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The Compressed Particle's Vector And Derived Applications

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Abstract
The representation of particles, including photons, as Hyper-Complex Numbers (H-numbers) is shortly presented. The Planck relation and the concept of light's compressed vector led to the concepts of photon ST inaccuracy, Planck units and quantization of the Physical World. Starting from bases representation of a H-number, the matter waves equation and Heisenberg's Uncertainty Relationships are also derived.

Keywords: *Particles, Hyper-complex numbers, geometrized unit system, ST modified, compressed state, inaccuracy, associated wave, Planck units, Heisenberg's Uncertainty Relations, Quantization.*

Introduction

The theory of the Hyper-Complex numbers (further H-numbers) and their applications in physics have been extensively presented in various articles [1],[2], [3], [4]. As was shown in the reference [2], it appears that this model conciliates the special relativity and quantum behavior. The present contribution illustrates how the H-numbers model of an ideal particle leads to new applications.

I. Kinematics of material particles

A. The ideal particle as a Hyper-Complex number

As stated in the reference paper [2], an ideal (or point) particle is characterized by four parameters: time (t), mass (z), momentum (y) and position in space (x). Using a cartesian coordinate system, these four basic parameters of the particle become the coordinates of a point P, and define a vector $\overrightarrow{OP}$, in a four-dimensional space called 'physical world'.

The physical world is not static because the time is flowing continuously in one direction. An observable particle will follow a curve in this four-dimensional space, called the **evolution line**. The Axiom 1, introduced in the reference [2], paragraph 3.2., states that the physical world and the set H of hyper-complex numbers are isomorphic groups. A point P belonging to the physical world has a corresponding H-number p, which is named the affix of the point P. Reciprocally the point P (or the vector $\overrightarrow{OP}$) represents the image of the number p. Thus, the particle is associated with an H-number having the following expression

$$p = t + iz + jy + kx \qquad (1.1)$$

The H-number representation of an evolution line is a hypercomplex-valued function of the real variable, t.

$$p(t) = t + iz(t) + jy(t) + kx(t) \quad (1.2)$$

The fundamental units of the H-numbers set are 1, i, j and k. The next table shows the rules of multiplication of these units.

Table 1. Units' Multiplication Table

x	**1**	**i**	**j**	**k**
1	1	i	j	k
i	i	-1	-k	j
j	j	-k	-1	i
k	k	j	i	1

Time and mass are scalar parameters but space (position in space) and momentum are typical 3D vectors, and consequently the moving particle will be represented by a Vector-Hyper-Complex number in an eight-dimensional space (as shown in the reference [3]). However, for simplicity reasons, we will consider further only unidimensional space and momentum. The four parameters of a particle are expressed using a common unit, meter^5, as shown in the table below.

Table 2. The conversion of international system of units (SI) to geometrized system of units (GU); c is the velocity of light and G gravitational constant, both expressed in SI

	GU	SI	SI→GU	GU→SI
Length	m	m	1	1
Time	m	s	c	c^{-1}
Velocity	dimensionless	ms^{-1}	c^{-1}	c
Mass	m	Kg	Gc^{-2}	$G^{-1}c^{2}$
Momentum	m	$Kgms^{-1}$	Gc^{-3}	$G^{-1}c^{3}$
Force	dimensionless	N	Gc^{-4}	$G^{-1}c^{4}$

A H-number has also an exponential representation as shown in the same reference [2], paragraph 2.4., equation 2.18:

$$p = \rho e^{i\alpha + j\beta + k\chi} \qquad (1.3)$$

Where, ρ is norm of p, α is the imaginary argument, β co-imaginary argument, and χ co-real argument (see reference [2], paragraph 2.5.). The magnitude of p, or of the vector $\overrightarrow{OP}$, is defined as:

$$\overrightarrow{OP} = |p| = \sqrt{t^2 + x^2 + z^2 + y^2} \qquad (1.4)$$

B. Light particles

According to relation (1.2), a material particle with constant mass (z), moving with constant velocity (v) is represented by the following H-number function:

$$\mathrm{p(t) = (t + iz)(1 + kv)} \qquad (1.5)$$

The above function is changing continuously with the time parameter and its image depicts a straight line as evolution line in the physical world.

