Relativistic Doppler Effect and Wave-Particle Duality

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Abstract

In the framework of Special Relativity and in agreement with the electromagnetic approach used to connect Quantum and Relativistic effects in Bridge Theory, a first order wave-equation able to emulate the wave-particle duality in terms of local electromagnetic interactions is proposed. The equation is able to describe the propagation of a fermion as a de Broglie's wave and simultaneously to characterise the wave in motion with a mass energy as a particle. The solution of the wave-equation proposed is compatible with the de Broglie's idea of an empty pilot wave, carrying information about the dynamical properties of a particle materialising during each local interaction with the observers.

Keywords – *duality, wave-equation, wave-particle, Quantum Mechanics, Klein-Gordon equation, Special Relativity, Bridge Theory*

I. INTRODUCTION

The history of Physics of the last century shows how two apparently incompatible behaviours such as those of a wave and of a particle, which are coexisting in the diffraction of photons in the Young's double-slit experiment and in the diffraction of electrons in the Davisson and Germer experiment, are responsible of one of the conceptual fundaments of the Quantum Physics as the wave-particle duality.

In an article of history of physics [1], Louis de Broglie writes about his wave-particle duality hypothesis of the autumn 1923 as of an attempt to arrive to a theoretical synthesis of waves and particles in such a way the corpuscular behaviour can appear as a peculiarity of a pilot wave able to control its propagation. He affirms that the theoretical description of duality would have not success without the contemporary introduction of Special Relativity (SR), essential to describe the high-speed phenomenology of the particles and the wave mechanics to describe the propagation. On the other hand, the joint use of so different deterministic and indeterministic concepts, essential in the theoretical develops of all the contemporary quantum-relativistic theories, could be a conceptual obstacle to accept unconditionally the hypothesis of duality as a real physical phenomenon.

Indeed, by accepting Copenhagen's interpretation, the phenomenological incompatibility in the contemporary description of a particle as a relativistic local object and as a wave of probability density is overtaken by the acceptance of the idea that nature is made in this way and that duality is a fundamental principle without further physical explanations.

The wave-particle incompatibility has been analysed in the past by Sachs [2] [3] from the point of view of SR and by many other authors from the point of view of the non-covariance problem, particularly evident in the usual QM formalism [4]-[11].

A first not usual step towards an attempt to revise the quantum physical phenomenology in electromagnetic terms was proposed by the author and by G. Dematteis in ref. [12], the work was based on a conjecture concerning the role of the transversal component of the Poynting vector of an electromagnetic dipole to localise a finite amount of energy and momentum [13] inside its first wavefront. The conjecture [13], demonstrated in the works [14] and [15] with the complete and consistent theoretical evaluation of Sommerfeld's constant only with the use of electromagnetic and mathematical-statistical methods, allowed to identify the energy and the moment located inside the source zone of a dipole as those of a photon exchanged between the two interacting particles.

Evidence that the Bridge Theory (BT) is capable of being a true theoretical and conceptual bridge unifying quantum and relativistic phenomena has been obtained in ref. [16] and [17], thus allowing to connect and develop seemingly incompatible phenomena [18] and [19] equipping Relativity and Quantum Mechanics with a valid basic support.

The possibility of linking seemingly so different phenomenologies with a single theory that can formally adapt to all others, gives us the opportunity to explain why quantization and relativity coexist synergically in nature representing such a success. In addition, BT is compatible with the principle of duality in which microscopic and macroscopic interactions between aparticle and the laboratory and between a body and the laboratory are both made using electromagnetic waves, but manifest themselves with quantum effects on a microscopic scale and with classical behaviours, typical of

Newtonian mechanics [19], in the macroscopic world.

In this work, the use of the relativistic Doppler effect, adapted to the case of a particle with respect to an observer, describes the interaction as an electromagnetic dipole in which the wave-particle behaviours of a fermion in motion coexist in the electromagnetic field of the dipole wave.

Considering the production of a large number of virtual Dipolar Electromagnetic Sources (DEMS) in vacuum, distributed along the particle's path, the model is able to produce an overlap of waves in space-time that describe the particle as a wave carrying only information about the initial energy and the moment of the particle. This description is in accordance with BT.

This result justifies the existence of an empty wave function that probabilistically describes the propagation of the particle in space-time. On the other hand, each dipole produced has the role of an observer who performs a measurement on the particle by means of the reciprocal exchange of a photon. The wave that satisfies the differential equation of the first proposed order, propagates like a particle in motion at a speed always lower than the speed of light in a vacuum and with a mass energy corresponding to that of the particle. In this sense, the proposed wave equation describes both the relativistic deterministic behaviour of the moving particle and its wavelike behaviour.

