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Analytical Method for Determining the Characteristics of an Aquifer Based on Analysis Parameters of the Gushing Well

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Abstract

The article developed a new method for determining the parameters of an underground aquifer based on the characteristics of a single gushing well based on the use of a solution with three-dimensional filtration to assess the value of the hydraulic resistance of the near-wellbore formation zone, that is, an alternative to the Dupuis formula obtained for flat filtration. In addition, new analytical expressions are obtained in work: the dependence of the density of geothermal water on the depth in the well ρ(h), the dependence of the volumetric compressibility coefficient B(h), and the dependence of the reservoir pressure of water in the well on its depth.

Keywords: *gushing well, porous skeleton, aquifer, volumetric compressibility coefficient of the water, hydraulic permeability, Dupuis equation, skin-factor.*

I. INTRODUCTION

When assessing the prospects of hydrothermal deposits, in addition to water temperature, very important parameters are the hydraulic permeability of the porous skeleton of an aquifer, which, along with reservoir pressure and effective reservoir thickness, forms the flow rate of water gushing from the well. Determining a porous skeleton's permeability using a rock core in laboratory conditions is a long and detailed procedure that requires special experimental equipment. Therefore, it is very important to develop a method for the full-scale assessment of the skeleton permeability. This is also important for optimizing the well's flow rate according to the criterion of the fountain's useful work and the subsequent organization of the optimal mode of its operation.

II. OVERVIEW

Estimates of the hydraulic characteristics of wells are traditionally based on the equation of the French hydro engineer Dupuit (Dupuit A.J., 1854), which he obtained to calculate the inflow of water from the aquifer into cylindrical ground wells on the earth's surface [1]:

$$Q = \frac{kg}{\nu} \cdot \frac{\pi}{\ln(R_K / r_C)} \cdot (H^2_K - H^2_C), \qquad (1)$$

where Q − is the inflow of water into the well (m^3/s);
H_C − piezometric water level in the well (m);
H_K − the piezometric level at the boundary of the feed circuit (m);
R_K − is the radius of the feed loop (or "depression funnel") (m);
r_C − radius of the circular section of the well (m);
υ − is the kinematic viscosity of water (m^2/s);
g − acceleration of gravity (m/s^2);
k − is the hydraulic permeability of the soil (m^2).

Nowadays, shallow wells are usually used instead of wells. Figure 1. a graphical diagram of a shallow well is shown in a section along the axis with the draw-down funnel designation and 2 observation wells within the recharge loop.

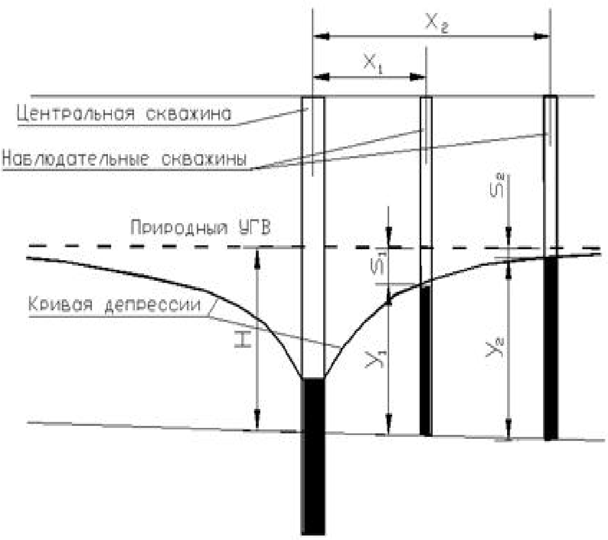

Fig. 1. Draw-down funnel for a shallow well (or well)

H − initial groundwater level (m), Уj − piezometric groundwater level at various distances Xj from the well (m) axis.

Thus, the Dupuis formula's expression in the form (1) describes the traditional depression funnel in wells or shallow wells with a water level below the earth's surface. An analysis of formula (1) shows that it is the presence of a "depression funnel" that leads to a sharp increase in the hydraulic resistance of water filtration in the near-wellbore zone due to a decrease in its piezometric level, which leads to a quadratic dependence of water inflow into the well. And to assess the potential productivity (flow rate) of gushing wells, a different modification of the Dupuis formula is usually used, which was obtained within the framework of the "flat problem" of filtration [2] and is written as follows:

$$Q = \frac{kh}{\mu B} \cdot \frac{2\pi}{\ln(R_k / r_C)} \cdot (P_K - P_C), \qquad (2)$$

where Q − is the flow rate of the flowing well (m^3/s);
P_K − reservoir pressure at the boundary of the feed loop (MPa);
P_C − bottom hole pressure in the wellbore (MPa);
k − is the permeability of the porous formation skeleton (m^2);
μ − dynamic viscosity of geothermal water (Pa·s);
B − is the dimensionless coefficient of volumetric compressibility of water, $B(P_C) = \rho(P_C)/\rho_o$,
ρ_o − is the density of water at atmospheric pressure;
h − is the effective thickness of the aquifer (m);
R_K - radius of the feed loop (depression funnel) (m);
r_C - wellbore radius along the bit (m).

