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Shock Waves and It's Interaction with Two-Phase Flow

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Abstract: *This paper is especially based on the importance and existence of shock waves in nature. More precisely, I have described why we need to discuss the propagation of shock in two-phase flow. Again I studied the shock propagation in two-phase flow in the presence of a magnetic field. For the study, I have taken the two phases: one is perfectly conducting non-ideal gas, and the other is small solid particles(pseudo fluid.)I have also derived the conservation equations for a mixture of perfectly conducting non-ideal gas and non conducting small solid particles.*

Keywords*: shock waves, perfectly conducting non-ideal gas, small solid particles, magnetic field.*

I. INTRODUCTION

The shock wave is a propagating disturbance, often produced in nature during explosion, earthquake, electric discharge, and when an object moves with supersonic speed. Like an ordinary wave, shock carries energy. It can propagate through a medium (solid, liquid, gas, or plasma) or, in some cases, in the absence of a material medium, through a field such as an electromagnetic field.

The shock wave is mainly characterized by an abrupt, nearly discontinuous change in pressure, temperature, density, flow-velocity, and Mach-number of the medium across it. During the propagation of shock, the total energy is preserved, but there is always an entropy gain across it. The energy and strength of shock dissipate relatively quickly with distance.

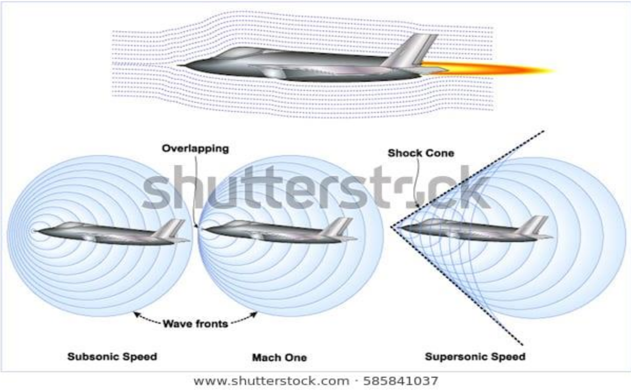
We see that the shock has some similarities with an ordinary wave, but some major physical properties of shock differ completely from an ordinary wave. The initial disturbance produced in the medium that causes a shock wave is always traveling faster than the medium's phase velocity. Another main characteristic that divides a shock wave from an ordinary wave is the thermodynamics of changes in the pressure and temperature due to the wave. For an ordinary wave, the gas's compression and rarefaction do not entail a change in the gas's entropy. Thus, an ordinary wave is a reversible process, but there is always an entropy gain across it in a shock wave. This is the reason that the shock is not a reversible process. Due to the above dissimilarities of shock from an ordinary wave, a conclusion may be made about the shock that to spell it "shock wave" is only customary. It is not a wave. It is a surface across which flow-variables are discontinuous. That's why it is called a discontinuity surface.

A more formal definition of shock may also be given: 'A shock is a region of very small thickness propagating in a gas across which the flow-variables change abruptly.' In many practical problems of shock in inviscid fluid, the shock has been considered as a surface of discontinuity of very small thickness. Whitham [1], Sedov [2], Zeldovich and Raizer [3], Laumbach and Probstein [4], Korobeinkov [5], Gretler [6], Steiner and Hirschler [7], Vishwakarma and Nath [8], Vishwakarma, Patel, and Chaube [9], Vishwakarma and Nath ([10], [11]), Singh et al. [12], Singh [13], Vishwakarma and Patel [14], Vishwakarma and Srivastava [15], Nath and Sahu [16] and many other have studied the problem of a shock for inviscid fluid by assuming shock to be a discontinuity surface of very small thickness.

The definition of shock comprises some other physical features as: "A shock wave separates two regions in space, the upstream, cold, low pressure, and low-density gas and the downstream, hot, high-pressure gas." When a shock propagates, it leads to compression, heating, and acceleration of the medium and contributes to kinetic energy dissipation.