The light particles are moving with a velocity of the magnitude 1, and consequently a simple representation of a photon could be:

$$p(t) = (t + iz)(1 \pm k) \qquad (1.6)$$

The corresponding H- numbers are known as OI (zero interval) numbers [2].

The H- number associated to a light particle has no exponential representation. In this case ρ = 0 but χ = ± ∞ and the extended Euler's formula does not apply anymore.

C. Space-Time diagram

The Space-time diagram (ST), or Minkowski space-time modified, has been presented in different published papers (see references [2], [4]). The points belonging to this diagram represent projections of image vectors on the plane (t, x). A curve in the ST diagram is the projection on this plane of one particular evolution line in the physical world. Further we will use the denomination introduced by Hermann Minkowski i.e., world lines. Being a projection, the world line shows only the evolution of space x in time, or the path of the ideal particle in ST.

The Figure 1 shows that this diagram is split by light world lines (ρ =0) in four wedges. The first and the third wedge represent the future and respectively the past. The wedge 2 and the wedge 4 belong to the region between light cones, known as “elsewhere”. In Special Relativity wedges 1 and 3 are also known as time-like zone and the corresponding moving particle are subluminal or tardyons. Wedges 2 and 4 build the space-like zone and the particle moving inside are superluminal or tachyons. The numeric representation for a point in time-like zone is:

$$p = t + kx = \pm\rho(\cosh\chi + k\sinh\chi) \qquad (1.7)$$

For a point in the space-like zone, the corresponding affix will be written as:

$$p = t + kx = \pm\rho k(\cosh\chi + k\sinh\chi) \qquad (1.8)$$

The norm of p is ρ (see reference [2], paragraph 2.5, formula 2.19), defined as $= \sqrt{|(t^2 - x^2)|}$, where $|a|$ signifies the absolute value of the real number a.

The point P in the diagram below, represents the image of the $p = 2 + kx1.7$.

Alternatively, the image can also be the vector OP.