Sometimes this Dupuis formula is written in the opposite form [3]:

$$\Delta P_{KC}/Q = R_h = [(\mu B/k)\cdot\ln(R_K/r_C)]/(2\pi h), \quad (3)$$

where R_h − is the hydraulic resistance of filtration in the near-wellbore zone,
ΔP_{KC} − is the pressure difference at the boundary of the feed loop and in the well, MPa.

Figure 2. presents a graphical diagram of a planar problem of water filtration from a reservoir into a well in a cylindrical coordinate system.

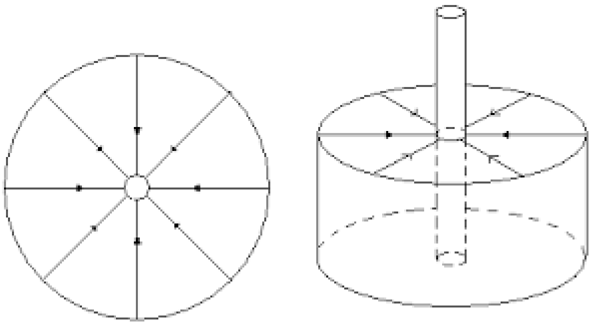

Fig. 2. Scheme of radial filtration of water from the reservoir to the well.

An analysis of the Dupuis equation (2) for wells showed that fluid filtration was considered within the framework of a "plane two-dimensional problem," which, using central symmetry, was reduced to a one-dimensional case. Thus, the Dupuis formula is a one-dimensional relationship between the fluid flow into the well and the pressure drop (between the reservoir and the well), which was obtained using the radial symmetry of fluid flow in a porous medium, which made it possible to integrate Darcy's law within the cylindrical coordinate system.

The exact non-stationary solution for planned radial filtration in a large-scale aquifer ($r >> R_K$), which was completely penetrated by a well, has long been known. It was obtained by the American hydrogeologist Theis (Theis C.V., 1935) [4] and is expressed in terms of the integral exponential function Ei ($-r^2/at$) in the following form:

$$Q_{(t)} = \frac{km}{\mu\beta} \cdot \frac{4\pi}{-Ei(-r^2/4at)} \cdot \Delta P_{(r)}, \qquad (4)$$

where ΔP(r) − is the difference between the water pressure in the well at $r_o = r_C$ and the water pressure in the aquifer at a distance $r >> R_K$ (Pa);
m − effective thickness of the aquifer (m);
β − coefficient of volumetric compressibility of the aquifer;
a − coefficient of piezoconductivity of the aquifer, $a = k/\beta$ (m^2/s);
t - the time from the beginning of good operation (s).

Analysis of the Theiss formula (4) shows that to obtain the flow rate Q's numerical values, it is necessary to integrate it along the range from $r = r_o$ to $r \to \infty$. Since the function Ei($-r^2/at$) is not integrated into quadratures, this problem must be solved numerically, which leads to large errors in the absence of real reservoir parameters. Determining the potential production rate of a well in such a complex and imprecise way is impractical.

Therefore, usually, when evaluating a real (imperfect for opening) a gushing well [5], the corrected Dupuis formula is used in the form (2):

$$Q = \frac{kh_R}{\mu B} \cdot \frac{2\pi}{\ln(R_{\hat{E}} / r_C) + S} \cdot \Delta P_{KC}, \qquad (5)$$

where h_R − is the length of the casing filter in the well, usually $h_R << h$ (m);
S – skin-factor (dimensionless hydraulic parameter).

It is accepted that it characterizes the additional hydraulic resistance to fluid flow in the porous skeleton of the aquifer's near-wellbore zone due to imperfect opening of the reservoir and other secondary effects (compression of the porous skeleton, the turbulence of water flow, etc.).