The occurrence of shock in nature is commonly associated with astrophysical, geophysical, and space research phenomena like a supernova explosion, motion in the interstellar medium, solar flares, sand storms, aerodynamic ablation, coal-mine blast, nozzle flow, and many others.

A phenomenon of artificially generated shock in the laboratory can be visualized by taking a piston's uniform motion into an open-ended tube filled with gas. A simple physical explanation of the shock formation, in this case, is the following. Suppose the piston's continuous motion is approximated by a set of forward-moving pulses, each of a short duration. When the piston makes the first short movement forward, a small disturbance is propagated toward the gas at sound speed. This small amplitude wave or sound wave heats the gas slightly, and because the square of the local speed of sound is proportional to the temperature, the second pulse will be propagated at a speed slightly more than the first one. Similarly, the third pulse will be propagated slightly more than the second, and so on. Thus the discrete pulses cause a train of sound waves of ever-increasing velocity to be propagated through the gas. The tendency is for faster moving rearmost waves to catch up with the slower moving foremost ones. The sound waves coalesce to form a more powerful shock front moving at a greater speed than the local speed of sound.



II. SIGNIFICANCE OF THE STUDY OF SHOCK WAVE PROPAGATION IN NON-IDEAL GASES

Shock waves are often produced in nature by the strong explosion, lightning, earthquake, and any other phenomena that create a violent change in pressure, density, and temperature. Since a gas behaves like an ideal one under high temperature and low-pressure condition, because of violent change in pressure, density, and temperature, the assumption of perfect gas as a medium for propagating shock is no more valid. Also, as the strength of shock increases, the effects of non-idealness of the gas becomes significant. So, there is always a need to study the propagation of shock in non-ideal gases.

The study of shock wave propagation in a non-ideal gas is of great scientific interest in many problems because of their wide application in astrophysics, oceanography, atmospheric science, hypersonic aerodynamic, hypervelocity impact, and many others. Many researchers have investigated the problem of shock propagation in non-ideal gas, especially Landau and Lifshtiz [17], Anisimov and Spiner [18], Ranga Rao and Purohit [19], Wu and Roberts [20], Ojha and Tiwari [21], Roberts and Wu [22], Singh and Singh [23], Madhumita and Sharma [24], Arora and Sharma [25], Pandey and Sharma [26], Vishwakarma, Patel, and Chaube [9], Vishwakarma and Nath [10], Singh et al. [12], Vishwakarma and Singh [27], Singh and Nath [28], Nath [29], Pandey and Singh [30], Singh and Singh [31] and Nath and Sahu [32].

III. EQUATION OF STATE OF A NON-IDEAL GAS

When a strong explosion occurs, the substance's motion's character depends essentially on its equation of state. Such a motion was studied originally for the case of an ideal gas; subsequently, examples of solutions of the explosion problem were given for certain real, thermodynamically imperfect media (Sedov [2], Korobeinikov, Melnikova and Ryazanov [33], Kochina and Melnikova [34]). It should, however, be noted that the study of explosions in the media differing from the ideal gas did, as a rule, involve empirical equations of state, which only describe the behavior of the medium satisfactorily in a certain, limited interval of densities. Almost every one of these empirical equations was incorrect for the low-density region. In the limit, when $\rho \to 0$, it either did not reduce to the equation of state for the ideal gas or yield an incorrect first term of the so-called virial expansion for the pressure in powers of density.

We know from statistical physics, at low densities, the equation of state can be written in the form (Landau and Lifstiz [17])

$$p = \rho RT\,[1 + \rho B(T) + \rho^2 C(T) + \cdots \ldots\ldots],$$

R is the gas constant, and B(T) and C(T) are virial coefficients, determining if the molecule's interaction potential is known. In the high-temperature range, the coefficients B(T) and C(T) tend to constant values equal to $\bar{b}$ and (5/8) $\bar{b}^2$, respectively. For gases $\bar{b}\rho$ << 1, $\bar{b}$ being the internal volume of the molecules, and therefore it is sufficient to consider the equation of the state in the form (Anisimov and Spiner [18], Ranga Rao and Purohit [19])

$$p = \rho RT(1 + \rho\bar{b})\,. \qquad (3.1)$$

Wu and Roberts [20] and Roberts and Wu [22] have used an equivalent equation of state to study the shock wave theory of sonoluminescence.