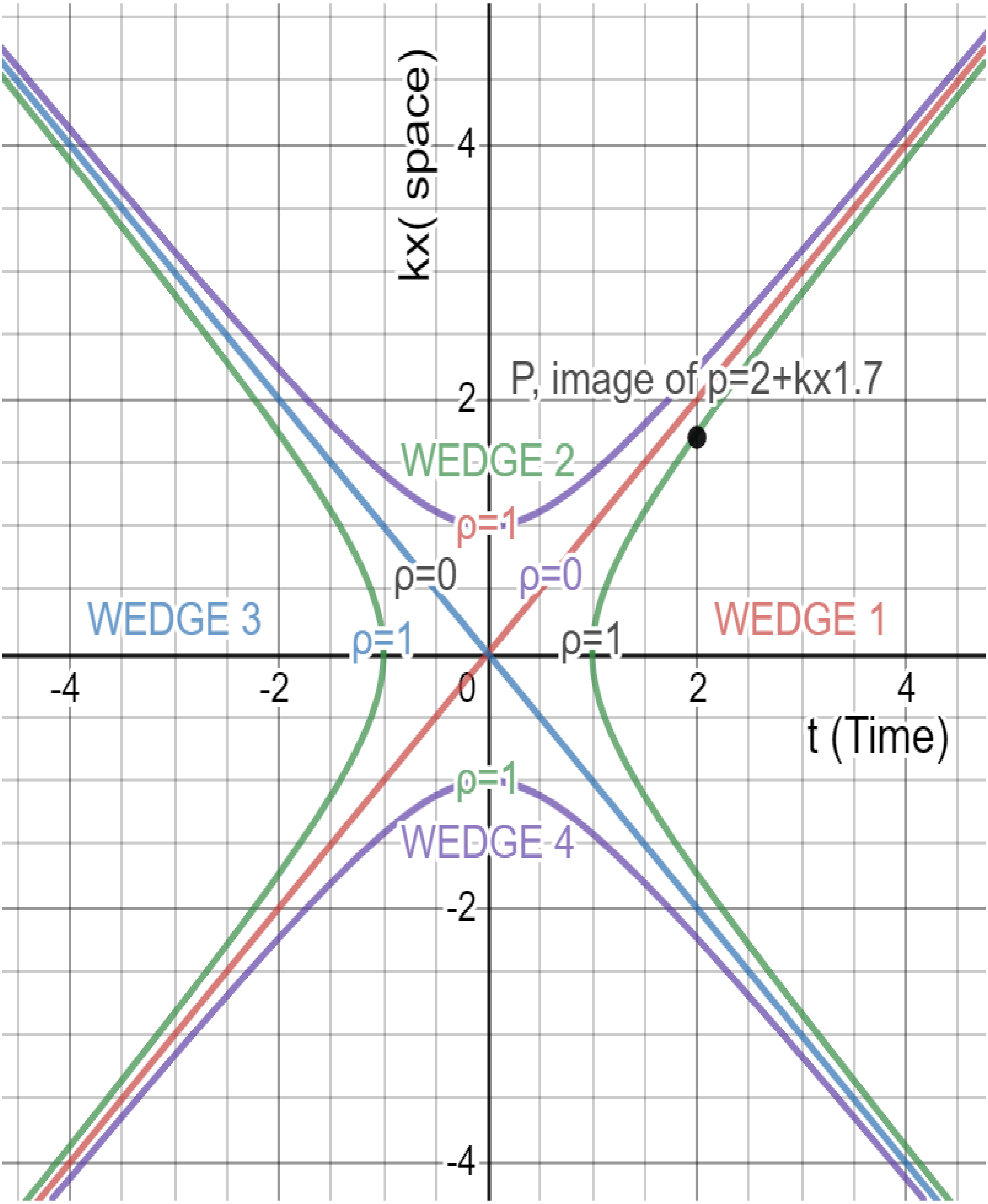

Figure 1. Minkowski space-time diagram modified

The Figure 2 gives a more detailed representation of the particle's motion using the non-static ST plane diagram. The physical world should be seen as a ST plane with its cosmic space S containing moving particle and islands of masses along with their momenta. We should be aware, that the continuous ST is permanently expanding since the Big Bang, which happened approximately 13.8 billion years ago.

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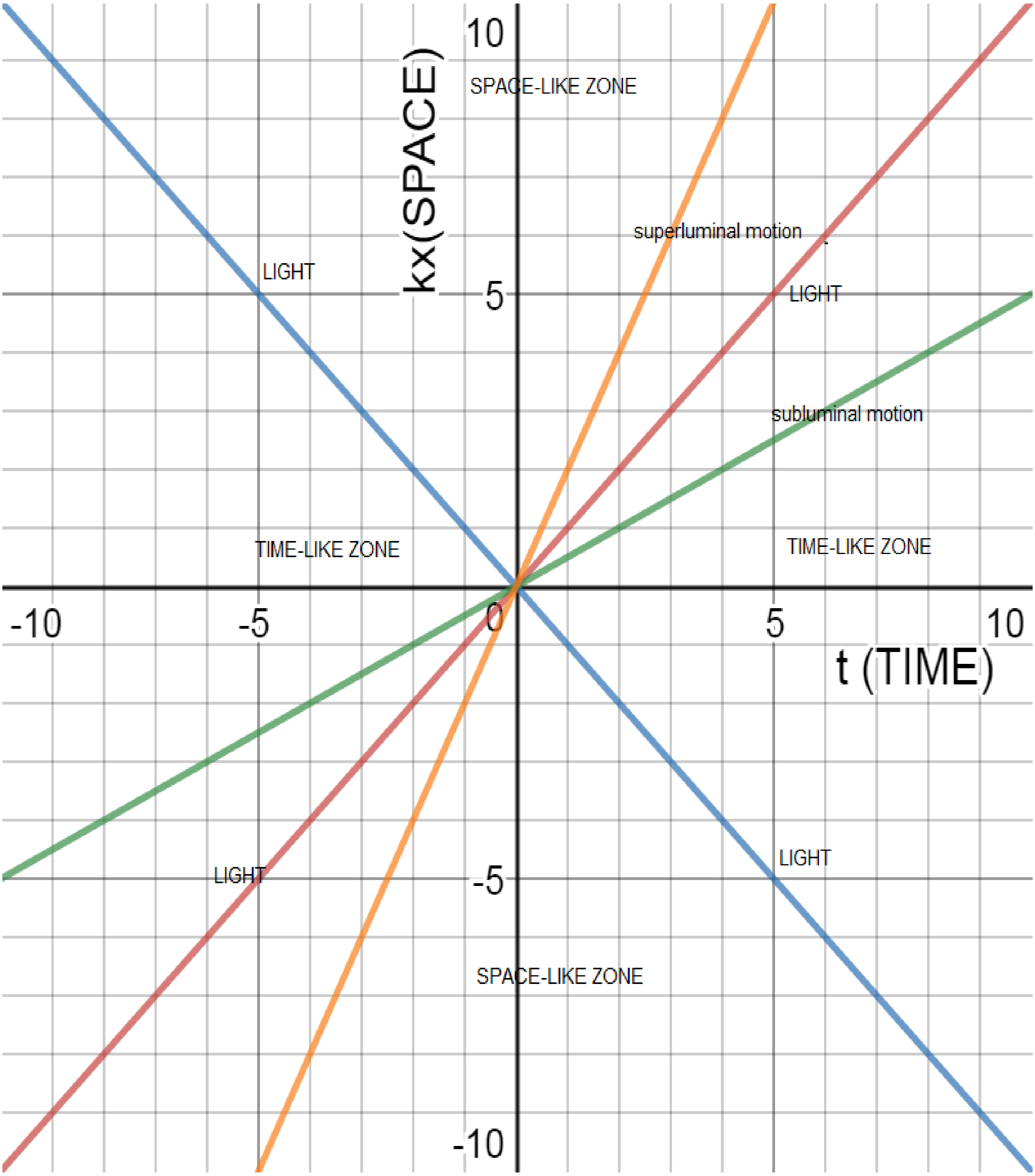

