

# New Method of Thermal Testing Shell-and-Tube Heat Exchangers

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## Abstract

The article presents new methods of thermal testing of shell-and-tube heat exchangers according to copyright certificates SU No. 1377558 and No. 1589021, in which for the first time, two parameters of a heat exchanger are jointly determined: the heat transfer coefficient  $k$  and the conditional angle  $\varphi$  between the directions of flow of heat carriers in the heat exchanger. This angle  $\varphi$  (rad) characterizes the efficiency of the mutual flow of heat carriers, which increases with an increase in the angle  $\varphi$ : for forward flow, i.e., the most ineffective scheme,  $\varphi = 0$ , and for counter-flow, i.e., the most efficient scheme  $\varphi = \pi$  (180 degrees). With the determination of a new parameter  $\varphi$ , this test method allows a more accurate determination of the heat transfer coefficient  $k$  – due to the separation of the contributions to the efficiency of the heat exchanger of both factors: intensive ( $k$ ) and extensive factor ( $\varphi$ ). The paper also presents the theoretical foundations of the universal thermal calculation method (UMTC) of heat exchangers-recuperators. The efficiency of the coolant flow circuit is set just by the conditional angle  $\varphi$ .

**Keywords:** shell-and-tube heat exchanger, heat carrier, thermal tests, heat transfer coefficient, the efficiency of the current circuit, counter-flow index, conditional angle.

## I. INTRODUCTION

Heat exchangers (HE) are the main class of devices used to transfer heat from one medium to another in various fields of technology: in heat power engineering, heat engineering, and heat supply systems, as well as in metallurgical, chemical, and other technological processes. Shell and tube heat exchangers differ from other types by creating a casing in which the tube bundle is installed. One of the coolants in it is pumped through the tube bundle, and the other through the space between the tube bundle and the casing.

The HE development procedure includes its design, thermal calculation, and thermal testing of the sample. Thermal tests of heat exchangers are usually carried out in their operation's design mode, while traditionally only one mode parameter is determined: the heat transfer coefficient of the heat exchanger  $k$  (W/m<sup>2</sup>K). At the same time, it is important to establish that the thermal test data correspond to the results of its thermal calculation since the heat transfer coefficient  $k$  demonstrates the relationship between the transferred heat flux and the heat transfer area of the tube bundle, that is, between the thermophysical and economic efficiency of HE. Indeed, to increase the

intensive factor of efficiency of HE ( $k$ ), it is necessary to increase the speed of the coolants. Hence, the energy consumption for their pumping, i.e., in general, for maintenance of maintenance during operation, while to increase  $\varphi$ , i.e., The efficiency of the coolant flow circuit requires only an innovative contribution from the designer at the design stage of the heat exchanger.

## II. OVERVIEW

At present, for the experimental determination of the heat transfer coefficient of HE –  $k$ , its thermal tests are carried out in the design mode of operation (at the calculated values of the flow rates and temperatures of both coolants at the entrance to the HE) so that it is possible to find out the real characteristics of the HE and establish whether the calculated value corresponds  $k$  - experimental data [1].

Thermal tests in the traditional way are carried out in the form of a single experiment carried out by pumping two coolants through the HE: one through the channels of the tube bundle, and the other through the annular space in the HE casing, as well as measuring the flow rates and temperatures at the inlet and outlet of the HE in a steady state. Heat transfer mode. In this case, the heat transfer coefficient is determined by the formula:

$$k = Q/(\Delta t_m \cdot F), \text{ (W/m}^2\text{K)} \quad (1)$$

where  $Q$  is the heat flux transferred in the HE, (W);  
 $F$  – the surface area of heat transfer in HE (m<sup>2</sup>);  
 $\Delta t_m$  – temperature head (K),  $\Delta t_m = f(t'_H, t''_H; t'_C, t''_C)$ , H – "Hot," C – "Cold."

However, the traditional method can be used to test only such HE, for the design circuits of the heat carrier current, the analytical expression for the temperature difference is known:  $\Delta t_m = f(t'_H, t''_H; t'_C, t''_C)$ , and in explicit form, and the number such cases are few. In addition, the experimental value of  $k$  obtained by the traditional method includes a large methodological error, the source of which is the difference between the calculated current circuit (based on which the analytical expression  $\Delta t_m$  was obtained) from the actual circuit, which is implemented in HE. This difference is caused by the presence of "secondary" effects that are not taken into account by the design circuit of the current (irregularities of the temperature, velocity, etc. fields along the sections of the coolant flows), which can be significant, especially for complex current circuits.

