SSRG International Journal of Applied Physics
ISSN: 2350 – 0301 /doi:10.14445/23500301/ IJAP-V7I3P111

Volume 7 Issue 3, 70-78, Sep-Dec 2020

$\Upsilon(4S),\Upsilon(5S)$ mesons as sources of B- mesons

A. M. Yasser[1, *], M. Ismail Hapareer[2, **] , M. A. Allosh[1] , G. S. Hassan[2]

[1]*Physics Department, Faculty of Science at Qena, South Valley University, 83523 Qena, Egypt*
[2] *Physics Department, Faculty of Science at Assuit, Assuit University, 71515 Assuit, Egypt*

Abstract: *A detailed study is presented about sources of B-mesons by strong decay of $\Upsilon(4S),\Upsilon(5S)$ triplet-spin states in S-wave $b\bar{b}$ bottomonium mesons. We introduce the start point to institute the strong decay of $b\bar{b}$ bottomonium mesons physics to get the sources of B-mesons that we can pass from during their rare decay to a new world of physics, the so-called BSM(Beyond Standard Model) or New Physics(NP) and probe its deeps; that's an important because a Theory of Everything(ToE) is striven to it from through New Physics theories. We use the $^3P_0$ model or so-called quark pair creation model (QPC) to calculate the strong decay of $\Upsilon(4S),\Upsilon(5S)$ triplet-spin states in S-wave $b\bar{b}$ bottomonium mesons. Strong decay ratio is calculated for every state. The results show a good fit with other theoretical results and with recent experimental data.*

Keywords: *sources of B-mesons, S-wave bottomonium mesons, Strong decay ratio , New Physics, BSM, QPC quark pair creation model*

I. INTRODUCTION AND MOTIVATION

The most recent challenge that occupies particle physics scientists is BSM (Beyond Standard Model) or co-called New Physics (NP). This challenge Leads us to revolution in the particle physics then revolution in physics science and all human science. The studying of BSM (Beyond Standard Model) leads us to appearance of New Physics (NP) that discuss the update issues of the physics beyond modern physics. Really, in 2012, the finding of the Higgs boson via the CMS[1] and ATLAS[2] experiments at the CERN gave the extended-awaited experiment proof that the Standard Model (SM) is completed. But, the model includes a great number of parameters that must to be input from experiment, the model does not supply any illustration for dark matter, dark energy, the matter-antimatter asymmetry of the Universe, neutrino oscillations and also doesn't combine Gravity. So that, most high energy physists think that the SM, although its great success, is not completed theory of nature. So that, the high energy physists will probably put the searches of new physics on the experimental program of CERN Large Hadron Collider through the next decades.

In fact, the beyond Standard Model theories that involve the numerous standard model extensions of from during Supersymmetry, like the Minimal Supersymmetric Standard Model (MSSM) and Next-to-Minimal Supersymmetric Standard Model (NMSSM), and also wholly novel exegesis, such M-theory, String theory, Supersymmetry, loop quantum gravity and extra dimensions, as a consequence of so-called beyond standard model (BSM) new physics theories, have been suggested, most of which foretell the being of new, until now unseen, massive particles[3]. Where, These other BSM theories foretell heavy versions of the Z and W bosons[4], a fourth-generation of quarks[5], etc. The Supersymmetry (SUSY) is the most commonly theory of BSM , which suggests a SUSY consort for every the established SM particles, and a varity set of five Higgs particles which involve a doublet of charged Higgs scalars[6]. A theory of Everything(ToE) is striven to it by previous theories where it fully interprets and relates together all familiar physical phenomena and expects the result of any experiment that could be run in principle. In fact, that's single of the most vigorous fields of research in both experimental and theoretical particle physics.

In the SM, don't found Flavor Changing Neutral Currents (FCNC) that immediately transform s→ d or b → s, etc. But, BSM physics at a high mass criterion that intermediates the Flavor Changing Neutral Currents (s → d, b → d, b → s and c → u) that are at perform role in particle-antiparticle mixing. Accordingly, the process is intermediated by heavy hypothetical particles: in the state of B^0-$\bar{B}^0$ mixing where meson-antimeson mixings come with the category of flavour-changing neutral current (FCNC) processes, a considerable difference relative to the mixing-induced CP-violating asymmetry in $B^0 \to K_s\,\phi$ and that in $B^0 \to K_s\,J/\psi$ would be a evident indication of BSM, new physics, that is as well the case relative to numerous other penguin-mediated rare decays, like $B^0 \to K_s\,\pi^0$, $B^0 \to K^0\,\phi$ and $B^0 \to K^0\,\acute{\eta}$[7]-[9] etc. It is worth notice, the very rare decay $B_s \to \mu^+ + \mu^-$ is noted as the first observation[10]. Rare B decays play an important role to find of New Physics (NP) impacts. Flavor physics measurements affect BSM(New Physics) researches at the TeV energy-frontier[11],[12]. In addition to being sentient to new physics, rare B decays can as well be used to define some of the CKM matrix elements (e.g V_{td} and V_{ts}) or to give ideas about the top quark.

From the above it becomes clear that the important role of B meson physics in finding the new physics (NP) add to its important role in Standard Model to study phenomena such as CKM matrix elements and CP violation (matter is greater than antimatter). So that, It is very necessary to research about the sources of $B\bar{B}$ mesons. In fact, the strong decay of $b\bar{b}$ bottomonium mesons give us this goal. Amongst these bound states, the resonance of $\Upsilon(4S)$ is particularly interesting because it is the lightest bound state that it is heavy sufficient to decay to a B-meson pair[13]. The first detectors to extensively study B-physics at the resonance of $\Upsilon(4S)$ were CLEO in Cornell and ARGUS at DESY (Deutsches-Elektronen-Synchrotron) in Hamburg[14]-[20]. Over the ARGUS detector running time (1982 : 1992)[18]-[20], the B°-$\overline{B}^{\circ}$ mixing first observation was most notably[21]. Thanks to the pure $e^{+}e^{-} \rightarrow \Upsilon(4S) \rightarrow B\overline{B}$ environment, also, relative to Bs physics, BelleII runs at the $\Upsilon(5S)$, furthermore the production rate of $B\overline{B}$ pairs is reached to be of the order 10^{12} per year.