Figure 2. The Space-Time plane with world lines of uniform moving particles

The time parameter t is a continuously changing variable, and the motion could be defined as the change in position of a particle over the time. The present analysis considers only motions forward in time.

The world lines of different moving particles (see also reference [4]) are displayed in the figure above. The violet line and the blue line represent the motion of light particles. In the four-dimension ST they build a super-cone of light which separates the time-like zone and the space-like zone. The green line represents the motion of a subluminal particle, and the orange line shows a superluminal one.

II. Compressed state

A. The magnitude of a photon vector; Inaccuracy's definition

The evolution line of an ideal particle is a time function as shown in the paragraph 1.1, equation (1.2). Every point P, belonging to the evolution line, has an affix p. The magnitude of the H-number p is a positive real number $|p|$, as defined in the equation (1.4).

The Plank relation generally applies to light particles(waves). This formula, takes the following form, as shown in reference [2], paragraph 4.1., equation (4.3):

$z = \frac{H}{T}$, where T is the period of the corresponding light-wave and z is the mass of the photon.

The constant $H = \frac{hG}{c^3} = 1{,}64113 \times 10^{-69} m^2$, where h is the Planck constant in SI, represents the Planck constant expressed in GU units-system.

Let us now consider the evolution line of a photon having the mass z. This evolution line is represented, as function of time, by the formula (1.6).

Definition: The evolution line of a light particle is said to be in the compressed state, if the time parameter is equal to T. The corresponding vector image will be named compressed.

$$p_{lightcomp} = (T + iz)(1 \pm k) = \left(\frac{H}{z} + iz\right)(1 \pm k) = \left(T + i\frac{H}{T}\right)(1 \pm k) \qquad (2.1)$$

$$p_{lightcomp} = \overrightarrow{OP}_{comp} = t_{comp} + iz_{comp} + jy_{comp} + kx_{comp} = \frac{H}{z} + iz \pm jz \pm k\frac{H}{z} \qquad (2.2)$$

There is a direct interelation between the number shown in the formula (2.2) and the associated plane light wave (see reference [2], paragraph 4.1).

$$Y = Acos\frac{2\pi z}{H}(t \pm x) = Acos2\pi\left(\frac{t}{t_{comp}} + \frac{x}{x_{comp}}\right) \qquad (2.3)$$

The magnitude of the compressed vector associated to a photon is:

$$\left|\overrightarrow{OP}\right|_{comp} = \left|p_{lightcomp}\right| = \sqrt{T^2 + \frac{H^2}{T^2} + T^2 + \frac{H^2}{T^2}} = \sqrt{2}\sqrt{T^2 + \frac{H^2}{T^2}} \qquad (2.4)$$

The expression shown in the formula (2.4) represents the inaccuracy of the photon's location on its evolution line.