But another (generalized) method of thermal calculation of HE has long been known [2]. The current



circuit's efficiency is expressed not by an individual formula for  $\Delta t_m$  but by an individual constant in a generalized expression for the average temperature head of all shell-and-tube HE. In the book [3], Belokon N. presented such a method of thermal calculation of HE, in which an expression of the temperature head  $\Delta t_m$ , generalized for various types of HE, was proposed, which, in addition to temperatures, also contained the efficiency factor of the HE current circuit – the so-called "counter-flow index"  $\mathbf{p}$  equal to zero for the least efficient current circuit (forward flow) and equal to one for the most effective current circuit (counter-flow).

The generalized expression for the temperature head  $\Delta t_m$ , in which the efficiency of the HE current circuit is determined only by the value of its "counter-flow index"  $\mathbf{p}$  (according to the Belokon formula), has the following form:

$$\Delta t_m = (\Delta t_G - \Delta t_S) / \ln(\Delta t_G / \Delta t_S), \quad (2)$$

where  $\Delta t_G$  and  $\Delta t_S$  are "Greatest" and "Smallest" of the temperature difference at the inlet to the HE and the outlet from the HE:  $\Delta t_G = \Delta t_{AR} + \frac{1}{2}(\Delta T)$ ,  $\Delta t_S = \Delta t_{AR} - \frac{1}{2}(\Delta T)$ ;  $\Delta t_{AR} = \frac{1}{2}(t'_H + t''_H) - \frac{1}{2}(t'_C + t''_C)$  – arithmetic mean temperature difference (K);

$\Delta T = \sqrt{[(\delta t_H + \delta t_C)^2 - 4\mathbf{p}(\delta t_H \delta t_C)]}$  – "characteristic" temperature difference of heat carriers,  $\delta t_H = t'_H - t''_H$ ,  $\delta t_C = t'_C - t''_C$ , (K),

$\mathbf{p}$  – is the "counterflow index" of the HE current circuit, for the most efficient circuit (counterflow:  $\downarrow\uparrow$ )  $\mathbf{p} = 1$ , for the least efficient current circuit (forward flow:  $\uparrow\uparrow$ )  $\mathbf{p} = 0$ , for all other circuits intermediate in efficiency ( $0 \leq \mathbf{p} \leq 1$ ), [3].

However, Belokon's formula for  $\Delta t_m$  did not get widespread in the practice of thermal calculations since he proposed a very inaccurate method for calculating the counter-flow index ( $\mathbf{p}$ ) for various circuits of coolant flow in shell-and-tube HE.

### III. FORMULATION OF THE PROBLEM

Analysis of Belokon's formula showed that the generalized formula for the heat transfer equation based on the generalized expression for  $\Delta t_m$  contains not one unknown ( $\mathbf{k}$ ) but two ( $\mathbf{k}$  and  $\mathbf{p}$ ). This makes it possible to create a fundamentally new method for HE's thermal testing with the determination of two unknowns ( $\mathbf{k}$  and  $\mathbf{p}$ ) simultaneously. Thus, in the case of using the Belokon's formula as the average temperature head  $\Delta t_m$  in the heat transfer equation, the latter can be used in thermal tests of HE with any scheme of the mutual current of heat carriers and even those HE for which the analytical expression  $\Delta t_m = f(t'_H, t''_H; t'_C, t''_C)$ .

The heat transfer equation for shell-and-tube HE with a generalized expression of the temperature head in a dimensionless form (in this case, it is better to use  $\Delta t_{AR}$  as the scale of reduction) has the following form:

$$Q / (\Delta t_{AR} \cdot \mathbf{k}F) = \Delta t_m / \Delta t_{AR} \equiv X / \text{Arth}X = y(X), \quad (3)$$

where  $X$  – is a dimensionless variable,  $X = (\Delta t_G / \Delta t_{AR}) \cdot \sqrt{[(1 + R)^2 / 4 - \mathbf{p}R]}$ ,

$R$  – dimensionless ratio,  $R = \delta t_{\min} / \delta t_{\max}$ ,  $\delta t_{\min} = \min(\delta t_H, \delta t_C)$ ,  $\delta t_{\max} = \max(\delta t_H, \delta t_C)$ .

This equation contains only two unknowns ( $\mathbf{k}$  and  $\mathbf{p}$ ) since as a result of the experiment, all other quantities included in it ( $Q$ ,  $\Delta t_{AR}$ ,  $\Delta t_G$ ,  $R$ ) can be measured and calculated for a known value of  $F$ . Therefore if HE tests within the framework of two experiments in different modes of its operation and under the condition,  $\mathbf{k}_1 = \mathbf{k}_2$  and  $\mathbf{p}_1 = \mathbf{p}_2$ , then two equations with two unknowns can be obtained. By solving such a system of equations, the values of both unknowns can be determined. In this case, the equality  $\mathbf{k}_1 = \mathbf{k}_2$  in both experiments is due to the fact that both the flow rates and the initial temperatures of both coolants in both experiments are kept the same.