Practical calculations of fluid inflow from an aquifer to a well are currently mainly developed within the framework of a one-dimensional logarithmic dependence of the Dupuis type (2) and (5). Here we should mention Musket's theoretical work performed using the methods of displaying sinks and their superposition and Shchurov's work performed by the graphic-analytical method using EHDA. Recently, Veliev [5] obtained formulas for the skin-factor in the form of a sum of components: $S = \Sigma S_i$; complex analytical expressions were obtained for the members of the sum S_i to describe various configurations of schemes of the imperfect opening of a porous aquifer by a well.

However, the analysis of the last work of Muzafalov [6] showed that the skin effect is nothing more than a "compensator" of errors arising from the inaccurate determination of the radius of the supply circuit, which is compensated by the authors by introducing the coefficient S into the original Dupuis equation with a dubious physical meaning. It is obvious that the skin-factor S is nothing more than a correction for the error in determining the "radius of the feed loop" and the "depression funnel," as well as for neglecting the three-dimensionality of the real fluid filtration in the porous skeleton of the near-wellbore zone of the real, i.e., imperfectly opened aquifer (in the case when h_R << h).

Thus, the uncertainty introduced by the skin factor's presence in calculating water inflow from the reservoir into the wellbore according to the Dupuis formula does not allow using it to determine the porous skeleton's permeability in natural conditions. And the lack of information on the hydraulic resistance of the aquifer's near-wellbore zone makes it difficult to assess a flowing well's characteristics.

III. FORMULATION OF THE PROBLEM

Analysis of the Dupuis formulas showed that they describe stationary filtration (solutions do not depend on time). Although in real cases, there is just a non-stationary emptying of the aquifer in the surface layer of the soil or a decrease in the reservoir pressure of water in a porous underground reservoir of a finite volume. Therefore, it is impossible to establish a good feed loop (or depression funnel) radius in the general case. For example, in the "groundwater table" near the well (after it has been dug out), the piezometric water level is constantly decreasing. At the same time, the depression funnel increases, and the radius of the feed loop grows. That is why the procedure for determining the radius R_K for the supply circuit (or "depression funnel") has not been established for the Dupuis formulas: it is not formulated either physically or mathematically. It allows any arbitrariness when calculating by formulas (1) and (2). And to determine the pressure P_K and use the Dupuis formula, it is necessary to drill at least one more well near the one whose potential productivity must be determined. Therefore, when analyzing the inflow of fluid from an aquifer to the filter of a gushing well, the Dupuis equation is of purely academic interest. Indeed, this formula, firstly, includes the parameters that characterize the wells: "depression funnel," "radius of the supply circuit" R_K of the well (formed by the depression funnel), i.e., parameters that are not and cannot be in the aquifers of deep wells. And secondly, if next to the well on the surface of the earth, it is enough to drill another - two wells in the ground and measure the piezometric level of groundwater, then drill a well a kilometer or two deep to measure the pressure value at the boundary of the good feeding circuit - very difficult and expensive.

The logarithmic nature of the dependence of the inflow to the well on the ratio of the two radii, which is obtained by integrating the differential filtration equation in a cylindrical coordinate system, does not allow the use of infinite limits of integration. Therefore, within the framework of the "flat problem," one cannot get away from the concepts of "depression funnel" and "feed loop." To take advantage of the infinite limits of integration and go beyond such non-constructive concepts as "depression funnel" and "supply circuit," it is necessary to solve the filtration problem in a three-dimensional setting. This will allow determining the permeability and hydraulic resistance of the near-wellbore zone.

IV. RESEARCH RESULTS

A. Evaluation of the permeability of the aquifer

To obtain a three-dimensional analytical solution to this problem, one can use a thermal-hydraulic analogy, that is, use the solution obtained for the heat flow from an infinite array to a bounded cylinder that simulates a casing filter. In this case, you can use the analogy of fluid flow in a porous structure (in accordance with Darcy's law) with the propagation of heat in an isotropic heat-conducting medium (in accordance with Fourier's law). For the propagation of heat in an infinite heat-conducting medium [7] under similar boundary conditions, there is already a solution for the thermal resistance between the rod (radius R and length L) and the array in the parameter range 100 > L/R > 20:

$$R_{35} = [Arch(L/R) - \sqrt{1+(R/L)^2} + R/L]/(2\pi\lambda\cdot L). \quad (5)$$

Thermal resistance is included in the heat flux expression, which depends on the temperature difference between the rod and the surrounding mass: $q = \Delta t/R_{35}$. In this expression for R_{35}, it is necessary to replace the thermal conductivity λ by the ratio of permeability to viscosity $k/\mu B$ and obtain an expression for the hydraulic resistance to the fluid flow:

$$R_h = (\mu B/k)\cdot[Arch(L/R) - \sqrt{1+(R/L)^2} + R/L]/(2\pi\cdot L). \quad (6)$$

In this case, R and L's values are equal, respectively, to the radius and length of the filter installed at the lower end of the casing.