The mass of a photon expressed in meter is, for most practical cases, very small compared with the period T. Let us illustrate this, considering the highest energy gamma rays ever detected [6], the so called Ultra High Energy Gamma Rays. These are photons having energies higher than 100 T eV and wavelength shorter than 10^{-20} m. This leads to:

$t_{comp} = x_{comp} = T = \lambda = 10^{-20} m$, and

$z = \frac{H}{T} = 1{,}64113 \times 10^{-49}$m

Replacing in (2.4) and neglecting z, it obtains the magnitude of the compressed vector of an Ultra High Gamma Ray photon, **which represents also its Space-Time inaccuracy**:

$\left|\overrightarrow{OP}\right| comp = 1.4142x10^{-20} meter$

The diagram attached below illustrates the inaccuracy (grossly exaggerated) of the photon's position on its path at a certain travel time.

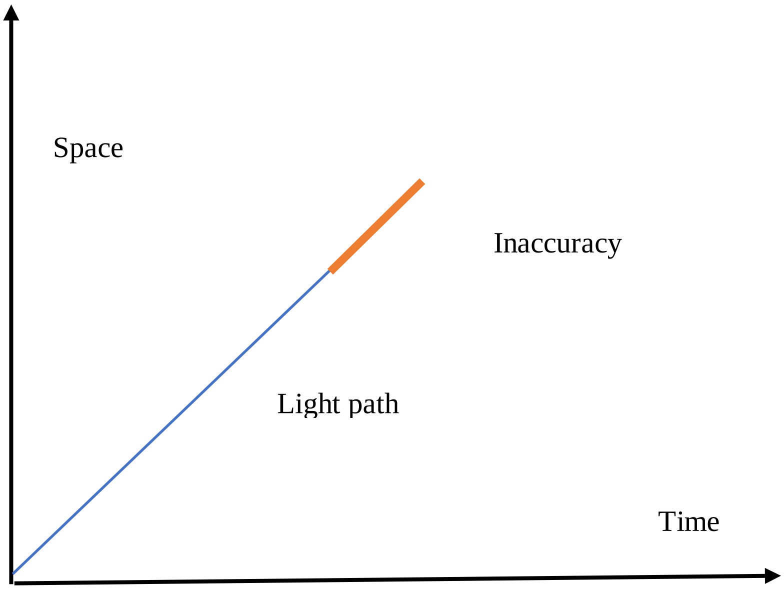

Figure 3. The Light Path and the inaccuracy of the photon's location

B. The magnitude of a compressed vector belonging to an arbitrary particle, and the corresponding matter wave

The H-number shown in (1.1) can also be written, as a sum of four light- affixes. For this purpose, we will use the bases numbers as defined in the reference paper [2], equation (2.61).

$$p = p(b + b_r) = \frac{1}{2}[p(1+k) + p(1-k)]$$

$$p = \frac{1}{2}[(t+iz)(1+k) + (t+iz)(1-k) + (x+iy)(1+k) - (x+iy)(1-k)] \quad (2.5)$$

The equation (2.5) can be rewritten as it follows:

$$p = \frac{1}{2}(p_{light1} + p_{light2} + p_{light3} + p_{light4}) \quad (2.6)$$

If the corresponding light vectors are in the compressed state, then the particle's vector can be also defined as compressed and it obtains:

$$p_{comp} = t_{comp} + iz_{comp} + jy_{comp} + kx_{comp} = \frac{1}{2}\sum_{N=1}^{4} p_{lightcompN}$$

After the processing it obtains:

$$p_{comp} = t_{comp} + iz_{comp} + jy_{comp} + kx_{comp} = \frac{H}{z} + iz + jy + k\frac{H}{y} \quad (2.7)$$

The formula (2.7) is essential and leads to some important derivations.

Starting from the light wave equation (2.3) it obtains the corresponding wave for any kind of particle:

$$Y = A\cos 2\pi\left(\frac{t}{t_{comp}} + \frac{x}{x_{comp}}\right) = A\cos\frac{2\pi}{H}(tz + xy) \quad (2.8)$$

The relation (2.8) represents de Broglie's matter wave as shown in the reference paper [2], paragraph 4.3.