From thermodynamics, we have

$$\left(\frac{\partial E}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p\,, \qquad (3.2)$$

where E is the internal energy per unit mass of the gas, and v is the specific volume.

Using the equation of state (3.1) in equation (3.2), we get $\left(\frac{\partial E}{\partial v}\right)_T = 0$, which shows that the internal energy E is a function of temperature T only. Therefore,

$$E = C_v T, \qquad (3.3)$$

where C_v is the specific heat at constant volume.

Using equation (3.2) in the first law of thermodynamics, we have

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial T}\right)_p, \qquad (3.4)$$

where C_p is the specific heats of the gas at constant pressure.

Using equation (3.1) in equation (3.2), we get

$$C_p - C_v = \frac{R(1+\rho\bar{b})^2}{1+2\rho\bar{b}} \cong R, \qquad (3.5)$$

neglecting second and higher powers $\bar{b}\rho$.

Equation (3.5) implies that

$$C_v = \frac{R^*}{(\gamma-1)} \quad . \qquad (3.6)$$

Then equations (3.1), (3.3) and (3.6) give the internal energy E as a function of p and ρ, in the form

$$E = \frac{p}{\rho(\gamma-1)(1+\rho\bar{b})}\,. \qquad (3.7)$$

The speed of sound.'a_s' maybe calculated from equation (3.1) as follows

$$a^2{}_s = \left(\frac{dp}{d\rho}\right)_s = \frac{(1+2\rho\bar{b})}{(1+\rho\bar{b})}\frac{\gamma p}{\rho}\,. \qquad (3.8)$$

The equation of the energy of the non-ideal gas whose equation of state is in the form of equation (3.1), is given by (Singh and Singh [23])

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-a^2{}_s\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0, \quad (3.9)$$

where $a^2{}_s$ is given in equation (3.8).

The equations of continuity and momentum are the same as for a perfect gas.

IV. FUNDAMENTAL EQUATIONS FOR A GAS-PARTICLE MIXTURE

In our entire work, we have studied the two-phase flow (Rudinger [35], Murray [36], Marble [37], Pai [38] and Vishwakarma and Nath [11]) of a mixture of a non-ideal gas and small solid particles for analyzing the problem of shock wave propagation under different conditions.

The study of shock propagation in the gas-particle mixture is of great importance due to its application in many astrophysical, geophysical, nuclear science, and space research phenomena like a coal-mine blast, bomb blast, lunar ash flow, nozzle flow, metalized propellant rocket, supersonic flight in polluted air, sand storms, aerodynamic ablation, cosmic dusts and formation of shock during a supernova explosion.

Many researchers have been studied the problem of shock in dusty gases (a mixture of gas and small solid particles), particularly Pai et al. [39], Higashino, and Suzuki [40], Miura and Glass [41], Steiner and Hirschler [7], Vishwakarma.

To analyze some essential physical features of shock during its propagation, we have made some assumptions for the medium (Pai et al. [39], Higashino and Suzuki [40], Vishwakarma and Vishwakarma [45] and Vishwakarma and Nath [11]

i. For the propagation of shock, the medium is a mixture of non-ideal gas and small solid particles.
ii. The small solid particles are taken as pseudo-fluid and are assumed to be continuously distributed in the mixture.