Thus, as a result of two experiments, it is possible to obtain a system of two equations with two unknowns ( $\mathbf{k}$  and  $\mathbf{p}$ ):

$$\begin{cases} Y_1 = y(X_1(\mathbf{k}, \mathbf{p})) \\ Y_2 = y(X_2(\mathbf{k}, \mathbf{p})) \end{cases} \quad (4)$$

It is known that such a system of two equations with two unknowns has a solution that can be obtained analytically or numerically.

It can be assumed that the new test method will also provide an opportunity to refine the methods of thermal calculation since it will allow a more accurate interpretation of the results of thermal tests of HE.

## IV. RESEARCH RESULTS

### A. Solving the system of equations for thermal tests

Since the expression  $y(X)$  in this case – is nonlinear and cannot be reduced to a linear one, this system can be solved only by numerical methods. The analytical solution of this system, and hence the possibility of obtaining explicit expressions for the unknowns:  $\mathbf{k}$  and  $\mathbf{p}$  – can be obtained only by using an approximation of the nonlinear expression  $y = y(X)$  that can be reduced to a linear form. In this case, for a system of two linear equations, solutions can be obtained explicitly. The exact approximation of this nonlinear expression, which can be reduced to a linear form, was obtained by the author [4] in the form of Newton's binomial:

$$y(X) = X / \text{Arth}X \cong [1 - (13/15) \cdot X^2]^{1/2.6}. \quad (5)$$

The error of this approximation  $\Delta$  is equal to zero at  $X = 0$ , in addition, it increases with the growth of  $X$  and reaches 1% at  $X^2 = 0.7$  (with  $\Delta t_m / \Delta t_{AR} = 0.7$ ). That is, formula (5) has sufficient accuracy in interpreting the results of thermal tests.

And the linearized heat transfer equation in HE can be written in dimensionless form as follows:

$$(Q / \Delta t_{AR} \cdot \mathbf{k}F)^{2.6} = 1 - (13/15) \cdot (\Delta t_G / \Delta t_{AR})^2 \cdot [(1 + R)^2 - \mathbf{p}R]. \quad (6)$$

If we introduce the notation:  $f(\mathbf{p}, R) = (1 + R)^2 - \mathbf{p}R$  and  $b = (13/15) \cdot (\Delta t_G / \Delta t_{AR})^2$ , then in the accepted notation this equation will be rewritten more compactly:

$$a_i \equiv (Q/\Delta t_{AR} \cdot F)^{2.6} = k^{2.6} \cdot [1 - b_i \cdot f(\mathbf{p}_i, R)]. \quad (7)$$

### B. Development of a thermal test procedure according to CC. No. 1377558

Suppose in a prototype HE within the thermal tests framework; both experiments are carried out in a laminar flow regime of the coolant flowing in the tube bundle. In that case, it can be assumed that in both experimental regimes, the unknown  $k$  are the same. And the equality of values  $\mathbf{p}_1 = \mathbf{p}_2$  here is obvious by definition.

The solution of the system of two equations – the results of thermal tests, including the data of two experiments (with the calculated temperatures of both coolants at the inlet to the HE, but with different flow rates of the coolant flowing laminarily in the tube bundle) will solve the problem.

On 8.08.1985, the author registered a fundamentally new method of testing recuperative HE [5], which allows one to determine two characteristics of HE at once: the heat transfer coefficient of HE –  $k$  and the index of counter-flow of its current circuit –  $\mathbf{p}$ .

The HE tests by a new method are carried out in the form of two experiments, which differ from each other only in the values of the coolant flow rate flowing laminarily in the tube bundle of the tested HE. In this case, the values of the heat transfer coefficient in the HE  $k$  and the counter-flow index of its current circuit  $\mathbf{p}$  – are determined by the formulas:

$$k^{2.6} = (a_1 b_2 R_2 - a_2 b_1 R_1) / (B_1 b_2 R_2 - B_2 b_1 R_1) \quad (8)$$

$$\mathbf{p} = 1/2 + (B_1 a_2 - B_2 a_1) / (a_1 b_2 R_2 - a_2 b_1 R_1), \quad (9)$$

where  $a_i, b_i$  are the complexes of coolant parameters measured in the  $i$ -th experiment:

$(a_1, b_1)$  in the first and  $(a_2, b_2)$  in the second, respectively,

$a_i = (Q/\Delta t_{AR} \cdot F)^{2.6}; b_i = (13/15) \cdot (\Delta t_G / \Delta t_{AR})^2$ ,  $R = \delta t_{\min} / \delta t_{\max}$ ,

$Q$  – is the transmitted heat flux (W);

$F$  – is the heat transfer surface area ( $m^2$ );

$\Delta t_{AR}$  – arithmetic mean temperature head (K),

$R$  – dimensionless ratio,  $R = \delta t_{\min} / \delta t_{\max}$ ,  $\delta t_{\min} = \min(\delta t_H, \delta t_C)$ ,  $\delta t_{\max} = \max(\delta t_H, \delta t_C)$ .