The angular frequency and the wave number of this wave are:

$\omega = \frac{2\pi z}{H}$, and respectively $K = \frac{2\pi y}{H}$.

This wave equation is valid for any kind of particles, subluminal or superluminal (if exist). The mass-momentum relation of an ideal particle, takes the following forms (see reference [4], equations 2.11 and 2.22):

-for tardyons, $z^2 - y^2 = z_0^2$;

-for tachyons, $y^2 - z^2 = y_0^2$.

The parameter z_0 and y_0 are constant.

Doing the differentiation and computing, it obtains:

$$\frac{z}{y} = \frac{dy}{dz} \qquad (2.9)$$

The phase velocity and the group velocity of the wave are respectively:

$v_p = \frac{\omega}{K} = \frac{z}{y}$, and

$v_g = \frac{d\omega}{dK} = \frac{dz}{dy}$.

The formula (2.9) leads to the relation of wave's velocities:

$$v_g v_p = 1 \qquad (2.10)$$

III. Planck units and Heisenberg Uncertainty

A. The deduction of the Planck units

The magnitude of a compressed light vector shown in (2.4), is a function of T.

This function reaches a minimum value for:

$$T = \sqrt{H} = \sqrt{\frac{Gh}{c^3}} = L \qquad (3.1)$$

Consequently, the magnitude of a compressed $\overrightarrow{OP}$ vector representing **a light particle** in the physical world, is always bigger than 2L:

$$\left|\overrightarrow{OP}\right|\text{compressed} \geq 2L \qquad (3.2)$$

The expression (3.1) seems to be something déjà vu.

In his famous paper [7], published in 1899, Max Planck had been presented his revolutionary ideas concerning the natural measurement units**, but how these units were deduced was never specified by him**. They simply resulted from his genial intuition. Planck has not associated direct physical significations to his units.

The expression of the Planck length was given as $l_p = \sqrt{\frac{Gh}{c^3}}$, which is identical with the parameter L from the relation (3.1).

In order to obtain the minimum condition, parameters t_{comp}, z_{comp}, y_{comp} and x_{comp}, shown in the equation (2.2), must have all the same value L.

Thus, we can find the corresponding Planck units, and **elucidate their significations**, using the Table 2. The results are put together in the Table 3 below.

Table 3. Planck units in their "original" forms

Name	Parameter (m)	Expression (SI units)	Value
Planck Length	x =L	$l_p = \sqrt{\frac{hG}{c^3}}$	4.0511x10^{-35} m
Planck Time	t =L	$t_p = \sqrt{\frac{hG}{c^5}}$	1.3514x10^{-43} s
Planck Mass	z =L	$m_p = \sqrt{\frac{hc}{G}}$	5.4555x10^{-8} Kg
Planck Momentum	y =L	$p_p = \sqrt{\frac{hc^3}{G}}$	16.3535 Ns

The modern expressions of Planck units use the reduced Planck constant ħ instead of h.

$$\hbar = \frac{h}{2\pi} \qquad (3.3)$$

Consequently, these updated Planck units can be obtained dividing by a factor of $\sqrt{2\pi}$ the corresponding values shown in the Table 3. In

B. Heisenberg Uncertainty Relations

Classical physics is deterministic and assumes that all physical parameters have simultaneously exact values, which can be theoretically predicted and measured.

On the contrary, Heisenberg affirms, that for the case of simultaneous measurements of position and momentum of a particle, this is not true. Please see the following citation from his paper, shown as reference [8]: "…thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely". This is the original formulation of the first uncertainty relation. Using the notations introduced in the present paper, the corresponding mathematical relationship presented by Heisenberg can be written as it follows:

$$\Delta x \Delta y \sim H \qquad (3.4)$$

He also obtained a similar relation for time and energy (time and mass in our notation):

$$\Delta t \Delta z \sim H \qquad (3.5)$$

The symbol Δ has been not well defined and means a change in value of a certain variable.

Niels Bohr gave a different derivation of the above uncertainty relations, using De Broglie's matter waves hypothesis and his relations (see reference [9]).

According to Kennard, Heisenberg's relations were considered the core of the Copenhagen Interpretation of the quantum mechanics. Kennard gave also the exact mathematical formulation [10] of them:

$$\sigma_x \sigma_y \geq \frac{H}{4\pi}, \qquad (3.6)$$

where σ_x and σ_y are standard deviation of position, respectively standard deviation of momentum. Of course, the proper notation and the GU system were used above. The uncertainty relation for time and mass looks similarly.