Until now, we can say except our this research there aren't detailed studies of sources of B-mesons and strong decay of S-wave $b\overline{b}$ bottomonium mesons. In addition to there aren't any detailed studies of high wave levels (P-wave, D-wave, F-wave and G-wave) of $b\overline{b}$ bottomonium mesons. In this paper, we will present in detailed study about sources of B-mesons by the strong decay of $\Upsilon(4S), \Upsilon(5S)$ triplet-spin states in S-wave $b\bar{b}$ bottomonium mesons, that is start point to establish the strong decay of $b\overline{b}$ bottomonium mesons physics to get the sources of B-mesons that we can pass from during their rare decay to a new world of physics. This paper is a first step towards this very challenging goal. This paper is organized as follows. After the introduction, the theoretical frame work includes nonrelativistic quark model and $^{3}P_{0}$ model have been introduced. In Sec. 3 the results and discussion are introduced. Finally in Sec. 4, we give the summary and conclusion.

II. Theoretical frame work

In fact the quark model (QM)[22]-[42] is very prosperous in reproducing the behavior of observables such as the spectrum and the magnetic moments. Significant information on mesons are also supplied by the different decay modes (strong, electromagnetic and weak decays). Specially, the two-body strong decays are transitions to open-flavor final states, when the initial $b\overline{b}$ meson decays by $q\overline{q}$ pair-creation (q = u, d or s), and thereafter it splits into two open-bottom mesons in the final channel states. There are many models of strong decay for example Cornell model[43],[44] fluxtube model[45]-[47], microscopic models[48],[49], $3S_1$ model[50],[51] and $^{3}P_{0}$model (quark pair-creation model)[52] but the last model is the widespread and simplest model, It gives good describing for the strong decay phenomenon[52]-[56]. In this work, we will use Nonrelativistic potential quark model in calculating of the spectrum and $^{3}P_{0}$ model in calculating of the $b\overline{b}$ bottomonium meson strong decays.

A. Non-relativistic potential quark model

Here in this part, we give the mass predictions of the nonrelativistic quark model for bottomonium mesons as shown in tables I and B-mesons as shown in tables II. In our calculations, we employ the traditional "Coulomb plus linear" potential in addition to spin dependent corrections produced from vector gluon exchange and an efficacious scalar confinement interaction[57],[58]. We express about our potential in the following general formula.

$$\begin{aligned}\boldsymbol{V}(r) &= [l(l+1)/2\mu r^2] - 4\alpha_s/3r + br \\ &+ (32\pi\alpha_s\delta_\sigma(r)\boldsymbol{S}_q\boldsymbol{S}_{\bar{q}})/9m_q m_{\bar{q}} + 1/m_q m_{\bar{q}}\Big[[(1 \\ &+ m_q^2 + m_{\bar{q}}^2/4m_q m_{\bar{q}})4\alpha_s/3r^3 \\ &- \left(m_q^2 + m_{\bar{q}}^2/4m_q m_{\bar{q}}\right) b/r]\boldsymbol{L.S} \\ &+ 4\alpha_s/r^3\,\boldsymbol{T}\Big] \qquad (1)\end{aligned}$$

We can derive from previous equation a special formula to calculate spectrum of bottomonium mesons as a following:

$$\begin{aligned}\boldsymbol{V}(r) &= [l(l+1)/2\mu r^2] - 4\alpha_s/3r + br \\ &+ (32\pi\alpha_s\delta_\sigma(r)\boldsymbol{S}_q\boldsymbol{S}_{\bar{q}})/9m_b^2 + 1/m_b^2\,[(2\alpha_s/r^3 - b/2r)\boldsymbol{L.S} \\ &+ 4\alpha_s/r^3\,\boldsymbol{T}] \qquad (2)\end{aligned}$$

Because the meson consists of two small particle(two quark) so μ the reduced mass of meson is determined from

$$\mu = m_q m_{\bar{q}}/(m_q + m_{\bar{q}}) \qquad (3)$$

And also, δ_σ is determined from

$$\delta_\sigma(r) = \left(\sigma/\sqrt{\pi}\right)^3 e^{-\sigma^2 r^2} \qquad (4)$$

Also, $S_b S_{\overline{b}}$ is specified from

$$S_b S_{\overline{b}} = S(S-1)/2 - 3/4 \qquad (5)$$

where S is the total spin quantum number of the meson[59]. $L.S$ operator is the spin-orbit operator, it is a diagonal in a $|J, L, S\rangle$ basis with the matrix elements.

$$\langle L.S\rangle = [J(J+1) - (L(L+1) - S(S+1))/2] \quad (6)$$

T operator is the tensor operator[60], where

$$T = S_q.\hat{r}\,S_{\overline{q}}.\hat{r} - 1/3\,S_q.S_{\overline{q}} \qquad (7),$$

that operator has diagonal matrix elements as a following

$$\langle 3L_J|T|3L_J\rangle = \begin{cases} -\ L/6(2L+3), & J = L+1 \\ +\ 1/6\,, & J = L \\ -\ (L+1)/6(2L-1), & J = L-1 \end{cases} \qquad (8)$$

α_s , b, σ, and m_b are the parameters of $b\overline{b}$ bottomonium of mesons, they are taken to be 0.4036, 0.1624 GeV^2, 2.4948 GeV and 4.8097 GeV, respectively[61]-[63]. And, we take the parameters of B-mesons $\alpha_s(B)$, $\alpha_s(B_s)$, b, σ to be 0.749, 0.681, 0.0925 GeV^2, 0.7576 GeV, respectively by fitting, and $m_{u,d} = 0.33$ GeV, $m_s = 0.55$ GeV [57],[64]-[67] $m_b = 4.8097$ GeV [61]-[63].