Firstly, thermal tests using a new method allow obtaining an updated value of the heat transfer coefficient  $k$ , and secondly, the real value of the counter-flow index of the current circuit HE  $\mathbf{p}$ , which can be compared with its calculated value and used in thermal calculations of HE.

### C. Development of a thermal test procedure according to C.C. No.1589021

Suppose in a prototype HE, both experiments within the framework of its thermal tests are carried out in a given mode (at the calculated values of flow rates and temperatures at the HE inlet). In contrast, one experiment is carried out for a given direction of the tube bundle's coolant flow. The other experiment is carried out in the opposite direction of the coolant flow in the tube bundle. In that case, it can be assumed that in both experimental regimes, the unknown  $k$  are the same. In this case, the

counter-flow index  $\mathbf{p}$  when the direction of one of the coolants changes to the opposite (i.e., when it is reversed) changes in accordance with the relationship:  $\mathbf{p}_1 + \mathbf{p}_2 = 1$  [6, 7]. The following example can illustrate this dependence: if in a HE with a counter-flow circuit ( $\mathbf{p} = 1$ ) change the direction of flow of one of the coolants (reverse it), then we get a HE with a direct-flow circuit ( $\mathbf{p} = 0$ ), i.e., for HE there is a dependence:  $\mathbf{p}_2 = 1 - \mathbf{p}_1$ .

On 08.05.1988, the author registered another method of HE testing [8] according to the definition of  $k$  and  $\mathbf{p}$ , devoid of the disadvantages inherent in the first method, since it allows thermal testing of HE no longer in the laminar mode, but in the design mode of operation: i.e., at the calculated values of flow rates and temperatures of heat carriers at the inlet to the HE.

In this case, the system of two linear equations with two unknowns will be written:

$$\begin{cases} a_1 = k^{2.6} \cdot [1 - b_1 \cdot f(\mathbf{p}_1, R)] \\ a_2 = k^{2.6} \cdot [1 - b_2 \cdot f(\mathbf{p}_2, R)] \end{cases} \quad (10)$$

Where we substitute from the constraint equation into the second equation:  $\mathbf{p}_2 = 1 - \mathbf{p}_1$ .

Solving this system with respect to unknowns, we obtain analytical expressions for the required quantities:  $k$  and  $\mathbf{p}$  – through the parameters of the coolant flows measured in both experiments:

$$k^{2.6} = \{(a_1 b_2 + a_2 b_1) / [b_1 + b_2 - b_1 b_2 (1 + R^2) / 2]\} \quad (11)$$

$$\mathbf{p} = [(a_2 b_1 - a_1 b_2) / (a_2 b_1 + a_1 b_2)] \cdot [(1 + R)^2 / 4R] + [a_1 b_2 + (a_1 - a_2) / R] / (a_2 b_1 + a_1 b_2). \quad (12)$$

where  $a_i, b_i$  are complexes of coolant parameters measured in the  $i$ -th experiment:  $(a_1, b_1)$  – in the first and  $(a_2, b_2)$  – in the second, respectively,  $a_i = (Q/\Delta t_{AR} \cdot F)^{2.6}$ ,  $b_i = (13/15) \cdot (\Delta t_G / \Delta t_{AR})^2$ .

### D. Development of a universal method for thermal calculation of heat exchangers (UMTC).

After registration of inventions (CC USSR) No. 1377558 and No. 1589021, the author developed UMTC HE using the results of F. Trefni [9, 10, 11] and N. Belokon [2, 3]. From the method of F. Trefni, very successful designations were used in it: the efficiency of the flow circuit of the coolants in the HE was expressed through the current function, which is presented in the trigonometric form:

$$f_\varphi = (1 - \cos\varphi) / 2, \quad (13)$$

where  $\varphi$  is the conditional angle between the direction of the coolants: for forward flow  $\varphi = 0$ , for counterflow,  $\varphi = \pi$  (rad), and for all other schemes  $0 \leq \varphi \leq \pi$  (rad). And from N. Belokon's method, it used the most accurate generalization of a complex variable by "three points": forward flow ( $\mathbf{p} = 0$ ), counterflow ( $\mathbf{p} = 1$ ) and A. Underwood's scheme, intermediate inefficiency ( $\mathbf{p} = 0.5$ ). Both in F. Trefni's approach and N. Belokon's approach, the

perfection of HE current circuits are defined in the same way:  $0 \leq \mathbf{p} \leq 1$  and  $0 \leq f_\varphi \leq 1$ .