We should be aware that Kennard formulation does not express the result of simultaneous measurements and their physical content. They are results of quantum mathematical model and only show consistency with the original Heisenberg's empirical relations.

C. Alternative Heisenberg's Uncertainty Relations; Quantization

The expression (2.7) leads to alternative Heisenberg's Uncertainty relations. Identifying expressions from the left side with those from the right side of the equality it obtains:

$$x_{comp} y_{comp} = H \qquad (3.7)$$

The expression above represents the Heisenberg's Uncertainty relation for position and momentum. The relation for time and mass is:

$$t_{comp} z_{comp} = H \qquad (3.8)$$

The uncertainty relations (3.7) and (3.8) lead to the conclusion that the measurements of position and momentum, respectively time and mass(energy) of a particle cannot be simultaneously performed, unless you tolerate a certain imprecision.

Let us consider the following two obvious inequality:

$$t_{comp}^2 + z_{comp}^2 \geq 2t_{comp}z_{comp} \geq 2H \geq 2L^2$$

$$y_{comp}^2 + x_{comp}^2 \geq 2x_{comp}y_{comp} \geq 2H \geq 2L^2$$

Adding them it obtains, after the calculation:

$$\left|\overrightarrow{OP}\right| compressed = \sqrt{t_{comp}^2 + z_{comp}^2 + y_{comp}^2 + x_{comp}^2} \geq 2L \qquad (3.9)$$

The formula (3.9) represents the extension of (3.2) to any kind of particles.

Therefore, the value $2L=8.1022\times10^{-35}$ m, represents the minimum minimorum of their vector's magnitudes defining the pixel of the Physical World and leading to the hypothesis of its quantization. Neglecting the compressed mass and momentum we define the pixel in spacetime as having the same value, i.e., 2L

Some scholars consider the Planck length as the shortest unit of length possible in spacetime (ST), due to the impossibility of observing anything smaller. Thus, the whole ST will be quantified in pixel on the scale of the Planck length, but this seems not to be accepted by the followers of conventional physics.

More information concerning this subject can be found in the reference paper [11].

References

[1] Vlad L. Negulescu, Hypernumbers and their applications in Mechanics, Romanian Journal of Physics, 42(1997) 3-4.

[2] Vlad L. Negulescu, Hyper-Complex Numbers in Physics, International Journal of Theoretical and Mathematical Physics, 5(2) (2015) 28-43, http://article.sapub.org/10.5923.j.ijtmp.20150502.03.html

[3] Vlad L. Negulescu, Addition of Velocities, Forces and Powers using Vector H-number Representation, International Journal of Theoretical and Mathematical Physics 7(3)(2017) 57-60, http://article.sapub.org/10.5923.j.ijtmp.20170703.03.html

[4] Vlad L. Negulescu, Motion analysis of particles using the hyper-complex numbers representation, Open Access Journal of Mathematical and Theoretical Physics, 2(1) (2019) 15-20, https://medcraveonline.com/OAJMTP/OAJMTP-02-00047.pdf

[5] Geometrized Units System, http://en.wikipedia.org/wiki/Geometrized_unit_system

[6] Chinese Academy of Science, The Highest-energy gamma rays discovered by the Tibet ASgamma experiment, https://phys.org/news/2019-07-highest-energy-gamma-rays-tibet-asgamma.html

[7] Max Planck 1899 Natürliche Maßeinheiten (Der Königlich Preußischen Akademie der Wissenschaften) p 479

[8] Werner Heisenberg, Über den anschaulichen Inhalt der quantentheoretischen Kinematik and Mechanik, Zeitschrift für Physik, 43(1927) 172–198.

[9] Niels Bohr, The Quantum postulate and the recent development of atomic theory, Nature, (Supplement), 121 (1928) 580–590.

[10] E.H. Kennard, Zur Quantenmechanik einfacher Bewegungstypen, Zeitschrift für Physik, 44(1927)326–352.

[11] Alex Klotz, (2015-09-09). A Hand-Wavy Discussion of the Planck Length. Physics Forums Insights.