Propagation in the laboratory reference is, however, associated with an electromagnetic wave without mass energy, therefore empty, very similar to the de Broglie idea of the carrier wave or pilot wave.

II. THE RELATIVISTIC DOPPLER EFFECTAND USE OF THE GENERALIZED COMPTON –DE BROGLIE'S WAVELENGTH

According to the relativistic Doppler effect [20], considering an electromagnetic source positioned at infinite and an observer moving away from the source at a speed $\beta = v/c$ forming an angle θ with the direction of propagation of the wave, When the observer is reached by the wave front, the frequency received by the moving observer is given by

$$\nu' = \nu \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}} \qquad (1)$$

which is a function of the emitted frequency, of the angle and of the relative velocity. Inverting the Eq. (1) in terms of emitted photon $E_\Gamma = h\nu$ and received photon $E' = h\nu'$, the Eq. (1) becomes

$$E_\Gamma = E' \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \quad . \qquad (2)$$

The Eq. (2) describes the energy of the photon emitted by the source as a function of the energy received by the observer, i.e. is the measure of the energy emitted performed by the observer.

Considering the case where the electromagnetic source placed at infinity is replaced by a primary DEMS formed by a pair of fermions and the observer is replaced by another fermion in motion in the same conditions described for the Eq. (1), the primary DEMS emits an electromagnetic wave in space-time, when the wavefront reaches the observer, a secondary DEMS is causally produced [18] with the reciprocal exchange of a photon between the DEMS and the particle observer. Considering that in the mass centre of the DEMS the photon energy received by the observer is half the total energy of the secondary DEMS produced in the direct interaction of the two particles, i.e. $E' \equiv \varepsilon/2$, Eq. (2) in accordance with de Broglie's hypothesis describes an energy and a moment exchanged by the two interacting particles along their different line of sight for each possible inertial observer [12],[19]:

$$\begin{cases} E_\Gamma = \dfrac{\varepsilon}{2}\gamma\dfrac{1-\beta^2}{1-\beta\cos\theta} \\ P_\Gamma = \dfrac{\varepsilon}{2}\dfrac{\gamma\beta}{c}\dfrac{\frac{1}{\beta^2}-1}{\frac{1}{\beta}-\cos\theta} \end{cases} \qquad (3)$$

The Eq. (3) describes energy and momentum of the photon emitted by the primary DEMS towards the observer and coincides with the energy and the momentum directly exchanged between fermion and antifermion of the secondary DEMS produced. Using Compton's generalized wavelength as proposed in the reference [17]-[18]

$$\lambda_0 = \lambda_c \gamma (1-\beta\cos\theta) \qquad (4)$$

where $\lambda_c = 2hc/\varepsilon$ is the classical Compton's wavelength, the energy and the momentum (3) of the photon exchanged between the fermions of the secondary DEMS are $E_\Gamma = P_\Gamma c = hc/\lambda_0$. To verify that the energy of the Eq. (3) is a good estimation of the energy of the particle measured by an observer during an interaction, is considered a DEMS fermion-antifermion emitting a wave of frequency $\nu = c/\lambda_0$ along the sight line on which he is placed the observer. For symmetry, energy and frequency are equal to that measured exchanging the roles of the two interacting partners observer and particle.

The energy (3) has got two physical constraints: the velocity of the particle moving in vacuum cannot exceed the light speed, the angle of interaction or of observation between the sight line of the observer and the velocity along the trajectory of the particle is within the interval $0 < \theta \le \pi$, so when the particle moves at low energy with $\beta << 1$, the frequency of the DEMS converges to the Compton's value

$$\nu_c = \frac{\varepsilon}{2h} \qquad (5)$$

of one of the two interacting fermions with energy $\varepsilon/2$.

On the other hand, when a head to head interaction occurs at extreme high relativistic velocity of collision $\beta \cong 1$, the angle of interaction between the two particles is close to zero, using at limit the Eq. (3), the frequency converges to the de Broglie wave frequency:

$$\nu_{db} \cong \frac{\gamma\varepsilon}{h} \qquad (6)$$

of a particle with relativistic total energy $E = \gamma\varepsilon$ associated at both the interacting particles as if one of the two was at rest and the other in motion. So, it is possible to conclude that equation (3) is a good way to estimate the energy and the momentum exchanged between an interacting particle and an observer in each possible their dynamic condition.