B. The $^{3}P_{0}$ model

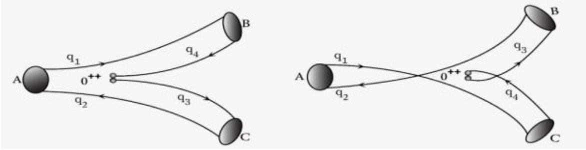

Fig.1 The two probable diagrams contributing to $A \rightarrow B + C$ process in the $^{3}P_{0}$ model of the meson strong decay (quark-antiquark pair-creation model).

In reality, the $^{3}P_{0}$ model or so-called QPC quark pair creation model was first time offered by Micu [52] and also developed by the Orsay group[53],[56],[68]-[70]. The $^3P_0$model was vastly used relative to the OZI-allowed hadron strong decays to two body[71]-[91].

We will give the theoretical frame work of the $^3P_0$ model of the meson strong decay as the following. In the meson strong decay process A → B + C, we can write

$$\langle \boldsymbol{BC}|\boldsymbol{\mathcal{T}}|\boldsymbol{A}\rangle = \delta^3(\vec{p}_{\boldsymbol{A}} - \vec{p}_{\boldsymbol{B}} - \vec{p}_{\boldsymbol{C}})\boldsymbol{\mathcal{M}}^{M_{J_A}M_{J_B}M_{J_C}} \qquad (1)$$

Where:

$\vec{p}_A$, $\vec{p}_B$ and $\vec{p}_C$: are a three-momentum of a meson A, B and C in the rest frame of a meson A.

M_{J_i} (i = A, B, C) : denotes an orbital magnetic momentum.

$\mathcal{T}$: is the transition operator that is introduced to dub a quark–antiquark pair creation from vacuum, which has the quantum number J^{PC} PC =0^{++}.

$\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$: is helicity amplitude.

1) The transition operator $\boldsymbol{\mathcal{T}}$ is given by:

$$\mathcal{T} = -3\gamma \sum_m \langle 1m1-m|00\rangle\sqrt{96\pi}\int d^3p_3 d^3p_4\, \delta^3\,(\vec{p}_3 + \vec{p}_4) \times \mathcal{Y}_{1m}\left(\frac{\vec{p}_3-\vec{p}_4}{2}\right)\chi^{34}_{1-m}\phi^{34}_0\omega^{34}_0 b^\dagger_3(p_3)d^\dagger_4(p_4) \quad (2)$$

that is formulated in a quite phenomenological way to describe from vacuum how the creation of a quark–antiquark pair, here the quark and antiquark are indicated by 3 and 4, respectively.

γ : is the one undetermined parameter of the model and it is dimensionless parameter that describes the creation strength of the $q\bar{q}$ from vacuum so, the model has super property because the model require only one normalization parameter for a pair creation process.

$\mathcal{Y}_{LM_L}(\vec{p}) = |\vec{p}|^L Y_{LM_L}(\theta_p, \phi_p)$: is the solid harmonics.

χ, ϕ, and ω : denote the spin, flavor, and color wave functions respectively, which can be treated separately.

2) The helicity amplitude $\boldsymbol{\mathcal{M}}^{M_{J_A}M_{J_B}M_{J_C}}$

Anywhere, utilizing the normalization as in the[57], [76], [92] and from eq.1 and eq.2 the helicity amplitude $\boldsymbol{\mathcal{M}}^{M_{J_A}M_{J_B}M_{J_C}}$ can be expressed as:

$$\begin{aligned}
&\boldsymbol{\mathcal{M}}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P}) \\
&= \gamma \sum \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle \langle L_B M_{L_B} S_B M_{S_B}|J_B \mathrm{M}_{J_B}\rangle \\
&\quad \langle L_C M_{L_C} S_C M_{S_C}|J_C M_{J_C}\rangle \langle 1m1-m|00\rangle \\
&\quad \left\langle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} \middle| \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m} \right\rangle \\
&\quad [\langle \phi^{14}_B \phi^{32}_C | \phi^{12}_A \phi^{34}_0 \rangle I(\vec{P}, m_1, m_2, m_3) \\
&\quad + (-1)^{1+S_A+S_B+S_C} \langle \phi^{32}_B \phi^{14}_C \phi | \phi^{12}_A \phi^{34}_0 \rangle \\
&\quad I(-\vec{P}, m_2, m_1, m_3)] \qquad (3)
\end{aligned}$$

here, we sum over M_{L_A}, M_{S_A}, M_{L_B} M_{S_B}, M_{L_C} , M_{L_C} , and m. The last factor has two terms for the two probable diagrams. In the first diagram, q_1 quark from meson A ends up in meson B and q_2 antiquark from meson A ends up in meson C. In the second diagram, q_1 quark from meson A ends up in meson C and q_2 antiquark from meson A ends up in meson B. Indices q_3 and q_4 point to the created quark and antiquark, respectively.

2.1) The momentum space integral $I(\vec{P}, m_1, m_2, m_3)$ is for the first diagram:

$$\begin{aligned}
I(\vec{P}, m_1, m_2, m_3) &= \sqrt{96\pi}\int d^3p\, \mathcal{Y}_{1m}(\vec{p})\psi_{n_A,L_A,M_{L_A}}(\vec{p} + \vec{P}) \\
&\quad \psi_{n_B,L_B,M_{L_B}}\left(\vec{p} + \frac{m_3}{m_1+m_3}\vec{P}\right) \\
&\quad \times \psi^*_{n_C,L_C,M_{L_C}}\left(\vec{p} + \frac{m_3}{m_2+m_3}\vec{P}\right) \quad (4)
\end{aligned}$$

Here, m_1, m_2 and $m_3 = m_4$: are the constituent quark masses. We put $\vec{P} \equiv P_B = -P_C$ in the centre of-mass frame of A meson. For the second diagram, we simply replace B ⟷ C, $m_1 \leftrightarrow m_2$ and $\vec{P} \longrightarrow -\vec{P}$ in Eq.3, by that we obtain second term of Eq.3.