Pseduo-fluid When a large number of small solid particles flow in a fluid, and the velocity of the fluid is sufficiently high, such solid particles' behavior becomes similar to the ordinary fluid. We may consider these solid particles as pseudo-fluid.

iii. The solid particles are assumed to be spheres of identical mass m_p , radius r_p and specific heat C_s (Pai [38]).
iv. Temperature and velocity of small solid particles are equal to those of the non-ideal gas for the equilibrium flow, i.e., the velocity of solid particles u_{sp} that of the gas u_g and that of the mixture, u are equal. The temperature of solid particles T_{sp} and that of the gas T_g are equal, and they are also equal to the temperature of the mixture.
v. The mixture's viscous stress and heat conduction are assumed to be negligible during the propagation of shock.

Since the medium is a mixture of two fluids: one is real fluid (gas), and another is pseudo fluid (dust particles), we have two definitions of density: the species density and partial density.

We consider an element of the mixture of the non-ideal gas and small solid

particles with total mass M = M_g+M_{sp} and with a total volume of $V=V_g+V_{sp}$ where subscript 'g' refers to the gas, and subscript 'sp' refers to the solid particles.

Here we define the number density of solid particles n_{sp} which is the number of solid particles per unit volume at a point in the flow-field. The volume occupied by the solid particles V_{sp} is

$$V_{sp}=n_{sp}\,\bar{v}_{sp}V, \quad (4.1)$$

where $\bar{v}_{sp}=\frac{4}{3}\wedge r_p^3$ is the volume of a solid particle in the mixture. Values without subscript are used for the whole mixture.

The mass of solid particles in volume V of the mixture is

$$M_{sp}=m_{sp}\,n_{sp}\,V\ , \quad (4.2)$$

where m_p is the mass of a solid particle.

The species density of solid particles is defined as

$$\rho_{sp}=\frac{M_{sp}}{V_{sp}}=\frac{m_{sp}}{\bar{v}_{sp}}\ . \quad (4.3)$$

Thus the species density of solid particles is a constant for a given problem.

The partial density of solid particles is defined as

$$\bar{\rho}_{sp}=\frac{M_{sp}}{V}=m_{sp}\,n_{sp}=Z\,\rho_{sp}=\rho_{sp}\bar{v}_{sp}\,n_{sp}, \quad (4.4)$$

Z represents the volume fraction of solid particles, which is one of the most important factors in treating the problem of two-phase flow of a gas-particle mixture.

From the above equation, we have the volume fraction of solid particles as follows

$$Z=n_{sp}\bar{v}_{sp}=\frac{V_{sp}}{V}. \quad (4.5)$$

Similarly, the species and partial densities of the gas can also be defined as

$$\rho_g=\frac{M_g}{V_g} \quad \text{and}$$

$$\bar{\rho}_g=\frac{M_g}{V}=\frac{M_g}{V_g}\,\frac{V_g}{V}=\frac{M_g}{V_g}\,\frac{(V-V_{sp})}{V}=(1-Z)\,\rho_g. \quad (4.6)$$

Equation of state and thermodynamics-

When a fluid is flowing, there should be a definite relation between the flow-variables pressure, density, and temperature that characterize the fluid properties to investigate these flow variables' dependency. The first law of thermodynamics gives a relation between these flow variables (pressure p, density ρ, and temperature T) known as the equation of state for a perfect gas.

$p=\rho R^*T,$

We have one state equation for each species in the mixture of a non-ideal gas and small solid particles. The equation of state for the small solid particles is simply,

$$\rho_{sp}=\text{constant}. \quad (4.7)$$

The equation of state for the non-ideal gas in the mixture is taken to be (Anisimov and Spiner [18], Ranga Rao and Purohit [19], Vishwakarma and

Nath [11])

$$p_g = R^* \rho'_g (1 + b\, \rho'_g) T = R^* (1-Z)[1 + b(1-Z)\rho_g] T, \quad (4.8)$$

where p_g and ρ'_g are the partial pressure and partial density of the gas in the mixture, T is the temperature of the gas and the solid particles as the equilibrium flow condition is maintained, R^* Is the specific gas constant, and b is the internal volume of the gas molecules. Because of the intermolecular force of interaction present among the gas's component molecules, an actual gas's deviations from the ideal state results in this equation. The non-ideal gas density is assumed to be so small that the triple, quadruple, and higher-order collisions among the gas molecules are negligible. Therefore the gas molecules interact through binary collisions only.