III. DUAL DESCRIPTION OF A PARTICLE

In agreement with BT, when a charged particle #1 crosses a medium inducing a great number of virtual DEMS, it interacts directly or indirectly with each real or virtual anti-charged particle in the space-time around the trajectory of the particle as proposed in reference [19]. The effect is to produce a distribution of DEMS with which to share all the energy and the momentum of the particle in motion. Let the vacuum be a polarizable field in which are produced electromagnetic active frames S_j, the fermion travelling in the vacuum is observed in each other frame as a generalised Compton - de Broglie's wave. In other terms, each DEMS formed by the couple of frames $S_1 S_j$ particle-observer produced during the direct interaction of the particle #1 with an observer in the Lab-frame *j*, emits a wave carrying away a fraction of the total energy and momentum of the moving particle. The DEMS distribution produced by the moving particle in the medium emit waves which overlapping in space-time originates a wave that carries information on the energy and the moment of the incident particle. In this sense, the vacuum polarisation it produces the wave behaviour of the particle but also its space-time particle-localization. For two interacting particles with total energy and momentum

$$\begin{cases} E = \hbar\omega = \gamma\varepsilon \\ \mathbf{P} = \hbar\mathbf{k} = \gamma\beta\dfrac{\varepsilon}{c}\hat{\boldsymbol{\kappa}} \end{cases} \qquad (7)$$

the energy exchanged along a direction $\hat{\boldsymbol{\kappa}}$ coinciding with the sight line of an observer assumed at rest, can be described using the energy (3) in the form:

$$E_\Gamma = \frac{E^2 - P^2c^2}{2\left(E - \hat{\boldsymbol{\kappa}}\cdot\mathbf{P}c\right)} \qquad (8)$$

Since the electromagnetic momentum exchanged with the observer along the sight line is $\hat{\boldsymbol{\kappa}}\cdot\mathbf{P}$, the not exchanged energy is

$$E - \hat{\boldsymbol{\kappa}}\cdot\mathbf{P}c = \frac{\varepsilon^2}{2E_\Gamma} \qquad (9)$$

corresponding to a not exchanged four-momentum

$$\frac{\not{E}}{c} = \frac{E}{c} - \hat{\boldsymbol{\kappa}}\cdot\mathbf{P} \qquad (10)$$

The equation (9) corresponds to the total energy of the DEMS, i.e. the not emitted energy along the sight line of the observer. In terms of total momentum associated to the centre of mass of the DEMS, the Eq. (10) using the Eq. (9) gives

$$\frac{\not{E}}{c} = \chi\frac{\varepsilon}{2c} \equiv \hbar K \qquad (11)$$

where the ratio $\chi = \varepsilon/E_\Gamma$ is every inside the interval $0 < \chi \le 2$, the extremes are the lower and upper limits corresponding respectively to a particle moving with relativistic velocity at very high energy and to a particle at rest in the lab.

Considering the Compton and the de Broglie frequencies as the lower and upper limits in frequency given by the Eq. (6) and (7), using the Eq. (11) the residual energy of the particle has a value within the range $0 < \not{E} \le \varepsilon$, where the lower limit is converging to zero when the Lorentz's factor γ converges to infinite at very high energy of interaction and the upper limit, equal to the rest energy of the DEMS, is achieved when the particle is stationary respect the observer. The case $\not{E} = 0$ can be considered achievable in two different situations: the first when a particle in motion at a very high speed is interacting with only one observer in space-time with which engages all the energy; the second when a particle interacts with a sufficient number of observers engaging all its available energy.

In both cases, when a particle interacts with the universe, it engages all its available energy, i.e. considering the sum of the energy exchanged by the particle towards the observers in all the directions, the residual energy of the particle is zero in favour of many possible states of the particle. Let

$$\hat{p}_\mu = i\hbar\partial_\mu = i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla\right) \qquad (12a)$$

and

$$\hat{p}^\mu = i\hbar\partial^\mu = i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t} - \nabla\right) \qquad (12b)$$

to be the covariant and contravariant four-momenta operators to obtain a wave-equation describing the interaction of the particle with the observer and the electromagnetic wave emission of the DEMS, defining a covariant and contravariant four-vectors $\kappa^{\mu}=(1,-\cos\theta,\ 0,\ 0)$ and $\kappa_{\mu}=(1,\ \cos\theta,\ 0,\ 0)$ giving the metrics of the sight line connecting the incoming particle with the observer in the approach and in the removal phase of the particle,the Eq. (10) can be rewritten in a double equivalent form as a first-order differential equation

$$\kappa_{\mu}\hat{p}^{\mu}\psi \equiv \kappa^{\mu}\hat{p}_{\mu}\psi = \hbar K\psi \qquad (13)$$

Let the left side operators of the Eq. (13) be the residual four-momentum of the interacting particles, since during the interaction cannot be exchanged more energy and momentum than the de Broglie's one, the momentum $\hbar K$ is the sum of all four space-time components of the momenta. The Eq. (13) in both forms give

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\cos\theta\frac{\partial}{\partial x}+iK\right)\varphi=0 \quad (14)$$

with the wave-function is defined as $\varphi=\hbar\psi$. The choice to assign at the wave-function the action dimensions throughout the Planck's action is in accordance with the references [12-15] in which is shown that Planck's constant is produced by DEMS during its creation.