We apply the momentum-space simple harmonic oscillator (SHO) wavefunctions that is written as:

$$\psi^{SHO}_{nLM_L}(\vec{p}) = R^{SHO}_{nL}(p) Y_{LM_L}(\theta_p, \phi_p) \qquad (5)$$

The radial wavefunctions are written as:

$$R^{SHO}_{nL}(p) = [(-1)^n(-i)^L/\beta^{5/2}]\sqrt{2n!/\Gamma(n + L + 3/2)} \times p^L\, L_n^{L+\frac{1}{2}}(p^2/\beta^2)\, e^{-p^2/(2\beta^2)} \qquad (6)$$

$L_n^{L+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right)$: It is an associated Laguerre polynomial. We apply the SHO wavefunctions of the meson with quantum numbers $n^{2S+1}L_J$ in spectroscopic notation. In fact, It is used ψ^{SHO}_{n-1,LM_L} for its momentum-space wavefunction.

2.2) *The colour matrix element is expressed by*

$$\langle \omega^{14}_B \omega^{32}_C | \omega^{12}_A \omega^{34}_0 \rangle = \langle \omega^{32}_B \omega^{14}_C | \omega^{12}_A \omega^{34}_0 \rangle = \frac{1}{3} \qquad (7)$$

It doesn't appear in Eq.3 because it cancels the factor of 3 in Eq.2.

2.3) *The flavour matrix element*

Here, we can easily find the flavour matrix element by the flavour wavefunctions of A, B and C mesons and the flavour wavefunction of the created $q\bar{q}$ pair that is written as

$$\phi_0 = 1/\sqrt{3}\,\left(u\bar{u} + d\bar{d} + s\bar{s}\right) = 1/\sqrt{3}\begin{pmatrix} 10000 \\ 01000 \\ 00100 \\ 00000 \\ 00000 \end{pmatrix} \qquad (8)$$

The flavour matrix element for the first term and the second one in Eq.3 are written as:

$$\langle \phi^{14}_B \phi^{32}_C | \phi^{12}_A \phi^{34}_0 \rangle = Tr[\phi^T_A \phi_B\, \phi^T_0 \phi_C] \qquad (9)$$

$$\langle \phi^{32}_B \phi^{14}_C | \phi^{12}_A \phi^{34}_0 \rangle = Tr[\phi^T_A \phi_C \phi^T_0\, \phi_B] \qquad (10)$$

2.4) *The spin matrix elements*

The spin matrix elements of the first diagram and second one are given by the Wigner 9j symbols terms[93] as the following:

$$\begin{aligned}
&\left\langle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} \middle| \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m} \right\rangle \\
&= (-1)^{S_A+S_B}\sqrt{3(2S_A + 1)(2S_B + 1)(2S_c + 1)} \\
&\quad \textstyle\sum_{S,M_S} \langle S_B M_{S_B} S_C M_{S_C} | S\mathrm{M}_S \rangle
\end{aligned}$$

$$\langle S_A M_{S_A} 1-m | S M_S \rangle \begin{Bmatrix} 1/2 & 1/2 & S_A \\ 1/2 & 1/2 & 1 \\ S_B & S_B & S \end{Bmatrix} \quad (11)$$

Here, the formula of spin matrix element for the second diagram is written as:

$$\left\langle \chi^{32}_{S_B M_{S_B}} \chi^{14}_{S_C M_{S_C}} \middle| \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m} \right\rangle = (-1)^{1+S_A+S_B+S_C} \left\langle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} \middle| \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m} \right\rangle \quad (12)$$

3) The partial wave decay amplitude $\mathcal{M}^{LS}(P)$

By applying the Jacob-Wick formulation, the helicity amplitude[94],[95] $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$, in Eq. 3, becomes partial decay amplitude $\mathcal{M}^{\mathrm{LS}}$ that it is written as

$$\mathcal{M}^{LS}(P) = [\sqrt{4\pi(2L+1)}/(2J_A+1)] \sum_{\mathrm{M}_{J_B},\mathrm{M}_{J_C}} \langle L0SM_{J_A} | J_A M_{J_A} \rangle \langle J_B M_{J_B} J_C M_{J_C} | S M_{J_A} \rangle \times \boldsymbol{\mathcal{M}}^{M_{J_A} M_{J_B} M_{J_C}}(\mathrm{P}_{\hat{z}}) \Big|_{M_{J_A} = M_{J_B} + M_{J_C}} \quad (13)$$

Where $\vec{\mathrm{L}} = \vec{\mathrm{J}}_{\mathrm{A}} - \vec{\mathrm{S}}$ and $\vec{\mathrm{S}} = \vec{\mathrm{J}}_{\mathrm{B}} + \vec{\mathrm{J}}_{\mathrm{C}}$, from that, we obtain

$$|J_A - S| \le L \le J_A + S \quad (14)$$

$$|J_B - J_C| \le S \le J_B + J_C \quad (15)$$

here the outgoing momentum of meson B or C meson, $\vec{\mathrm{P}} \equiv \mathrm{P}_{\hat{z}}$, in the centre-of-mass frame of meson A is taken along the $\hat{z}$-axis

4) The partial decay width

By utilizing (RPS) relativistic phase space[67],[83], the partial decay width is written as

$$\Gamma^{\mathrm{LS}} = 2\pi \mathcal{S} P \frac{\mathcal{E}_B(P)\mathcal{E}_C(P)}{\mathrm{M_A}} |\mathcal{M}^{LS}(P)|^2 \quad (16)$$

