1/r$. Amplitudes of its EGM density and EGM tension are proportional to its frequency:

$$|\rho_0(x,\omega)|=\frac{\omega}{4\pi r},$$
$$\|\Phi_0(x,\omega)\|=\frac{1}{4\pi r}\sqrt{\omega^2+\frac{1}{r^2}}$$

The energy-momentum density of an elementary photon is equal to

$$\mathbf{\Xi}_{\Phi}^{0}=w_{\Phi}^{0}+iP_{\Phi}^{0}=0{,}5\mathbf{\Phi}_0\circ\mathbf{\Phi}_0^{*}=$$
$$=\frac{1}{2\pi\|x\|^2}\left\{\omega+e_x\left(i\omega+\frac{1}{\|x\|}\right)\right\}\circ\left\{\omega+e_x\left(i\omega-\frac{1}{\|x\|}\right)\right\}=$$
$$=\frac{1}{\pi\|x\|^2}\left\{\omega^2+\frac{1}{2\|x\|^2}+i\omega^2 e_x\right\}$$

From here we get photon energy density w_{Φ}^{0} and Pointing vector P_{Φ}^{0}, which determines the direction of the photon energy propagations at a fixed point in space:

$$w_{\Phi}^{0}=\frac{1}{\pi\|x\|^2}\left(\omega^2+\frac{1}{2\|x\|^2}\right),$$
$$P_{\Phi}^{0}=\frac{\omega^2}{\pi\|x\|^2}e_x \quad (12)$$

VI. FREE PHOTONS AS PLANE HARMONIC EGM WAVES

Let's consider the solutions of the homogeneous photon equation (7):

$$\nabla_{\omega}^{-}\mathbf{\Phi}(x)=0 \quad (13)$$

As follows from (9), its solution satisfies to homogeneous Helmholtz equation

$$\left(\omega^2+\Delta\right)\mathbf{\Phi}(x,\omega)=0 \quad (14)$$

and can be represented as

$$\mathbf{\Phi}(x,\omega)=\nabla_{\omega}^{+}\left(\sum_{j=0}^{4}a_j\varphi_j(x,\omega)e_j\right) \quad (15)$$

where the scalar potential $\varphi_0(x,\omega)$ and vector potential $\varphi_j(x,\omega)e_j$ are arbitrary solutions of the homogeneous Helmholtz equation.

A simple solution to this equation is plane harmonic wave (taking into account the time exponent):

$$\varphi_j(\mathbf{x},\omega)=\exp(i(\mathbf{k},\mathbf{x})),\quad \|\mathbf{k}\|=\omega \quad (16)$$

which propagate in the direction of the vector $\mathbf{k}$ at a speed of 1 (a_j are arbitrary complex constants). We consider each of these terms separately.

The biamplitude of the free planar harmonic photons generated by potential for j=0 has the form

$$\mathbf{\Phi}_0^{k}(x,\omega)=\nabla_{\omega}^{+}\varphi_0=\omega\varphi_0+\operatorname{grad}\varphi_0=$$
$$=\omega e^{i(\mathbf{k},x)}(1+ie^{\mathbf{k}}) \quad (17)$$

where $e^{\mathbf{k}}=\mathbf{k}/\omega$.

EGM density, EGM tension and their amplitudes are

$$\phi_0^{\mathbf{k}}=\omega e^{i\omega(e^{\mathbf{k}},x)},\quad \left|\phi_0^{\mathbf{k}}\right|=\omega,$$
$$\Phi_0^{\mathbf{k}}=i\mathbf{k}e^{i\omega(e^{\mathbf{k}},x)},\quad \left\|\Phi_0^{\mathbf{k}}\right\|=\omega.$$

Note, in this EGM wave vectors of electric and gravimagnetic fields are parallel : $E\|H$. The density of its energy-momentum is

$$\Xi_0^{\mathbf{k}}(x,\omega)=0,5\mathbf{\Phi}_0^{\mathbf{k}}\circ\mathbf{\Phi}_0^{\mathbf{k}}*=\omega^2(1+ie^{\mathbf{k}})$$

Its energy density and Pointing vector are constant, independent of x:

$$w_0^{\mathbf{k}}=\omega^2,\quad P_0^{\mathbf{k}}=\omega^2 e^{\mathbf{k}} \qquad (18)$$

It is plane longitudinal harmonic EGM wave.