EXAMPLE.

The standard length of the filter section is L = 3 m [8], and the casing diameters are Ø100, Ø150, or Ø200 mm. On this basis, it is possible to calculate one of the components of the hydraulic resistance of the near-wellbore zone R_h, presented in square brackets […] for a single-section filter.

The calculation data for the values [...] in (6) based on [8] are given in Table 1.

Table 1.

The results of calculating the expression in brackets

Ø Pipe, mm	Ø 100	Ø 150	Ø 200
L/R Ratio	60	40	30
Value […]	3,79	3,41	3,06

Since the size ratio of a single-section filter L/R with Ø100 mm is the same as the L/R ratio for a two-section filter with Ø200 mm, the value in […] equal to 3.7885 can be used to calculate this case.

If L/R ≥ 100, the expression in brackets […] in formula (6) degenerates:

$$lim[Arch(L/R) - \sqrt{1+(R/L)^2} + R/L] \rightarrow$$
$$\rightarrow ln(2L/R) - 1, [7].$$

Usually, a real flowing well is subjected to field tests not to calculate its "potential productivity" but to estimate the hydraulic resistance of the near-wellbore zone of an aquifer. And on this basis, taking into account the size of the casing pipe, the filter, and the thickness of the aquifer, determine the permeability of the skeleton and optimize the good operation mode: determine the optimal flow rate and temperature of the geothermal water pumped back.

From formulas (3) and (6), one can easily express the permeability of the skeleton *k* based on the expression of the hydraulic resistance of the near-wellbore zone ($R_h = \Delta P_{KC}/Q$) and the ratio of the sizes of the casing filter:

$$k = (\mu B)\cdot(\Delta P_{KC}/Q)\cdot[Arch(L/R) - \sqrt{1+(R/L)^2} + R/L]/(2\pi\cdot L). \quad (7)$$

Having previously calculated the value of μB, one can also find the permeability *k*.

B. Determination of water compressibility, reservoir, and bottom hole pressure

It is known that its compressibility can be neglected at low water pressures (less than 50 atm). Still, at depths greater than 0.5 km, it must already be taken into account, that is, to calculate the water density $\rho = \rho(p)$.

To take into account the compressibility of water, one can use the well-known Tate equation [8], which characterizes the dependence of the density of a liquid on pressure:

$$\rho\ (p) = \rho_o\cdot(1 + p/C)^{0.14}, \quad (8)$$

where ρ_o − is the density of water (10^3 kg/m^3) at atmospheric pressure;
p − is the absolute pressure of compressed saline water (MPa);
C − is an empirical constant for water C = 320 MPa.

Using the Tate equation, it is easy to determine the volumetric compression ratio of geothermal water at the level of casing filter *B*:

$$B = \rho(P_C)/\rho_o = (1 + P_C/C)^{0.14}. \quad (9)$$

The magnitude of the reservoir pressure can be estimated, for example, at zero production rate in a shut-in well, and it is equal to the sum of the pressure of the liquid column P_H in the lower section of the filter plus the pressure according to the manometer at its wellhead $(p_w)_{max}$:

$$R_K = P_H + (p_w)_{max}, \quad (10)$$

The bottom hole pressure can also be indirectly estimated since it is equal to the difference between the pressure in the filter P_H and the pressure loss due to viscous friction during the flow of water in the casing ΔH_{fr} (Pa):

$$P_C = P_H + \Delta H_{fr}(Q) + \rho_o g\cdot h_F, \quad (11)$$

where ΔH_{fr} − is the pressure loss during the laminar flow of geothermal water in the casing pipe is calculated using the Hagen-Poiseuille formula:

$$\Delta H_{fr} = 32\mu w\cdot(l/d^2), \quad (12)$$

w − is the average water velocity in the casing (m/s);
μ − dynamic viscosity of geothermal water (Pa·s);
l and *d* − are the casing (m) 's length and diameter, usually $l = h$.
h_F – fountain height (m).

A big problem in determining the volumetric compressibility factor *B* is calculating the pressure of the water column in the casing since here we are dealing with a nonlinear problem ($P_H \neq \rho_o gh$), because following the Tate equation $\rho = f(p)$, and the pressure, in turn, is $p = \varphi(h)$. Usually, this problem is solved numerically, but it can also be solved analytically.