Where

$\mathcal{S}$ is a symmetry factor is written as

$$\mathcal{S} = \frac{1}{1+\delta_{\mathrm{BC}}} = \begin{cases} \frac{1}{2} & ;\ \mathrm{B} = C \\ 1 & ;\ \mathrm{B} \neq C \end{cases} \quad (17)$$

P, the momentum is given by terms of the A, B and C mesons masses as following

$$P = \frac{\sqrt{[\mathrm{M_A}^2 - (\mathrm{M_B}+\mathrm{M_C})^2][\mathrm{M_A}^2 - (\mathrm{M_B}-\mathrm{M_C})^2]}}{2\mathrm{M_A}} \quad (18)$$

$\mathcal{E}_{\mathrm{B}}$ and $\mathcal{E}_{\mathrm{C}}$ is determined from.

$$\mathcal{E}_B = \sqrt{\mathrm{M}_\mathrm{B}^2 + P^2}, \quad \mathcal{E}_C = \sqrt{\mathrm{M}_\mathrm{C}^2 + P^2} \quad (19)$$

here $\mathrm{M_A}$, $\mathrm{M_B}$ and $\mathrm{M_C}$ are the masses of mesons A, B and C respectively.

5) Finally, the total strong decay width

for a given decay mode of meson A is written as

$$\Gamma = \sum_{\mathrm{L,S}} \Gamma^{\mathrm{LS}} \quad (20)$$

III. Numerical Results and Discussion

In the previous part, techniques were constructed to describe the bottomonium meson strong decay widths and we explained it. In this part, these techniques will be applied and examined to see if it is in a good agreement with the experimental data or not. We use the $^3P_0$ quark pair creation model to get theoretical decay widths of higher bottomonium mesons. Then those results will be compared with the recently published experimental data of Particle Data Group (PDG2020)[96]. In fact, calculating the spectra of the higher bottomonium family, as we mention previously using nonrelativistic quark model which can be used in the QPC model to study the strong decay of the bottomonium family.

Here, we input the parameters that make $^3\mathrm{P}_0$ model is the applicable. In fact, these parameters represents our modification of the model to perform calculations of the strong decay of bottomonium mesons. These input parameters include γ the strength of quark pair creation from the vacuum, the parameter γ in the QPC model is determined by fitting with the experimental data[46]. Thus, there is no free.

parameter in the QPC model. After fitting, We get $\gamma = 0.64$. In our work, we use the substitution of the pair-creation strength, γ, with an effective one γ^{eff} [97]- [104],

$$\gamma^{eff} = (m_n/m_i)\,\gamma$$

where $\mathrm{m_i}$ is the mass of the produced quark and $\mathrm{i} = \mathrm{n}$ (u or d), s, c, and b and to suppress heavy quark pair creation.

Here, the oscillator parameter $\beta = 0.5$ GeV, whose is from the literature [57], [100], [105]-[107].
$\mathrm{m_{u,d}} = 0.33$ GeV, $\mathrm{m_s} = 0.55$ GeV [57],[64]-[67] and
$m_b = 4.8097$ GeV[61]-[63].

Table I: The Predicted masses of bottomonium mesons for nS states by Our Nonrelativistic Quarkmodel. The experimental data values are taken from PDG2020 [96]

Bottomonium meson	**state**	**Predicted masses by Our Nonrelativistic model**	**Measured masses by PDG2020 [96]**
ϒ(4S)	$4\,^3S_1$	10604	10579.4±1.2
ϒ(5S)	$5\,^3S_1$	10834	$10885.2^{+2.6}_{-1.6}$
ϒ(6S)	$6\,^3S_1$	11041	11000±4

Table II: The Predicted masses of B mesons that produced from strong decay of bottomonium by our Nonrelativistic Quark model.
The experimental data values are taken from PDG2020.

B meson	**state**	**Predicted masses by Our Nonrelativistic model (MeV)**	**Measured masses(MeV) by PDG2020 [96]**
$B^{\pm}$	1^1S_0	5271	5279.34 ± 0.12
B^0	1^1S_0	5271	5279.65 ± 0.12
B^*	1^3S_1	5323	5324.70 ± 0.21
B_s	1^1S_0	5368	5366.88 ± 0.14
B_s^*	1^3S_1	54133	$5415.4^{+1.8}_{-1.5}$

Studying the strong decay width of Υ(4S) and Υ(5S) states in the S-wave bottomonium meson states

In table I above the $B\bar{B}$ threshold, there are three bottomonium states to determine the parameters in our model, that three states well established in the PDG (Particle Data Group) with quantum number $J^{PC} = 1^{--}$. They are the so-called Υ(4S), Υ(10860) and Υ(11020), being the last two natural candidates for the Υ(5S)and Υ(6S), respectively.
A good understanding of their strong decay properties is the starting point for our study of the strong decay properties of other bottomonium states.

In fact, there are two sectors of S-wave bottomonium mesons Υ(nS) and η(nS). Υ(nS) sector is triplet-spin state while η(nS) sector is singlet-spin state. In higher bottomonia mesons above the $B\bar{B}$ threshold, both sectors decay strong type into $B\bar{B}$ mesons. Υ(nS) sector starts strong decays from Υ(4S) state and after that Υ(5S) state and Υ(6S) state while η(nS) sector starts strong decays from η(5S) state and after that η(6S) state. And we will analyze the strong decay results for Υ(4S) state then Υ(5S) state for every state alone in the next sections in this paper to introduce a clear picture about them.

1) Υ(4S) state

We begin with Υ(4S) triplet-spin state, that state is above $B\bar{B}$ mesons threshold. Υ(4S) decays into B^+B^- mesons and $B^o\bar{B}^o$ mesons. Υ(4S) is assigned as 4S state with notation
$n\,^{2S+1}L_J = 4\,^3S_1$ and quantum numbers $J^{PC} = 1^{--}$.