Moreover, a higher degree of generalization was used in the UMTC than in the method of N.Belokon and the method of F.Trefni. This was ensured by the fact that the author, for the first time, introduced a new criterion for the thermal efficiency of HE:

$$E = Q/(\Delta t_{AR} \cdot kF), \quad (14)$$

The value of which is maximum and equal to one only for the counter-flow circuit and then only when the water equivalents of the heat transfer fluids ( $W = G \times C_p$ ) are equal to each other, that is, when  $W_H = W_C$  ( $R = 1$ ). In all other cases, it is less than one ( $0 < E \leq 1$ ). It is in this unique case that the mathematical expression for the mean logarithmic temperature difference of a HE with a countercurrent current circuit degenerates into an expression for the arithmetic mean:  $\lim(\Delta t_m)_{\downarrow \uparrow} \rightarrow \Delta t_{AR}$ . Thus, the introduced criterion characterizes the TO current circuit's efficiency and the perfection of the thermal mode of its operation, which is the most effective in the case of an equality of the water equivalents of the coolants  $W_H = W_C$  ( $R = 1$ ). It is appropriate to remember here that in the mode of operation of HE at  $R \rightarrow 0$ , i.e., in the presence of condensation or evaporation of one of the heat carriers, the advantages of all current circuits are leveled and heat transfer in any TO no longer depends on the efficiency of its current circuit ( $\mathbf{p}$  and  $f_\varphi$ ).

Secondly, the new efficiency criterion  $E$ , which is an analog of the correction factor  $f$  from the Bauman method (in formula  $(\Delta t_m)_i = f_i(P, R) \times (\Delta t_m)_{\downarrow \uparrow}$ ), but is a function of only one variable (albeit complex), and is determined analytically, and not from the nomogram. Moreover, for each specific thermal mode of HE, the value of criterion  $E$ , all other things being equal, is the same for both the design and verification procedures of the UMTC and, in both cases, is expressed through hyperbolic functions [12, 13]. In the design methodology of the UMTC of HE, criterion  $E$  is expressed through the inverse hyperbolic function:

$$E = \Theta / \text{arth} \Theta, \quad (15)$$

where  $\Theta$  is a scalar temperature variable,  $\Theta = |\bar{\Theta}_\Sigma|$ .

In the verification methodology of the UMTC, the efficiency criterion  $E$  is also expressed through the hyperbolic function:

$$E = (thS)/S, \quad (16)$$

where  $S$  is a scalar consumption variable,  $S = |\bar{S}_\Sigma|$ .

And, thirdly, to generalize N.Belokon's interpolation by three points for the counter-flow index  $\mathbf{p}$  ( $\mathbf{p} = 0$  for forwarding flow,  $\mathbf{p} = 1$  for counter-flow, and  $\mathbf{p} = 0.5$  for A.Underwood's scheme) for temperature and flow rate variable with the extremely successful analogy of the " $\varphi$ -current" F.Trefni (in which  $f_\varphi = 0$  for forwarding flow and  $f_\varphi = 1$  for counter-flow) in the UMTC, it turned out to be convenient to interpret a complex scalar variable (both

temperature  $\Theta$  and consumption  $S$ ) within the framework of complex calculus. In this case, the contributions of both coolants to the expression of criterion  $E$  are complex quantities. The complex variable for both methods (design and verification) is equal to the modulus of the so-called "complex average" since, in this case, it will depend not only on the scalar values of the contributions but and on the mutual direction of the flows of heat carriers, determined by F.Trefni in the current circuit HE by the angle  $\varphi$  ( $0 \leq \varphi \leq \pi$ , rad).

In this case,  $\Theta$  is equal to the modulus of the complex radius vector  $|\bar{\Theta}_\Sigma|$ , which is the half-sum of the radius vectors of two heat carriers:

$$\bar{\Theta}_\Sigma = (\bar{\Theta}_H + \bar{\Theta}_C)/2, \quad (17)$$

Where  $\bar{\Theta}_j = \Theta_j \cdot \exp(i\varphi)$ ,  $\Theta_j$  – is the modulus of the temperature variable;  $\varphi_j$  is the angle between the direction of the radius vector and the actual coordinate axis.

This angle is formed by the general direction of the coolant flow in the annular space (real), coinciding with the axis of the HE casing and the conventional (imaginary in the general case) direction of the flow in the tube bundle,  $i$  is an imaginary unit,  $i = \sqrt{-1}$ . The modulus of the half-sum of the complex radius vectors of the two coolants in this case is:

$$\Theta = (1/2) \cdot \sqrt{(\Theta_H^2 + 2\Theta_H \cdot \Theta_C \cdot \cos\varphi + \Theta_C^2)}. \quad (18)$$

Similarly, the generalized variable  $S$  is equal to the modulus of the complex radius vector ( $S = |\bar{S}_\Sigma|$ ). The expression  $|\bar{S}_\Sigma|$  is the modulus of the half-sum of the complex radius vectors of both heat carriers:

$$\bar{S}_\Sigma = (\bar{S}_H + \bar{S}_C)/2, \quad (19)$$

where  $\bar{S}_j = S_j \cdot \exp(i\varphi_j)$ ,  $S_j$  – is the modulus of the flow variable,  $\varphi_j$  – is the angle between the general direction of the coolant flow in the annular space (real), coinciding with the axis of the HE casing and the conventional (imaginary in the general case) direction of the coolant flow in the tube beam, here  $i$  is the imaginary unit,  $i = \sqrt{-1}$ .