By Ref. [21], the Eq. (14) gives:

a) for relativistic energy and forward emission with angle $\theta \cong 0$ the Eq. (14) yields the final equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}+iK\right)\varphi=0 \qquad (15a)$$

with solution $\varphi_I=\hbar e^{i\left[\left(\frac{\omega}{c}-K\right)ct+kx\right]}$;

b) for relativistic energy and transversal emission, the angle $\theta \cong \pi/2$ yields the equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+iK\right)\varphi=0 \qquad (15b)$$

with solution $\varphi_{II}=\hbar e^{-iKct}$.

This condition corresponds to the DEMS production with the exchanging of momentum $\hbar K$;

c) for relativistic energy and backward emission during the omega phase of the DEMS the angle $\theta=\pi$ yields the equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+iK\right)\varphi=0 \quad (15c)$$

with solution $\varphi_{III}=\hbar e^{i\left[\left(\frac{\omega}{c}-K\right)ct-kx\right]}$.

Considering the previous three solutions obtained for an incoming wave-particle (a) which interacts directly forming a dipole exchanging a photon in (b) and after moving away as an wave-particle in (c), taken together they describe a perturbation of the action in space-time as a wave that comes from infinity from the past on a negative timeline (a) before the formation of the dipole in the present at zero time in (b), and after it moves back to infinity in the future along a positive timeline in (c), three steps that globally describe what happens during the DEMS formation.

In agreement with the definition of the source zone limits of the DEMS given in [14], using the direct interaction distance between fermion and antifermion $R=(1+\frac{t_d}{T})\lambda$ defined by the delay time between the real position of the particle and the effective position due to the electric signal propagation, since the delay time is every closed within the interval $0 \le t_d \le T$, the particles interaction is active only within a time $-\frac{T}{2} \le t \le 0$ forcing the interaction to occur from a distance $\frac{3}{2}\lambda$ with a delay time $t_d=\frac{T}{2}$, up to a distance λ at the delay time $t_d=0$. The wavelength of the DEMS is corresponding at the minimum distance between the two interacting particles. After, the particle goes towards infinite.

The action-wave:

$$\varphi=\begin{cases}\hbar e^{i\left[\left(\frac{\omega}{c}-K\right)ct+kx\right]} & -\frac{T}{2}\le t<0^- \\ \hbar e^{-iKct} & 0^- \le t \le 0^+ \\ \hbar e^{i\left[\left(\frac{\omega}{c}-K\right)ct-kx\right]} & 0^+ < t \le \frac{T}{2}\end{cases} \qquad (16)$$

is describing the trajectory of an impinging particle in space-time as measured by an observer at rest coinciding with the anti-particle forming the DEMS. When the two particles interact directly the DEMS emits an electromagnetic wave which propagates in space-time achieving external observers. When an observer is achieved by the wave, the amount of momentum exchanged cannot exceed the momentum of the impinging particle, i.e. $P_{\Gamma}=\hbar\left(\frac{\omega}{c}-K\right)$.

The residual amount of four-momentum $\hbar K$ is equivalent to the measure of the total momentum of the impinging particle in the Lab. Using the Eq. (10), and (11) the real part of the wave (16) is

$$\operatorname{Re}\varphi=\begin{cases}\hbar\cos\left(\dfrac{E-\not{E}}{\hbar}t+\dfrac{P}{\hbar}x\right) & -\dfrac{T}{2}\le t<0^{-}\\ \hbar\cos\left(-\dfrac{\not{E}}{\hbar}t\right) & 0^{-}\le t\le 0^{+}\\ \hbar\cos\left(\dfrac{E-\not{E}}{\hbar}t-\dfrac{P}{\hbar}x\right) & 0^{+}<t\le\dfrac{T}{2}\end{cases}\qquad(17)$$

Each of the three components of the Eq. (17) describes a part of a one-way wave with a phase described by the de Broglie's energy and momentum and by the residual energy after the DEMS formation. The propagation speed of the wave of action in space-time is

$$\beta_w=\frac{E-\not{E}}{Pc}.\qquad(18)$$

which is varying within the interval $0\le\beta_w<1$ decreasing up to zero with the growing of the residual energy Fig.1.