The partial pressure of the gas p_g and the total pressure of the mixture p are related by the equation

$$p_g = (1-Z)p \,. \quad (4.9)$$

Therefore from equation (4.8)

$$p = R^* \rho_g [1 + b(1-Z)\rho_g] T \,. \quad (4.10)$$

The density of the mixture as a whole is given by

$$\rho = \bar{\rho}_{sp} + \rho_g = Z\rho_{sp} + (1-Z)\rho_g. \quad (4.11)$$

The mass concentration of solid particles in the mixture is defined as

$$K_p = \frac{M_{sp}}{M} = Z\frac{\rho_{sp}}{\rho}. \quad (4.12)$$

In equilibrium flow, K_p is constant in the whole flow field. Therefore from (4.8)

$$\frac{Z}{\rho} = \text{constant}. \quad (4.13)$$

Also, we have from equation (4.11) and (4.12)

$$Z = \frac{K_p}{G(1-K_p)+K_p}, \quad (4.14)$$

where $= \frac{\rho_{sp}}{\rho_g}$.

Also, from equation (4.10), (4.11), and (4.12), we obtain the equation of state of the mixture of non-ideal gas and small solid particles as (Pai [39])

$$p = \frac{1-K_p}{1-Z}\left[1 + b\rho(1-K_p)\right]\rho R^* T \,. \quad (4.15)$$

For thermodynamic equilibrium condition, the internal energy E per unit mass of the mixture is related to the internal energies of two species by the following relation:

$$\rho E = Z\rho_{sp} C_{sp} T + (1-Z)\rho_g C_v T$$

or

$$E = \left[K_p C_{sp} + (1-K_p)C_v\right] T \,, \quad (4.16)$$

where C_{sp} is the specific heat of the solid particles, C_v is the specific heat of the gas at constant volume.

The internal energy per unit mass of the mixture may be written as

$$E = [K_p C_{sp} + (1-K_p)C_v]\mathrm{T} = C_{vm}\mathrm{T}, \quad (4.17)$$

where C_{sp} is the specific heat of the solid particles, C_v is the specific heat of the gas at constant volume and C_{vm} is the specific heat of the mixture at constant volume.

Therefore from equation (4.17), we have the specific heat of the mixture at constant volume is

$$C_{vm} = K_p C_{sp} + (1-K_p)C_v \,. \quad (4.18)$$

Also, the specific heat of the mixture at constant pressure is

$$C_{pm} = K_p C_{sp} + (1-K_p)C_p. \quad (4.19)$$

where C_p is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai et al. [39], Pai [38], Marble [37]),

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma \frac{1+\delta\beta'/\gamma}{1+\delta\beta'}, \quad (4.20)$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{K_p}{1-K_p}$, and $\beta' = \frac{C_{sp}}{C_v}$.

Now,

$$C_{pm} - C_{vm} = (1-K_p)(C_p - C_v) = (1-K_p)R^*, \quad (4.21)$$

where $R^* = (C_p - C_v)$ neglecting the term $b^2\rho^2$ (Landau and Lifshtiz [17], Singh [56]). The internal energy per unit mass of the mixture is, therefore, given by

$$E = \frac{p\,(1-Z)}{\rho\,(\Gamma-1)\,[1+b\rho\,(1-K_p)]}. \quad (4.22)$$

If we consider the mixture as a homogeneous medium, the first law of thermodynamics for the mixture gives

$$dQ = dE - \frac{1}{\rho^2} p\, d\rho, \quad (4.23)$$

where dQ is the heat addition of the mixture.