And the modulus of the half-sum of complex radius vectors is:

$$S = (1/2) \cdot \sqrt{(S_H^2 + 2S_H \cdot S_C \cdot \cos\varphi + S_C^2)}. \quad (20)$$

### ***E. Procedure for the thermal calculation of HE within the UMTC***

The basis of the method of thermal calculation of UMTC for recuperative HE is the heat transfer equation:

$$Q = \Delta t_m \times k \cdot F, \quad (21)$$

where  $Q$  – is the transmitted heat flux (W);  $F$  – is the heat transfer surface area ( $m^2$ );  $k$  – is the heat transfer coefficient ( $W/m^2K$ );  $\Delta t_m$  – temperature head (K).

A certain advantage of the UMTC is that in both cases (both in the design and in the verification procedure), the average temperature head  $\Delta t_m$  is preliminarily determined, and it is determined by a generalized formula, to some extent similar to the representation of R. Bauman, where instead of the correcting factor  $f_i(P, R)$ , the value of criterion  $E$  is used, and instead of the mean logarithmic temperature head  $(\Delta t_m)_{\downarrow\uparrow}$ , its arithmetic mean analog  $\Delta t_{AR}$  is used:

$$\Delta t_m = E \times \Delta t_{AR}, \quad (22)$$

where  $\Delta t_{AR}$  – is the arithmetic mean temperature head,  $E$  – is the criterion of HE's thermal efficiency. The  $\Delta t_{AR}$  and  $E$  values are determined in a form corresponding to one of the two thermal calculation methods.

### 1) Methodology for the design calculation of UMTC.

In the design calculation methodology, criterion  $E$  is defined in the form of F. Grashof:

$$E = \Theta / \text{arth} \Theta, \quad \Theta = \frac{1}{2} \sqrt{(\Theta_H^2 + 2\Theta_H \cdot \Theta_C \cdot \cos \varphi + \Theta_C^2)}, \quad (23)$$

where  $\Theta_i$  are dimensionless temperature variables:  $\Theta_H = \delta t_H / \Delta t_{AR}$ ,  $\Theta_C = \delta t_C / \Delta t_{AR}$ , and temperature drops of heat carriers:  $\delta t_H = t'_H - t''_H$ ,  $\delta t_C = t''_C - t'_C$ , (K).

And the arithmetic mean temperature head  $\Delta t_{AR}$  in this case, is calculated by the formula:

$$\Delta t_{AR} = (t'_H + t''_H) / 2 - (t'_C + t''_C) / 2; \quad (24)$$

### 2) Method of verification calculation of UMTC.

In the verification method of calculation, criterion  $E$  is determined in the form of G. Greber:

$$E = (thS) / S, \quad \text{где } S = \frac{1}{2} \sqrt{(S_H^2 + 2S_H \cdot S_C \cdot \cos \varphi + S_C^2)}, \quad (25)$$

where  $S_H$  and  $S_C$  are dimensionless consumption variables,  $S_H = kF / W_H$ ,  $S_C = kF / W_C$ .

And the arithmetic means temperature head  $\Delta t_{he}$  formula calculates  $t_{AR}$  in this case:

$$\Delta t_{AR} = (t'_H - t'_C) / [1 + E \cdot (S_H + S_C) / 2] \quad (26)$$

### 3) Calculation of the values of the current function $\cos \varphi_{\Sigma}$ for the heat exchanger-recuperator

To calculate the value of the current function  $\cos \varphi_{\Sigma}$  of heat exchangers with different flow patterns, the author used a special mathematical method [14], which consisted in expanding the expression of criterion  $E$  in the generalized expression of the UMTC and expressions  $E$  for each generalized solution of the corresponding class of heat exchangers and the subsequent equating the first members of the received ranks of Maclaurin. As a result

of this operation, expressions were obtained for the stream functions of various current circuits [15]:

#### 1. For heat exchangers with the parallel-mixed flow:

$$\cos \varphi = \Delta / N^2, \quad (27)$$

where  $N$  – is the total number of half-loops of the tube bundle element,  $N = N_{\uparrow\uparrow} + N_{\downarrow\uparrow}$ ;

$N_{\uparrow\uparrow}$  and  $N_{\downarrow\uparrow}$  are the number of half-loops with forwarding flow and counter-flow, respectively;

$\Delta$  – is the difference in the number of half-loops with forward and backward flow,  $\Delta = N_{\uparrow\uparrow} - N_{\downarrow\uparrow}$ .

\*) In the cases  $N = 1$  and  $N = 2$ , formula (29) is the exact result.