To study what occurs at the Eq. (14) and at the solutions (15a-b-c) when the energy of the incoming particle is extremely high, using the Eq. (11) it is necessary to consider when $\chi\to 0$ it follows $\not{E}\to 0$, considering only one interaction in time, the wave (16) can be written as

$$\varphi_{High}=\begin{cases}\hbar e^{i\left(\frac{E}{\hbar}t+\frac{P}{\hbar}x\right)} & t<0^{-}\\ \hbar & 0^{-}\le t\le 0^{+}\\ \hbar e^{i\left(\frac{E}{\hbar}t-\frac{P}{\hbar}x\right)} & t\ge 0^{+}\end{cases}\qquad(19a)$$

or by using the real part, with the Eq. (17)

$$\operatorname{Re}\varphi_{High}=\begin{cases}\hbar\cos\left(\dfrac{E}{\hbar}t+\dfrac{P}{\hbar}x\right) & t<0\\ \hbar & 0^{-}\le t\le 0^{+}\\ \hbar\cos\left(\dfrac{E}{\hbar}t-\dfrac{P}{\hbar}x\right) & t>0\end{cases}\qquad(19b)$$

The Eq. (19a) or (19b) describe in space-time the propagation of an empty wave. In this case the measuring of the four-momentum (10) of the incoming particle is zero, i.e. the particle does not carry residual energy and momentum but only the information of the energy and momentum (7) used in the DEMS produced. In agreement with reference [17] and [18], when the particle uses all its four-momentum in the production of the DEMS it behaves locally as a photon with a de Broglie's frequency associated to an empty wave in motion at speed of light. Vice versa, considering the case of a particle moving at low speed respect an antiparticle in the lab, in this case the Lorentz's factor is $\gamma\cong 1$ and the energy ratio $\chi\cong 2$, all the energy and momentum (3) are that of the interacting particles at rest with a wave propagating with a speed $\beta_w\cong 0$ characterised by a Compton wavelength (6). In this case the exchanged momentum with the antiparticle is $P_\Gamma=|1-\chi|\dfrac{\varepsilon}{2c}\cong\dfrac{\varepsilon}{2c}$ corresponding to the Compton momentum of the particle. The DEMS is vibrating with a Compton's frequency originating ripples of action in space-time

$$\operatorname{Re}\varphi_{Low}=\begin{cases}\hbar\cos\dfrac{\varepsilon}{2\hbar c}(x-ct) & t\le 0^{-}\vee t\ge 0^{+}\\ \hbar\cos\dfrac{\varepsilon}{2\hbar}t & 0^{-}<t<0^{+}\end{cases}\qquad(20)$$

IV. THE KLEIN-GORDON EQUATION

To know the energy and the momentum of a particle in motion in the space-time an observer is forced to interact electromagnetically with it producing a DEMS with energy and momentum equal to those of both the particles that produce the dipole. In fact, the DEMS is formed by the reciprocal interaction of the particle #1 with the target #2, the exchanged energies and moment a are identical in both the direction because in the centre of mass the total energy and momentum involved are the same. Using the Eq. (8), the total energy exchanged during the phase of approach with $\theta\cong 0$ can be estimated using

$$E+Pc=2E_\Gamma\qquad(21)$$

The factor 2 on the right side of the Eq. (21) is due to the double exchange of photons, once toward each of the two interacting partners particles. To consider the total amount of energy and momentum residual respect the DEMS formation, i.e. the resting energy in the centre of mass of the DEMS, it is necessary to consider the Eq. (9) in the same previous context, where with an incident angle $\theta\cong 0$ gives;

$$E-Pc=\varepsilon^2/2E_\Gamma\qquad(22)$$

Multiplying term by term the Eq. (21) and (22), the product gives the squared of the total resting energy of mass associated with the production of the DEMS

$$E^2-P^2c^2=\varepsilon^2\qquad(23)$$

As proved in ref. [19] the total resting energy $\varepsilon=2mc^2$ is associated to the Galilean momentum received by the zitterbewegung of the two interacting particles in space-time. The Eq. (23), in agreement with the invariant four-vector energy-momentum, corresponds the total rest energy of a pair of interacting particles exchanging reciprocally photons and emitting waves towards external observers, so using the equation (15) in the two cases (a) and (c), the Eq. (23) becomes

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}+\frac{\varepsilon^2}{\hbar^2}\right)\Phi=0 \qquad (24)$$

The solution of the Klein-Gordon Equation (KGE) (24) represents the wave describing a DEMS as a boson formed by the reciprocal interaction with the exchange of a photon between the two interacting fermions.