The biamplitude of a free photon generated by the j-th vector potential (j=1, 2, 3) has the next form

$$\begin{aligned}&\mathbf{\Phi}_j^{\mathbf{k}}(x,\omega)=\nabla_\omega^+\left(\varphi_j e_{[j]}\right)=\\&=-ik_j e^{i(\mathbf{k},x)}+\omega e^{i(\mathbf{k},x)}e_{[j]}+\mathrm{rot}\left(e^{i(\mathbf{k},x)}e_n\delta_j^n\right)=\\&=e^{i(k,x)}\left\{-ik_j e^{i(\mathbf{k},x)}+\omega e_{[j]}e^{i(\mathbf{k},x)}+\varepsilon_{lmn}e_l\partial_m e^{i(\mathbf{k},x)}\delta_j^n\right\}=\\&=e^{i(\mathbf{k},x)}\left\{-ik_j+\omega e_{[j]}+i\varepsilon_{mjl}k_m e_l\right\}=\\&==e^{i(\mathbf{k},x)}\left\{-ik_j+\omega e_{[j]}+i\left[\mathbf{k},\mathbf{e}_j\right]\right\}\end{aligned}$$

(19)

The EGM density and EGM tension and their amplitudes are

$$\begin{aligned}&\phi_j^{\mathbf{k}}(x,\omega)=-ik_j e^{i(\mathbf{k},x)},\quad\left|\phi_j^{\mathbf{k}}(x,\omega)\right|=\omega\left|\cos(\gamma_j)\right|,\\&\Phi_j^{\mathbf{k}}(x,\omega)=\omega\left(e_{[j]}+i[e^{\mathbf{k}},e_j]\right)e^{i(\mathbf{k},x)},\\&\left\|\Phi_j^{\mathbf{k}}(x,\omega)\right\|=\omega\sqrt{2-\cos^2(\gamma_j)},\end{aligned}$$

where $\cos(\gamma_j)=(e_{[j]},e^{\mathbf{k}})$.

Note, in this EGM wave the vectors of electric and gravimagnetic tensions (E and H) are perpendicular as in classic electrodynamics.

The density of its energy-momentum is

$$\begin{aligned}&\Xi_j^{\mathbf{k}}(x,\omega)=0,5\mathbf{\Phi}_j^{\mathbf{k}}\circ\mathbf{\Phi}_j^{\mathbf{k}}*=\\&0,5\left\{-ik_j+\omega e_{[j]}+i\left[\mathbf{k},\mathbf{e}_j\right]\right\}\circ\left\{ik_j-\omega e_{[j]}+i\left[\mathbf{k},\mathbf{e}_j\right]\right\}=\\&=0.5\left(k_j^2+\omega^2+\left\|\left[\mathbf{k},\mathbf{e}_j\right]\right\|^2\right)+i\omega k_j e_{[j]}=\\&0.5\left(\|\mathbf{k}\|^2+\omega^2\right)+i\omega k_j e_{[j]}=\\&=\omega^2\left\{1+i\cos(\gamma_j)e_{[j]}\right\}\end{aligned}$$

Its energy density and Pointing vector also are constant, independent of x:

$$w_j^{\mathbf{k}}(x,\omega)=\omega^2,\quad P_j^{\mathbf{k}}(x,\omega)=\omega^2\cos(\gamma_j)e_{[j]} \qquad (20)$$

If $e_{[j]}$ perpendicular to $\mathbf{k}$ ($\cos(\gamma_j)=0$), it is transversely polarized harmonic EGM wave with zero EGM density and Pointing vector. Such wave is named a tensional *wave*.

But if $(\mathbf{k},e_j)\neq 0$ this EGM wave has longitudinal part in electric and gravimagnetic tensions. Using these free harmonic photons and formula (14), we obtain a biquaternionic representation of plane harmonic photons:

$$\begin{aligned}&\mathbf{\Phi}^{\mathbf{k}}(x,\omega)=\sum_{j=0}^{4}a_j(\mathrm{k})\mathbf{\Phi}_j^{\mathbf{k}}=\\&=\sum_{j=0}^{4}a_j(\mathbf{k})\left(\phi_j^{\mathbf{k}}(x,\omega)+\Phi_j^{\mathbf{k}}(x,\omega)\right)\end{aligned} \qquad (17)$$

where $a_j(x)$ are arbitrary functions or constants. They are propagate in the direction of a wave vector k at the speed of 1 (in the original space-time with the speed of light c and frequency ωc). Its longitudinal part in a vector addend is a light pressure p(x,ω) which we observed in practice:

$$p(x,\omega)=\left|\sum_{j=0}^{4}a_j(\mathbf{k})\left(\Phi_j^{\mathbf{k}}(x,\omega),e^{\mathbf{k}}\right)\right|.$$