We have from the isentropic change of state of mixture $dQ = 0$, therefore using equations (4.22) and (4.15) in equation (4.23), we get

$$\frac{1}{\Gamma-1}\frac{dT}{T} = \frac{[1+b\rho(1-K_p)]}{(1-Z)}\frac{d\rho}{\rho}. \quad (4.24)$$

or

From equation (4.15), we have

$$\frac{dp}{p} = \frac{dT}{T} + \left[\frac{1}{(1-Z)\rho} + \frac{(\Gamma-1)b(1-K_p)}{(1-Z)} + \frac{b(1-K_p)}{1+b\rho(1-K_p)}\right] d\rho \,. \quad (4.25)$$

Now, equations (24) and (25) give

$$\frac{dp}{p} = \left[\frac{\Gamma}{(1-Z)\rho} + + \frac{(\Gamma-1)b(1-K_p)}{(1-Z)} + \frac{b(1-K_p)}{1+b\rho(1-K_p)}\right] d\rho \,, \quad (4.26)$$

or

$$p\left(\frac{\rho}{(1-Z)}\right)^{-\Gamma} \frac{1}{[1+b\rho(1-K_p)](1-Z)^{-(\Gamma-1)b(1-K_p)}} = \text{constant}. \quad (4.27)$$

We may calculate the equilibrium speed of sound of the mixture of non-ideal gas and small solid particles from equation (4.26), as

$$a = \left(\frac{dp}{d\rho}\right)_s^{1/2} = \left[\frac{\{\Gamma+(2\Gamma-Z)b\rho(1-K_p)\}P}{(1-Z)\{1+b\rho(1-K_p)\}\rho}\right]^{\frac{1}{2}}, \quad (4.28)$$

neglecting the term $b^2\rho^2$.

Equation of continuity-

We have one equation of continuity for each species in the gas-particle mixture that gives the conservation of that species' mass. Combining the equation of continuity of both the species, we have the equation of continuity for the one-dimensional motion of the mixture (Pai et al. [39], Pai [38])

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0, \quad (4.29)$$

where $j = 0, 1$ or 2 for the plane, cylindrical or spherical symmetry, ρ, and u are the density and the flow velocity of the mixture, r and t are space and time coordinates.

Equation of motion-

We have one equation of motion for each species in the gas-particle mixture, which gives the conservation of momentum for that species. If we combine the equation of motion of both the species, we may obtain the equation of motion for one dimensional, the unsteady flow of the mixture as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \quad (4.30)$$

where p = $p_g + p_p$ is the total pressure of the mixture. The viscous stress and heat conduction of the mixture is assumed to be negligible in the above equation.

Equation of energy-

We have one equation of energy for each species in the gas-particle mixture, which gives energy conservation for that species. Combining the equation of energy of both the species, we may obtain the equation of energy for the mixture as a whole.

The equation of energy for the unsteady, one-dimensional flow of the whole gas-particle mixture in which viscous stress and heat conduction are assumed to be negligible can be written as (Pai et al. [39], Pai [38])

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} - \frac{p}{\rho^2}\left\{\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right\} = 0, \quad (4.31)$$

where *E* is the internal energy of the gas-particle mixture.

V. MAGNETOGASDYNAMICS

If a conducting fluid moves in a magnetic field, electric fields are induced, and electric currents flow. The magnetic field exerts forces on these currents, which considerably modify the flow (Laundau and Lifstiz [17]). In many problems, the electric field's energy is much smaller than that in the magnetic field. We may express all the electromagnetic quantities in the magnetic field (Pai [57]). As a result, we consider only the interaction between the magnetic field and the gas-dynamic field. This analysis forms the subject matter of the well-known 'magneto gas dynamics.' This interaction is of prime importance in most astrophysical and geophysical problems and interstellar gaseous masses' behavior. As done in many problems, we have ignored Maxwell's displacement current. As usual, we also assume that the dissipative mechanisms such as viscosity and thermal conductivity are absent.