The total strong decay is Υ(4S) → $B\bar{B}$ that decay has two branching:
Υ(4S) → B^+B^- & Υ(4S) → $B^o\bar{B}^o$

The calculated total decay width by our $^3P_0$ model is ≅ 20.7 MeV, that value is good agreement with update experimental value in all over the world experimental data (PDG) Particle Data Group 2020 [96] where the experimental value ≅ 20.5±2.5 MeV. Also, our value is consistent with theoretical value ≅ 20.59 MeV in[108]. The strong decay properties for Υ(4S) state and Υ(5S) state higher bottomonia have been listed in Table III.

1.1) *Υ (4S)* $\rightarrow B^+B^-$

This is the first branching decay of Υ(4S) bottomonium mesons to the B charge mesons B^+B^-, the calculated partial decay width value $\Gamma^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 10.56 MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 50.98% by our $^3P_0$ model. This state has good agree with experimental partial decay width value $\Gamma^{exp}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 10.54 MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 51.4±0.6% and it has good agree with theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 10.41 MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(4S)\rightarrow B^+B^-}$ ≅ 50.54% by[108].

1.2) *Υ(4S)* $\rightarrow B^o\bar{B}^o$

This is the second branching decay of Υ(4S) bottomonium mesons to the B neutral mesons $B^o\bar{B}^o$, the calculated partial decay width value $\Gamma^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\,)\rightarrow B^o\bar{B}^o}$ ≅ 10.15 MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\rightarrow B^o\bar{B}^o}$ ≅ 49.01% by our $^3P_0$ model. This state has good agree with experimental partial decay width value $\Gamma^{exp}_{\Upsilon(4S)\rightarrow B^o\bar{B}^o}$ ≅ 9.96 MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(4S)\rightarrow B^o\bar{B}^o}$ ≅ 48.6±0.6%. However, it has good agree with theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(4S)\rightarrow B^o\bar{B}^o}$ ≅ 10.18 MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(4S)\rightarrow B^o\bar{B}^o}$ ≅ 49.46% by[108].
Finally, we find in PDG (Particle Data Group) the measured total strong decay width value ≅ 20.5±2.5 MeV is approximately equal to the measured total decay width of Υ(4S) bottomonium mesons state[96]. The other decays (Radiative – Hadronic – Annihilation – leptonic) of this state have very tiny decay width. That means the strong decay is the dominant decay relative to the reset of other decays (Radiative – Hadronic – Annihilation – leptonic-semileptonic).

And where, $\frac{\Gamma^{exp}_{\Upsilon(4S)\,)\rightarrow B^+B^-}}{\Gamma^{exp}_{\Upsilon(4S)\,)\rightarrow BB}} \cong 51.4\%$ that is good agreement with our $^3P_0$ model.
$\frac{\Gamma^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\rightarrow B^+B^-}}{\Gamma^{our\ ^3P_0\ \mathrm{model}}_{\Upsilon(4S)\rightarrow BB}} \cong 50.9\%$. That means Υ(4S) → B^+B^- the first branching decay is the dominant partial decay width relative to Υ(4S) state bottomonium mesons strong decay.

Here from previous, we can say that the strong decay of Υ(4S) state bottomonium mesons is the dominant decay and its branching decay Υ(4S) → B^+B^- is the dominant partial strong decay width.

Table III: The Predicted results of open-bottom strong decays for $\Upsilon(4S)$ state and $\Upsilon(5S)$ state higher bottomonia by Our $^3P_0$ and by[108]. The experimental data values are taken from PDG2020 [96].

initial meson	state	decay mode	Predicted results by Our $^3P_0$		Predicted results by [108]		Measured results by PDG2020 [96]	
			Width (MeV)	BR (%)	Width (MeV)	BR (%)	Width (MeV)	BR (%)
$\Upsilon(4S)$	4 3S1	B^+B^-	10.56	50.98	10.41	50.54	10.54	51.4±0.6
		$B^0\overline{B}^0$	10.15	49.01	10.18	49.46	9.96	48.6±0.6
		BB	20.71	100	20.59	100	20.5	> 96
		Total	20.71		20.59		20.5±2.5	~100
$\Upsilon(5S)$	5 3S1	BB	2.17	5.3	6.22	22.29	2.8±0.33	5.5±1
		$B\ B^*$	6.45	15.9	11.83	42.41	6.99±0.8	13.7±1.6
		$B^*\ B^*$	16.67	40.99	0.09	0.32	19.43±2.3	38.1±3.4
		B_sB_s	0.007	0.02	0.96	3.45	0.3±0.3	0.5±0.5
		$B_sB_s^*$	0.66	1.62	1.15	4.11	0.69±0.1	1.35±0.32
		$B_s^*B_s^*$	14.71	36.17	7.65	27.42	8.98±1.1	17.6±2.7
		Total	40.67		27.89		51^{+6}_{-7}	

2) Υ(5S) state

$\Upsilon(10860)$ state is candidate to be $\Upsilon(5S)$ state, with notation n $^{2S+1}L_J$= 5 $^3S_1$ and this state quantum numbers $J^{PC} = 1^{--}$. This state is above $B\bar{B}$ mesons threshold.

$\Upsilon(5S)$ state decays into:

$\Upsilon(5S) \rightarrow BB$ & $\Upsilon(5S) \rightarrow BB^*$ & $\Upsilon(5S) \rightarrow B^*B^*$

$\Upsilon(5S) \rightarrow B_sB_s$ & $\Upsilon(5S) \rightarrow B_sB_s^*$ & $\Upsilon(5S) \rightarrow B_s^*B_s^*$

We will calculated every partial decay alone using our $^3P_0$ model in calculating theoretical partial decay width value and calculating $\mathfrak{BR}$ branching ratio for every partial decay. Then, we do comparison of our results by experimental results in PDG2020[96]. After that, we compare our theoretical results with theoretical results of[108].

Our total theoretical strong decay width $\Gamma_{\Upsilon(5S)}^{our\ \text{total theo.}} \cong$ 40.67 MeV while experimental value $\cong 51^{+6}_{-7}$ MeV and the theoretical value in[108] $\cong$ 27.89 MeV. We conclude that our results close to experimental results more than Ref.[108]. The total experimental value of strong decay acts almost the total decay width value[96].