VII. BIQUATERNIONIC REPRESENTATION OF LIGHT

Consider the photons emitted by monochromatic charges-currents. Using the property of convolution differentiation, from formula (11) we obtain the following biquaternionic representation through elementary photons

$$\mathbf{\Phi}(x,\omega)=i\mathbf{\Theta}(x.\omega)*\mathbf{\Phi}_\omega^0(x,\omega) \qquad (18)$$

Here, the convolution for regular biquaternions has the next integral representation:

$$\mathbf{\Phi}(x,\omega)=i\int_{R^3}\mathbf{\Theta}(x-y,\omega)\circ\mathbf{\Phi}_\omega^0(y,\omega)dy_1dy_2dy_3$$

Those EGM charges and currents are emitters of elementary photons, the intensity of which is determined by their biquaternion densities.

The energy-momentum density of such photons is equal to:

$$\begin{aligned}&\Xi_\Phi(x,\omega)=0,5\mathbf{\Phi}\circ\mathbf{\Phi}^*=\\&=0,5\left(\mathbf{\Theta}*\mathbf{\Phi}_\omega^0\right)\circ\left(\mathbf{\Theta}*\mathbf{\Phi}_\omega^0\right)^*=\\&=0,5\left(\mathbf{\Theta}*\mathbf{\Phi}_\omega^0\right)\circ\left(\mathbf{\Phi}_\omega^{0*}*\mathbf{\Theta}^*\right)\end{aligned} \qquad (19)$$

Light contains a whole spectrum of frequencies and can be represented as a cloud of elementary photons in the form of the Fourier integral:

$$\mathbf{\Lambda}(x,\tau)=\int_{\omega 1}^{\omega 2}\mathbf{\Phi}(x,\omega)e^{-i\omega\tau}d\omega=$$
$$=i\int_{\omega 1}^{\omega 2}\mathbf{\Theta}(x.\omega)*\mathbf{\Phi}_{\omega}^{0}(x,\omega)e^{-i\omega\tau}d\omega \qquad (20)$$

where $(\omega 1,\omega 2)$ is the spectral interval of light (visible photons). Accordingly, it is also possible to determine the density of its energy-momentum:

$$\mathbf{\Xi}^{\mathbf{\Lambda}}(x,t)=\mathbf{\Lambda}(x,t)\circ\mathbf{\Lambda}^{\bullet}(x,t) \qquad (21)$$

In a similar way, we can build a cloud of free photons:

$$\mathbf{O}(x,\tau)=$$
$$=\sum_{j=0}^{4}\int_{\omega 1}^{\omega 2}e^{-i\omega\tau}\left\{\mathbf{\Omega}_j(x,\omega)*\int_{\|\mathbf{k}\|=\omega}b_j(\mathbf{k},\omega)\mathbf{\Phi}_j^{\mathbf{k}}(x,\omega)dS(\mathbf{k})\right\}d\omega,$$
$$\mathbf{\Xi}^{\mathbf{O}}(x,\tau)=0,5\mathbf{O}(x,\tau)\circ\mathbf{O}^{*}(x,\tau) \qquad (22)$$

where $b_j(x,\omega),\mathbf{\Omega}_j(x,\omega)$ are arbitrary regular functions and biquaternions that admit such a convolution. Apparently, such biquaternions describe *ball lightning*.

Note that these formulas can also be used for singular biquaternions, such as simple and dabble layers. Only in this case, convolutions must be taken according to the convolution rules for generalized functions [18].

VIII. CONCLUSION

The arbitrariness of the functions included in the determination of the solutions of the photon equation allows us to construct an infinite number of the most diverse solutions for photons, light, photon clouds from the formulas presented, which can be done by an interested reader. It would be useful to give such a construction to students and undergraduates as programming exercises, which would allow them to show their ingenuity and skills in the study of a wide variety of light phenomena.

At the end of the article, a few words about gravitational waves, for which so hunt for physicists-experimenters. As follows from the formulas for ether and photons, no pure gravitational waves exist. Any changes of gravitational field lead to changing of electromagnetic field.

Photons themselves are electro-gravimagnetic waves. They contain a gravitational wave as a component of the EGM wave. It is the gravitational component that determines the light pressure, well known in practice.

Also note that he proposed biquaternionic theory of ether and photons is very constructive. It allows determining its characteristics at any point in space-time what is impossible in models of quantum mechanics.

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