Since the energy and the momentum of a DEMS cannot exceed the total energy and momentum of the two interacting particles, by using the two-component solution for the free KGE [22]

$$\Phi=\begin{pmatrix}\phi^+\\ \phi^-\end{pmatrix} \qquad (25)$$

considering the virtual photon exchanged between two interacting particles as a polarized pair with energy and momentum, from the Eq. (23) using the Eq. (7) it follows

$$\begin{cases} E^{\pm}=\pm\gamma\,\dfrac{\varepsilon}{2}\\ P=\gamma\beta\,\dfrac{\varepsilon}{2c}\end{cases} \qquad (26)$$

with ($\pm$) is denoted the energies of the positive and negative particles, i.e. reciprocally particle and antiparticle, each described by two components wave functions as given in [23]

$$\phi^+=\frac{1}{2\sqrt{\frac{\varepsilon}{2}E^+}}\begin{pmatrix}\frac{\varepsilon}{2}+E^+\\ \frac{\varepsilon}{2}-E^+\end{pmatrix}e^{+i\frac{P\,x-E^+t}{\hbar}} \qquad (27a)$$

$$\phi^-=\frac{1}{2\sqrt{\frac{\varepsilon}{2}E^+}}\begin{pmatrix}\frac{\varepsilon}{2}+E^-\\ \frac{\varepsilon}{2}-E^-\end{pmatrix}e^{-i\frac{P\,x+E^-t}{\hbar}} \qquad (27b)$$

Using the Eq. (26), the Eq. (27a) and (27b) become

$$\phi^+=\frac{1}{\sqrt{\gamma\varepsilon}}\begin{pmatrix}(1+\gamma)\frac{\varepsilon}{2}\\ (1-\gamma)\frac{\varepsilon}{2}\end{pmatrix}e^{+i\frac{\gamma\varepsilon}{2\hbar c}(\beta x-ct)} \qquad (28a)$$

$$\phi^-=\frac{1}{\sqrt{\gamma\varepsilon}}\begin{pmatrix}(1-\gamma)\frac{\varepsilon}{2}\\ (1+\gamma)\frac{\varepsilon}{2}\end{pmatrix}e^{-i\frac{\gamma\varepsilon}{2\hbar c}(\beta x-ct)} \qquad (28b)$$

Following the reference [23], the Schrödinger "Zitterbewegung" could be described as a spontaneous polarisation, i.e. a quantum fluctuation of the vacuum induced at very low energy near electric fields as those produced by nuclei. The vacuum spontaneously polarises itself in a distribution of DEMS at very low kinetic energy $\beta\approx 0$:

$$\phi^+\cong\begin{pmatrix}1\\0\end{pmatrix}e^{-i\frac{\varepsilon}{2\hbar}t} \qquad (29a)$$

$$\phi^-\cong\begin{pmatrix}0\\1\end{pmatrix}e^{+i\frac{\varepsilon}{2\hbar}t} \qquad (29b)$$

each DEMS is represented by two stationary plane waves emitted along the dipole axis, associated respectively to a pair of particles with a positive and a negative energy and a near zero amount of momentum, the two components (29a) and (29b) represent two standing waves with Compton frequency (5):

$$\Phi=\phi^++\phi^-\cong\begin{pmatrix}e^{-i\frac{\varepsilon}{2\hbar}t}\\ e^{+i\frac{\varepsilon}{2\hbar}t}\end{pmatrix} \qquad (30a)$$

overlapping become

$$\Phi\cong e^{-i\frac{\varepsilon}{2\hbar}t}+e^{+i\frac{\varepsilon}{2\hbar}t}=2\cos\frac{\varepsilon}{2\hbar}t \qquad (30b)$$

The equation (30b) describes a trembling in time in each point of space which associated to a spontaneously polarised virtual pair with a total energy at rest ε.

It is easy to verify using Eq. (20) and (30b) as both the solutions of the two wave equations are able to describe the trembling of pairs particle-antiparticle:

$$\hbar\Phi\equiv 2\,\mathrm{Re}\,\varphi_{Low}(0,t)\cong 2\hbar\cos\frac{\varepsilon}{2\hbar}t \qquad (31)$$

The identity (31) verifies how the KGE is able to describe the behaviour of a pair of virtual fermions producing a DEMS, i.e. a virtual boson, but also proves as the relativistic first-order wave-equation (14) is able to describe a single fermion in each dynamic condition only if it is interacting with an antifermion. The particle is described as a wave in motion in space-time at the speed β_w of the particle at which is associated a residual energy and momentum.