#### 2. For heat exchangers with cross-mixed flow:

$$\cos \varphi = \pm (1 - \beta / N^2), \quad (28)$$

where the upper sign (+) means the total forward flow, and the lower sign (–) the total counter-flow,

a) for HE with a tube bundle of the "flat coil" type,  $N$  – is the number of half-loops of the tube bundle element, and constant  $\beta = 1$ ;

b) for HE with a tube bundle of the "twisted coil" type,  $N$  is the number turns of the tube bundle element, and the constant  $\beta = 1/2$ ;

\*) In case (b): for  $R = 1$ , formula (30) is an exact result.

### 4). Calculation of $\cos \varphi_{\Sigma}$ for rows and complexes of identical heat exchangers:

For rows of identical heat exchangers with a common forward flow:

$$\cos \varphi_{\Sigma} = 1 - (1 - \cos \varphi_i) / N^2, \quad (29)$$

Where  $\cos \varphi_i$  – is the universal characteristic of the HE current circuit,  $N$  – is the total number of identical HEs in the row.

For rows of identical heat exchangers with a common forward flow:

$$\cos \varphi_{\Sigma} = (1 + \cos \varphi_i) / N^2 - 1, \quad (30)$$

Where  $\cos \varphi_i$  – is the universal characteristic of the HE current circuit,  $N$  – is the total number of identical HEs in the row.

For "degenerate" complexes of identical heat exchangers:

$$\cos \varphi_{\Sigma} = \cos \varphi_i / \sqrt{N} \quad (31)$$

where  $\cos \varphi_i$  – is the universal characteristic of the HE current circuit,  $N$  – is the total number of identical HE in the complex.

### F. Simplified method for thermal testing of heat exchangers.

To the greatest extent, the HE current circuit's efficiency is manifested under the condition  $R \rightarrow 1$  ( $\delta t_H \rightarrow \delta t_C$ ), and in the least: at  $R \rightarrow 0$  ( $\delta t_S \rightarrow 0$ ). That is, for cases when a phase transition of the coolant (boiling or condensation) takes place on one of the two heat exchange surfaces in the HE, i.e., takes place:  $R = 0$  ( $\delta t_S = 0$ ), the current circuit practically does not affect the heat transfer in the HE. Therefore, thermal tests when determining a pair of unknowns ( $\mathbf{k}$  and  $\varphi$ ) should be carried out at  $R = 1$  ( $\delta t_H = \delta t_C$ ).

Thus, the most accurate determination of the TO current circuit's efficiency is possible only in one particular case  $R = 1$  [16, 17]. If, at the same time, the expression for formula (5) is rounded, then with a slight loss of accuracy in approximation, provided  $R = 1$ , we can write

$$y(X) = X/\text{Arth}X \cong (1 - X^2/1,2)^{0,4} \quad (32)$$

where  $X$  – is a dimensionless variable,

$$X = (\Delta t_G/\Delta t_{AR}) \cdot \sqrt{(1 - \mathbf{p})}, \text{ and } \mathbf{p} \equiv \sin^2(\varphi/2).$$

Then, similarly to the method of thermal tests according to CC. No. 1589021, we carry out two experiments: one with the calculated direction of the coolant flow in the tube bundle and the second – with the same parameters of the coolant at the inlet to the HE, but with the opposite direction of the flow of this coolant. In this case, the coupling equation will be written in the following form:  $\varphi_1 + \varphi_2 = \pi$ .

In this case, as a result of thermal tests, we obtain expressions for two unknowns in the following form:

$$\mathbf{k} = [(a_1 b_2 + a_2 b_1)/(b_1 + b_2 - b_1 b_2)]^{0,4} \quad (33)$$

$$\begin{aligned} \mathbf{p} &= (a_1 + a_2 b_1 - a_2)/(a_1 b_2 + a_2 b_1), \\ \varphi &\equiv 2\text{arcsin}(\sqrt{\mathbf{p}}), \end{aligned} \quad (34)$$

Where  $a_i$  and  $b_i$  are complexes of coolant parameters measured in the  $i$ -th experiment:  $(a_1, b_1)$  – in the first and  $(a_2, b_2)$  – in the second, respectively,  $a_i = (Q/\Delta t_{AR} \cdot F)^{2,5}$ ;  $b_i = (5/6) \cdot (\Delta t_G/\Delta t_{AR})^2$ .