VI. INTERACTION OF TWO-PHASE FLOW WITH MAGNETIC FIELD

When we discuss two-phase flow motion under the magnetic field's influence, it is not always necessary that both phases of the flow are conducted. It may be possible that only one phase of the flow is conducting.

In the entire thesis, we are concerned with the motion of the two-phase flow of gas-particle mixture under an azimuthal magnetic field's influence. We have discussed both the situations first when the whole gas-particle mixture is perfectly conducting and second: when only the gas is perfectly conducting. Still, the particles are non-conducting in the gas-particle mixture.

The equations of motion for a one-dimensional, unsteady flow of a perfectly conducting gas-particle mixture in the presence of an azimuthal magnetic field are given as (Christer and Helliwell [58], Saukari [69], Verma and Vishwakarma [60], Vishwakarma and Pandey [42]).

Equation of continuity-

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0, \quad (6.1)$$

where $j = 0, 1$ or 2 for the plane, cylindrical or spherical symmetry, ρ, and u are the density and the flow velocity of the mixture, r and t are space and time coordinates.

Equation of momentum-

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0, \quad (6.2)$$

where p is the pressure of the mixture, h is the azimuthal magnetic field, and μ is the magnetic permeability.

Equation of magnetic field-

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (j-1)\frac{hu}{r} = 0, \quad (6.3)$$

Here j=1 and 2 for cylindrical and spherical symmetry.

Equation of energy –

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} - \frac{p}{\rho^2}\left\{\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right\} = 0, \quad (6.4)$$

where *E* is the internal energy of the mixture.

Equations of continuity, magnetic field, and energy are the same as for the mixture of conducting gas and non-conducting solid particles, but the equation of momentum is quite changed as,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1-Z}{\rho}\left[\mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0, \quad (6.5)$$

where Z is the volume fraction of solid particles in the mixture.

VII. SHOCKWAVESINMAGNETOGASDYNAMICS

It is a well-known fact about the shock wave that the temperature becomes much high when it propagates through a gaseous medium. The gas gets ionized at such a high temperature, and the medium behaves like a very high electrically conducting medium. Hence, in this case, the effects of the magnetic field become significant. So, there is always a need to study the propagation of shock waves in a magnetized medium.

The study of the shock wave in gas-particle mixture under the influence of magnetic field has been of great scientific interest as it can be applicable while discussing the problem of a supernova explosion, motion in the interstellar medium, photo-ionized gas, solar winds, collisions between high-velocity clumps of interstellar gas and many others.

In the present thesis, we have analyzed the problem of shock wave propagation in a two-phase flow of a mixture of a non-ideal gas and small solid particles under the influence of a variable azimuthal magnetic field.

The jump conditions (the generalized Rankine-Hugoniot relations) across the shock front that relate the fluid properties behind the shock (downstream of the shock) to the fluid properties ahead of the shock (upstream of the shock) are derived from the principles of conservation of mass, magnetic flux, momentum and energy under the assumption that the shock front is a discontinuity surface with no thickness as (Pai [57], Sedov [2])

$$\rho_a U = \rho_b\ (U - u_b),$$
$$h_a U = h_b\ (U - u_b),$$
$$p_a + \rho_a U^2 + \frac{\mu h_a^2}{2} = p_b + \rho_b (U - u_b)^2 + \frac{\mu h_b^2}{2},$$

(7.1)

$$\frac{\Upsilon p_a}{(\Upsilon - 1)\rho_a} + \frac{U^2}{2} + \frac{\mu h_a^2}{\rho_a} = \frac{\Upsilon p_b}{(\Upsilon - 1)\rho_b} + \frac{1}{2}(U - u_b)^2 + \frac{\mu h_b^2}{\rho_b},$$

$$\frac{Z_a}{\rho_a} = \frac{Z_b}{\rho_b}.$$

Here U denotes the shock velocity; the subscript 'a' and 'b' denote the values of flow variables just ahead and just behind the shock.

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