V. THE EMPTINESS OF THE WAVE-FUNCTION

When a particle crosses space-time, it interacts with a large number of virtual particles polarised from the medium at zero energy. As seen in ref. [19] the effect is to produce a large number of DEMS with which the impinging particle shares all its energy (10). Each DEMS emits a wave, the overlapping of all the waves forms an empty wave signal carrying only the information of the energy and momentum of the particle in motion but not residual resting energy.

Let the exchanged energy and momentum (3) to be

$$E_\Gamma = P_\Gamma c = \hbar\Omega \tag{32}$$

every within the interval

$$\frac{\varepsilon}{2} \le E_\Gamma \le E = \hbar\omega \tag{33}$$

the sum of the energy (32) symmetrically exchanged during the interactions between a moving particle and all the other antiparticles, does not exceed each the energy of the upper limit of the range (33), corresponding to the de Broglie energy of each DEMS produced, i.e.

$$\hbar\sum_{j=1}^{2n}\Omega_j = 2n\hbar\omega \tag{34}$$

Considering the progressive sum of all the energies exchanged cannot exceed the total energy of the impinging particle, it is necessary to normalise the sum of the waves at the number of interactions occurred, so using the equations (16)

$$\Xi = \frac{1}{2n}\sum_{j=1}^{2n}\varphi_j(x,t) \tag{35}$$

that is

$$\Xi = \begin{cases} \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n} e^{i(\Omega_j t + kx)} & -\dfrac{T}{2} \le t < 0^- \\ \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n} e^{i(\Omega_j - \omega)t} & 0^- \le t \le 0^+ \\ \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n} e^{i(\Omega_j t - kx)} & 0^+ < t \le \dfrac{T}{2} \end{cases} \tag{36}$$

or using the Eq. (17), the real part of the wave (36) is

$$\mathrm{Re}\,\Xi = \begin{cases} \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n}\cos\left(\Omega_j t + kx\right) & -\dfrac{T}{2} \le t < 0^- \\ \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n}\cos\left(\Omega_j t - \omega t\right) & 0^- \le t \le 0^+ \\ \dfrac{\hbar}{2n}\displaystyle\sum_{j=1}^{2n}\cos\left(\Omega_j t - kx\right) & 0^+ < t \le \dfrac{T}{2} \end{cases} \tag{37}$$

which, considering Eq. (34), can be proven be equal to

$$\mathrm{Re}\,\Xi \cong \begin{cases} \hbar\cos\left(\dfrac{\sum_{j=1}^{2n}\Omega_j}{2n}t + kx\right) & -\dfrac{T}{2} \le t < 0^- \\ \hbar\cos\left(\dfrac{\sum_{j=1}^{2n}\Omega_j}{2n}t - \omega t\right) & 0^- \le t \le 0^+ \\ \hbar\cos\left(\dfrac{\sum_{j=1}^{2n}\Omega_j}{2n}t - kx\right) & 0^+ < t \le \dfrac{T}{2} \end{cases} \tag{38}$$

Using the Eq. (34) and the Eq. (7), considering only one interaction in time, the Eq. (38) yields

$$\mathrm{Re}\,\Xi \cong \begin{cases} \hbar\cos\left(\dfrac{E}{\hbar}t + \dfrac{P}{\hbar}x\right) & t < 0^- \\ \hbar & 0^- \le t \le 0^+ \\ \hbar\cos\left(\dfrac{E}{\hbar}t - \dfrac{P}{\hbar}x\right) & t > 0^+ \end{cases} \tag{39}$$

that corresponds to the result (19b) obtained for one interaction at very high energy. In this case the wave (39) is empty, in the sense that the wave not carries residual energy of mass but only action and information about the energy and the momentum of the particle in motion. Equivalently in complete exponential form

$$\Xi \cong \begin{cases} \hbar e^{i\left(\frac{E}{\hbar}t + \frac{P}{\hbar}x\right)} & t < 0^- \\ \hbar & 0^- \le t \le 0^+ \\ \hbar e^{i\left(\frac{E}{\hbar}t - \frac{P}{\hbar}x\right)} & t > 0^+ \end{cases} \tag{40}$$

The Eq. (40) can be considered a progressive wave moving on the X axis, the first part of the wave describes the movement of the particle in the past while it moves from infinity to the minimum interaction distance reaching it at zero time with action equal to the Planck's constant. The third part describes the movement of the particle on the X axis starting from the minimum distance in the present and going towards infinity in the future. On the whole, the wave moves progressively until it produces the dipole, then it goes back as a reflected wave, that is, it moves away from the dipole on the same axis of movement. The creation of the DEMS at time zero corresponds to a measurement process equivalent to the collapse of the wave function.