2.1) $\Upsilon(5S) \rightarrow BB$

The calculated value for this partial strong decay width value $\Gamma_{\Upsilon(5S)\rightarrow BB}^{our\ ^3\mathrm{P}_0\ \text{model}} \cong$ 2.7 MeV with the branching decay width ratio $\mathfrak{BR}_{\Upsilon(5S)\rightarrow BB}^{our\ ^3\mathrm{P}_0\ \text{model}} \cong$ 5.3% by our $^3P_0$ model while experimental partial decay width value $\Gamma_{\Upsilon(5S)\rightarrow BB}^{exp} \cong$ 2.8$\pm$0.33 MeV with branching decay width ratio $\mathfrak{BR}_{\Upsilon(5S)\rightarrow BB}^{exp} \cong$ 5.5$\pm$1%. Theoretical partial decay width value $\Gamma_{\Upsilon(5S)\rightarrow BB}^{theo.[108]} \cong$ 6.22 MeV with branching decay width ratio $\mathfrak{BR}_{\Upsilon(5S)\rightarrow BB}^{theo.[108]} \cong$ 22.29% in[108]. We can see our theoretical partial decay width value is a good agreement with experimental value more than theoretical value by[108].

2.2) $\Upsilon(5S) \rightarrow BB^*$

We calculate the theoretical value for this partial strong decay width, it is $\Gamma_{\Upsilon(5S)\rightarrow BB^*}^{our\ ^3\mathrm{P}_0\ \text{model}} \cong$ 6.45 MeV with the branching decay width ratio $\mathfrak{BR}_{\Upsilon(5S)\rightarrow BB^*}^{our\ ^3\mathrm{P}_0\ \text{model}} \cong$ 15.9% by our

$^3P_0$ model. While, we find the experimental partial decay width value $\Gamma^{exp}_{\Upsilon(5S)\to BB^*} \cong 6.99\pm0.8$ MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(5S)\to BB^*} \cong 13.7\pm1.6\%$. Also, we find theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(5S)\to BB^*} \cong 11.83$ MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(5S)\to BB^*} \cong 42.41\%$ by[108]. It is clear that our theoretical partial decay width value is a consistent with experimental value but it is different from theoretical value by [108].

2.3) $\Upsilon(5S) \to B^*B^*$

Theoretical calculated value by our $^3P_0$ model for this partial strong decay width value $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B^*B^*} \cong 16.67$ MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B^*B^*} \cong 40.99\%$ by our $^3P_0$ model while experimental partial decay width value $\Gamma^{exp}_{\Upsilon(5S)\to B^*B^*} \cong 19.43\pm2.3$ MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(5S)\to B^*B^*} \cong 38.1\pm3.4$ %. Theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(5S)\to B^*B^*} \cong 0.09$ MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(5S)\to B^*B^*} \cong 0.32\%$ by[108]. So that, we find our theoretical partial decay width value agree with experimental value very more than theoretical value by[108].

2.4) $\Upsilon(5S) \to B_sB_s$

The calculated value of this partial strong decay width $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_sB_s} \cong 0.007$ MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_sB_s} \cong 0.02\%$ by our $^3P_0$ model that value is the most small relative to our other partial decays values of $\Upsilon(5S)$ state. While experimental partial decay width value $\Gamma^{exp}_{\Upsilon(5S)\to B_sB_s} \cong 0.3\pm0.3$ MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(5S)\to B_sB_s} \cong 0.5\pm0.5\%$. Theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(5S)\to B_sB_s} \cong 0.96$ MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(5S)\to B_sB_s} \cong 3.45\%$ by[108]. We can see our result is a small relative to experimental strong decay width maximum value but very small relative to the theoretical value by[108].

2.5) $\Upsilon(5S) \to B_sB_s^*$

The calculated value of this partial strong decay width $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_sB_s^*} \cong 0.66$ MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_sB_s^*} \cong 1.62\%$ by our $^3P_0$ model that value is small relative to our other partial decays values of $\Upsilon(5S)$ state. The experimental partial decay width value $\Gamma^{exp}_{\Upsilon(5S)\to B_sB_s^*} \cong 0.69\pm0.1$ MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(5S)\to B_sB_s^*} \cong 1.35\pm0.32\%$. Theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(5S)\to B_sB_s^*} \cong 1.15$ MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(5S)\to B_sB_s^*} \cong 4.11\%$ by[108]. We can find our result is a agreement with experimental strong decay width value but small relative to the theoretical value by[108]. Our result is the nearest to experimental value.

2.6) $\Upsilon(5S) \to B_s^*B_s^*$

The calculated value for this partial strong decay width value $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 14.71$ MeV with the branching decay width ratio $\mathfrak{BR}^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 36.17\%$ by our $^3P_0$ model while experimental partial decay width value $\Gamma^{exp}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 8.98\pm1.1$ MeV with branching decay width ratio $\mathfrak{BR}^{exp}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 17.6\pm2.7\%$. Theoretical partial decay width value $\Gamma^{theo.[108]}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 7.65$ MeV with branching decay width ratio $\mathfrak{BR}^{theo.[108]}_{\Upsilon(5S)\to B_s^*B_s^*} \cong 27.42\%$ by[108]. Here, we find our theoretical partial decay width value is a sizable, it is different from experimental value and it is bigger than experimental value and theoretical value from [108].

We can conclude from the previous results in Table III that:

- There are a good agreement between our calculated results and experimental results more than theoretical results from [108] except the last partial strong decay width.
- $\Upsilon(5S) \to B^*B^*$ is dominant partial strong decay relative to our results of $\Upsilon(5S)$ state and also relative to the experimental results of $\Upsilon(5S)$ state.
- $\Upsilon(5S) \to B_s^*B_s^*$ is sizable partial strong decay relative to our calculated results and the experimental results of $\Upsilon(5S)$ state.
- The strong decay is dominant decay in $\Upsilon(5S)$ state.