#### EXAMPLE.

Let us analyze the results of model thermal tests of a shell-and-tube heat exchanger-recuperator with a parallel-mixed current circuit (Underwood loop circuit) in a simplified way. The area of the heat exchange surface (tube bundle loops) is  $F = 5 \text{ m}^2$ . The parameters of the design operating mode are as follows: the initial temperatures of the "hot" and "cold" streams of heat carriers, respectively:  $t'_H = 60^\circ\text{C}$ ,  $t'_C = 20^\circ\text{C}$  ( $\Delta t' = 20^\circ\text{C}$ ), the water equivalents ( $W_H$  and  $W_C$ ) of both heat carriers are as follows: coolant flowing in the annular space ( $G \times C_p$ ) = 10 [kW/K], and flowing in the tube bundle is the same ( $G \times C_p$ ) = 10 [kW/K]. Under these conditions, the heat transfer coefficient's value is assumed to be  $\mathbf{k} = 1$  [kW/m<sup>2</sup>K]. The ratio  $R = (G \times C_p)/(G \times C_p) = 1$ . Now we can

calculate the values of the quantities ( $Q$ ,  $\Delta t_{AR}$ ,  $\delta t_H$ ,  $\delta t_C$ ) included in the coefficients ( $a$ ,  $b$ ,  $R$ ) and in the formulas for  $\mathbf{k}$  and  $\mathbf{p}$ . They will be calculated by the Case-London method using the expression for the thermal efficiency coefficient  $\varepsilon$ (NTU). For a HE with an Underwood loop, which is subjected to thermal tests, the analytical expression for  $\varepsilon$  ([18], Tabl. 6.4, No. 16) has the following form:

$$(\varepsilon)^{-1} = (1/2) \cdot \{(1 + R) + \sqrt{(1 + R^2)/th[(1/2) \cdot \text{NTU} \cdot \sqrt{(1 + R^2)}]}\} \quad (35)$$

For the first mode, we obtain  $\varepsilon = 0.3244$ . By substituting the  $R$  and  $\text{NTU}$  values, you can calculate the values needed to calculate the coefficients:  $\delta t = \Delta t' \times \varepsilon = 40 \times 0.3244 = 13$  [K];  $\Delta t_{AR} = \Delta t' - 1/2(\delta t_H + \delta t_C) = 40 - 1/2(13 + 13) = 27$  [K];  $Q = \delta t \times W = 13 \times 10 = 130$  [kW]. For the second mode – reverse, we obtain similar results, since for this current circuit the counterflow index is  $\mathbf{p} = 0.5$  [2] and theoretically, when the direction of the flow in the tube bundle changes, the transferred heat flux should not change.

As a result, the calculation results are as follows:  $a = (Q \cdot \Delta t_{AR} \cdot F)^{2,5} = (130/27.5)^{2,5} = 0.9065$ ;  $b = (5/6) \times (13/27)^2 = 0.201$ . The data for the second reverse mode are similar. Substituting the values of complexes  $a$  and  $b$  for both modes into the formulas for  $\mathbf{k}$  and  $\mathbf{p}$ , we get:  $\mathbf{k} = 1.005$  [kW/m<sup>2</sup>K] and  $\mathbf{p} = 0.5002$  ( $\varphi = \pi/2$ ). Thus, the binomial approximation error in this case for  $\mathbf{k}$  is 0.5%, and for  $\mathbf{p}$  – 0.2%. If we use a more accurate approximation adopted in the framework of both C.C., then the approximation error there will be lower:  $\mathbf{k} = 1.003$  [kW/m<sup>2</sup>K] and  $\mathbf{p} = 0.5001$  ( $\varphi = \pi/2$ ), i.e. 0.3% and 0.1%. But even in a simplified version, the approximation accuracy is quite acceptable.

### V. CONCLUSIONS

The main advantage of the proposed method for thermal testing of shell-and-tube HE is that its use does not require a priori knowledge of the analytical expression for the temperature difference  $\Delta t_m$ .

As a result of carrying out thermal tests of HE using a new method, it is possible to determine at once two parameters of HE – together with the heat transfer coefficient ( $\mathbf{k}$ ), it is possible to determine the parameter of the efficiency of the HE current circuit – the conditional angle between the directions of the current of heat carriers  $j$  from the equality  $\sin^2(\varphi/2) \equiv \mathbf{p}$ .

It turned out that thermal tests by a new method make it possible to separate the influence of two factors on the efficiency of heat transfer in HE: the intensive factor ( $\mathbf{k}$ ), which characterizes the efficiency of using the heat exchange surface, and the extensive factor ( $\varphi$ ), which characterizes only the influence of the perfection of the mutual flow of heat carriers in the HE.

This information enables designers to improve the efficiency of heat transfer in the heat transfer fluid by trivially increasing the flow of heat transfer media around heat transfer surfaces and improving the mutual flow of heat transfer media.

It is important here that the improvement of heat transfer in HE within the intensive factor framework is

mainly due to an increase in the cost of servicing the HE (that is, for pumping coolants in it). The improvement in HE's heat transfer due to the intensive factor is due only to the innovative contribution of designers at the design stage HE.

As a result, we can say that the thermal testing methods of HE, according to C.C. No.1589021, are incomparably better than the current standard EN 305:1997, which is based on long-outdated foundations.

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