If we write down the expression for p (h) in integral form, we get:

$$p(h) = g\cdot\int\rho(h)dh \quad (13)$$

Substituting the pressure expression from (13) into the Tate formula and carrying out the appropriate transformations, we obtain the following:

$$(\rho/\rho_o)^{1/0.14} = (\rho/\rho_o)^{7.14} = 1 + (g/C)\cdot\int\rho(h)dh \quad (14)$$

If the resulting expression is differentiated concerning the coordinate h, then we obtain a first-order differential equation for the density $\rho(h)$:

$$(\rho^{7,14})' = (\rho_o^{7,14})\cdot(g\rho/C) \quad (15)$$

After bringing similar terms and separating the variables, we get:

$$\rho^{5.14}\, d\rho = (\rho_o^{7.14}/7.14)\cdot(g/C)\, dh \quad (16)$$

Integration of the equation in the boundaries: for ρ (ρo → ρ) and h (0 → h), gives:

$$\rho^{6.14} - \rho_o^{6.14} = (6.14/7.14)\cdot\rho_o^{7.14}\cdot(gh/C) \quad (17)$$

The transformation of the obtained expression (17) leads to the following formula for the volumetric compressibility coefficient of water *B* to the power of 6.14:

$$(\rho/\rho_o)^{6.14} = B^{6.14} = 1 + 0.86\cdot(\rho_o gh/C) \quad (18)$$

And finally, we write down the final expression for the coefficient *B*:

$$B = [1 + 0.86\cdot(\rho_o gh/C)]^{0.163} \quad (19)$$

The obtained formula for the coefficient of volumetric compressibility of water makes it possible to calculate with high accuracy the value of *B* from the good depth and allows solving the problem of assessing the permeability of a water-saturated reservoir by a simple analytical method.

If we express the dependence of water density on depth from formula (19) and substitute it into integral expression (13), then solving it is possible to obtain the value of reservoir pressure.

$$\rho(h) = \rho_o \cdot [1 + 0.86 \cdot (\rho_o gh/C)]^{0.163} \qquad (20)$$

So, based on the Tate formula, expressions were obtained for the density of geothermal water and the coefficient of its volumetric compressibility depending on the well's depth.

Having this dependence, it will be quite simple to find an expression for the bottom hole water pressure in the reservoir. Thus, it is necessary to substitute the expression for the dependence ρ(h) in formula (13) and integrate it within limits: from 0 → to H.

$$P_H = \int \rho_o \cdot [1 + 0.86 \cdot (\rho_o gh/C)]^{0.163} dh \qquad (21)$$

As a result of integration, we obtain the following dependence:

$$P_H = C \cdot [(1 + 0.86 \cdot (\rho_o gH/C)^{1.163} - 1] \qquad (22)$$

This dependence can be more briefly expressed through the coefficient of volumetric compressibility of water *B*:

$$P_H = C \cdot (B^{7.14} - 1) \qquad (23)$$

Where the *B*-factor is calculated for the lower section of the casing filter.

V. CONCLUSIONS

The article analyzes the current state of the analytical description of filtration in the near-wellbore zone of an imperfectly opened aquifer's porous skeleton. It is established that the primary Dupuis formula (1) was obtained for water wells in the ground on the earth's surface, and it is she who describes such a phenomenon as a "depression funnel."

The logarithmic nature of another expression of the Dupuis formula in the form (3), i.e., for the dependence of water inflow from the reservoir into a deep well, which was established when solving the "flat filtration problem," does not allow assessing the hydraulic parameters of an aquifer based on the results of analyzing the parameters of a single well.

It is very difficult to use the Theiss formula to analyze the test results of a single well due to the lack of a priori information on the aquifer's parameters.

Based on the analysis of the problem by the method of thermal-hydraulic analogy, an analytical solution to the problem of three-dimensional filtration of fluid from the aquifer into the casing filter for the case of $L/R \geq 20$. This made it possible to estimate the hydraulic resistance of the near-wellbore zone of the reservoir skeleton.

The use of the new formula eliminates arbitrariness in determining the radius of the feed loop (it is absent here at all) since, in a three-dimensional solution, there is no such thing as a "depression funnel" characteristic of a one-dimensional solution in the case of plane filtration in a cylindrical coordinate system.

Based on the Tate formula for the dependence of the compressibility of geothermal water on the pressure by compiling and integrating a differential equation, new analytical expressions are obtained:

− the formula for the dependence of the water density on the depth in the well ρ(h),

− the formula for the dependence of the coefficient of volumetric compressibility *B*(h)

− the formula for reservoir pressure of water in a well from its depth,

This makes it very easy to calculate all the necessary parameters for calculating the desired value of the aquifer skeleton's permeability based on the results of its opening with just one well.

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