From our previous calculations for the strong decay of $\Upsilon(4S)$ and $\Upsilon(5S)$ states :

In fact, we find to $\Upsilon(4S)$ and $\Upsilon(5S)$ states is a good source of BB, BB^*, B^*B^*, B_sB_s, $B_sB_s^*$ and $B_s^*B_s^*$. In the following, we will clear this idea.

- The source of *BB* mesons
  That is from $\Upsilon(4S)$ and $\Upsilon(5S)$ states during the following partial strong decays:
  $\Upsilon(4S) \to BB$, $\Upsilon(5S) \to BB$.
  Where according to our calculations the order of BB mesons sources by magnitude as the following:
  $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(4S)\to BB} \cong 20.71$ MeV > $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to BB} \cong 2.17$ MeV.
  But taking into consideration, the partial strong decay $\Upsilon(4S) \to BB$ is the most important source of *BB* mesons in the both two states $\Upsilon(4S)$ and $\Upsilon(5S)$ of the S-wave $b\bar{b}$ bottomonium mesons.
- *The source of BB^* mesons*
  That is from $\Upsilon(5S)$ state during the following partial strong decays:
  $\Upsilon(5S) \to BB^*$, Where: $\Gamma^{our\ ^3P_0\ \text{model}}_{\Upsilon(5S)\to BB^*} \cong 6.45$ MeV.
- *The source of B^*B^* mesons*
  That is from $\Upsilon(5S)$ state during the following partial

strong decays:
$\Upsilon(5S) \rightarrow B^*B^*$, Where: $\Gamma^{our\ ^3\mathrm{P}_0\ \mathrm{model}}_{\Upsilon(5S)\rightarrow B^*B^*} \cong 16.67$ MeV.

- *The source of B_sB_s mesons*
  That is from Υ(5S) state during the following partial strong decays:
  $\Upsilon(5S) \rightarrow B_sB_s$, Where: $\Gamma^{our\ ^3\mathrm{P}_0\ \mathrm{model}}_{\Upsilon(5S)\rightarrow B_sB_s} \cong 0.007$ MeV.

- *The source of $B_sB_s^*$ mesons*
  That is from Υ(5S) state during the following partial strong decays:
  $\Upsilon(5S) \rightarrow B_sB_s^*$, Where: $\Gamma^{our\ ^3\mathrm{P}_0\ \mathrm{model}}_{\Upsilon(5S)\rightarrow B_sB_s^*} \cong 0.66$ MeV.
- *The source of $B_s^*B_s^*$ mesons*
  That is from Υ(5S) state during the following partial strong decays:
- $\Upsilon(5S) \rightarrow B_s^*B_s^*$, Where: $\Gamma^{our\ ^3\mathrm{P}_0\ \mathrm{model}}_{\Upsilon(5S)\rightarrow B_s^*B_s^*} \cong 14.71$ MeV.

To here, we have finished our detailed studies on strong decay of two states Υ(4S) and Υ(5S) in the S-wave $b\bar{b}$ bottomonium mesons.

IV. SUMMARY AND CONCLUSION

In this paper, we presented in detailed study about sources of B-mesons by strong decay of two states, the $\Upsilon(4S)$ state and the $\Upsilon(5S)$ state of S-wave $\mathrm{b\bar{b}}$ bottomonium mesons. The principle purpose from this paper the accurate determination of B-mesons sources to place the start point to establish the strong decay of the $\mathrm{b\bar{b}}$ bottomonium mesons physics to get the sources of B-mesons where the rare B decays can be used to determine some of the CKM matrix elements (e.g V_{td} and V_{ts}) or to give ideas about the top quark in the Standard Model (SM) that is considered the top of pyramid of the modern physics and the rare B decays can as well be used to pass from them to a new world of physics that it is so-called BSM(Beyond Standard Model) or New Physics(NP) and probe its deeps. which is abled particle physics scientists to strive toward a theory of everything. This paper is a first step towards this very challenging goal.

In fact, we conclude from the previous that two states: the $\Upsilon(4S)$ state and the $\Upsilon(5S)$ state of S-wave $\mathrm{b\bar{b}}$ bottomonium mesons provide us with sources of BB, BB^*, B^*B^*, B_sB_s, $B_sB_s^*$, $B_s^*B_s^*$ mesons as following:

1) *A source of BB mesons* from Υ(4S) state and Υ(5S) state during the partial strong decays $\Upsilon(4S) \rightarrow BB$ and $\Upsilon(5S) \rightarrow BB$. But taking into consideration, the partial strong decay $\Upsilon(4S) \rightarrow BB$ is the most important source of BB mesons. It is consider a very good source of B-B mixing. While Υ(5S) is the reasonable source of B-B mixing mesons.

2) *Υ(5S) state is considered as a good source of B^*B^** pair and $B_s^*B_s^*$ pair, also, it is the reasonable source of BB^* mesons, but it is a small source of $B_sB_s^*$ pair and it is a very small source of $B_sB_s$pair to the extent that we may see them with great difficulty in the B-factories collaboration in all over world.

We use the $^3P_0$ model or so-called QPC quark pair creation model to calculate the strong decay of the $\Upsilon(4S)$ state and the $\Upsilon(5S)$ state in theS-wave $\mathrm{b\bar{b}}$ bottomonium mesons. Strong decay ratio is calculated for every state. The results show general agreement between the predictions of our $^3P_0$ model and the available recent experimental data(PDG 2020) also their general agreement with available other theoretical results but our predictions is the nearest to recent experimental data. We advise using $^3P_0$ model for obtaining the strong decay of $b\bar{b}$ bottomonium mesons states and other heavy mesons width because it is easy, saves the time and is accurate where that clears from our calculations and compare them by the available recent experimental data(PDG 2020).

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