VI. DUALITY

Using the Eq. (40), the wave-solution (16) can be rewritten as

$$\varphi = e^{-iKct}\Xi \qquad (41)$$

from which the dimensionless wave-function ψ solution of the Eq. (13) can be considered formed by two independent terms

$$\psi = \frac{2\pi}{h} e^{-iKct}\Xi \qquad (42)$$

the first exponential function is associated to the corpuscular characteristics of the particle carrying an amount of momentum (11) which corresponds to the Galilean four momentum of the particle [19], i.e. at the momentum due to the Zitterbewegung of the particle at rest measured in the frame of the observer, the Ξ wave-function (40) is instead associated at the wave behaviour of the particle. In this case it is necessary to specify what is its role. When an electromagnetic signal achieves an external inertial observer, the interaction produces a direct DEMS with the exchanging of a photon with an amount of energy and momentum (7). In this sense the wave-function (40) is associated to the electromagnetic transferring of energy-momentum between primary and secondary DEMS, i.e. the Eq. (40) carries all the information to produce a secondary DEMS. Considering the equation (13) in the three cases (15a), (15b) and (15c) is possible rewrite the three equation in the matrix form

$$\boldsymbol{\kappa}\partial_{\mathbf{K}}\varphi = \mathbf{0} \qquad (43)$$

where the matrix

$$\boldsymbol{\kappa} = \begin{pmatrix} i & -i & 0 & 0 & -1 \\ i & 0 & 0 & 0 & -1 \\ i & 1 & 0 & 0 & -1 \end{pmatrix} \qquad (44)$$

is associated to the space-time metric of the interaction fermion - antifermion and the operator

$$\partial_{\mathbf{K}} = \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & K \end{pmatrix} \qquad (45)$$

describes as a vector the conservation of the Galilean momentum exchanged from particle and observer.

VII. CONCLUSIONS

The complexity of nature has frequently required new approaches in order to reduce the number of models or theories used to describe the natural phenomena. In this article is proposed a new wave-equation able to emulate the wave-particle duality of a fermion during its approach to an antifermion with which it produces a dipole electromagnetic source (DEMS). The fundamental equation here used, takes origin from the Doppler effect in Special Relativity [20] revised in conceptual terms in agreement with the Bridge Theory (BT) approach [16], developed in references [17]-[19] starting from the proof of the Poynting Vector Conjecture [13].

The fundamental result of this work is to explain through the Eq. (43) the double wave-particle description of a fermion as the result of an electromagnetic entanglement between the fermion with an antifermion. The direct interaction produces a DEMSthat allows both interacting particles in the laboratory to be revealed as if they were corpuscles with a localized mass energy, DEMS acts as a reciprocal measure of particle energy performed by each particle on the other. When the electromagnetic interaction begins, the wave associated with the moving particle describes its trajectory within the source zone of the DEMS. The track is delimited by an ideal cylinder with a maximum length of 3λ per interaction, where the wavelength is that of the DEMS produced. When a fermion crosses in space-time a polarisable medium, the distribution of DEMS it tracks the path with a trajectory with a thickness growing with the amount of energy transferred by the particle in motion to the polarisable media in which it moves producing DEMS.

In this work is proven that without interactions the particle propagates as a wave emitted by a primary DEMS in agreement with the de Broglie's description. The wave obtained is an empty wave similar to that used in Quantum Mechanics. This result gives to the assumption in Quantum Mechanics of the existence of a probability density wave of a particle a new physical foundation.

In this model the de Broglie wave is associated at the electromagnetic energy and at the momentum emitted by a primary DEMS under form of photon, corresponding to the energy carried by the electromagnetic fields of the primary wave that achieves the observer. The frequency of the photon exchanged satisfy the relativistic Doppler's effect.

Is also shown as the Klein-Gordon equation (KGE) is able to describe the DEMS as a boson, because it describes the simultaneous and reciprocal interaction between the pair of fermions. The use of KGE proves how the vacuum at ZPE produces the trembling of the virtual pairs of particles, a phenomenon perfectly described under form of a zitterbewegung originated by the continue casual interaction with the Higgs field from the new equation proposed. The trembling is able to generate the Galilean momentum of the particles [19].

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