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Original Article

Some Studies on the Validation of the Revised Second Law of Black Hole Thermodynamics

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Abstract* — Four different laws of black hole thermodynamics are named Zeroth Law, First Law, Second Law and Third Lawl. As quantum effect violates Hawking's Area Theorem and thereby the Second Law, we needed to revise the second law, which emerged as the Revised Second Law of Black Hole Thermodynamics. This includes into consideration the entropy due to thermal radiation or quantum radiation, usually called Hawking Radiation surrounding the black Hole. Including this component of entropy, it is possible to validate the non-decreasing entropy stated by the Second Law. Here we are establishing that even the unrevised or original version of the Second Law of Black Hole Thermodynamics can be validated for some giant or supergiant black holes. For an abnormally massive black hole, when its huge gravity performs a huge amount of surrounding matter accretion, the rate of matter accretion exceeds a certain critical value. For those black holes, the loss of Mass due to Hawking Radiation cannot balance the increase of Mass due to matter accretion. This results in a continuous gain of Mass by the deposit of accreted Mass in the black holes, resulting in an increase of the Mass and thereby the area of the Black Hole. As the area is directly proportional to entropy, the entropy of the black Hole continuously increases. This is only the entropy due to the geometric structure of the Black Hole. Thus for some giant or supergiant black holes where the rate of surrounding matter accretion dominates over quantum effect thermal evaporation, the original or unrevised Second Law also gets validated – without adding the entropy due to thermal radiation. So from small black holes up to some medium-sized ones, where the rate of matter accretion is below a critical value, we only need to revise the Second Law by adding the entropy due to quantum effect. On the contrary, if the size of the black Hole is huge enough such that the rate of matter accretion is beyond a certain critical value, even the original, unrevised Second Law of non-decreasing entropy due to the geometric structure of the black Hole is validated.

Keywords — *Entropy, Black hole, Black hole thermodynamics, Primordial black holes, Giant and supergiant black holes, Quantum effects, Particle antiparticle pair, Hawking radiation, Matter accretion.*

I. INTRODUCTION

One of the latest wizards or puzzles of astronomy and space science is the Black Hole [1,2,3,4]. They have tremendous importance in the galaxy [5,6]. Astronomers and astrophysicists are nowadays deeply concentrating on this quite uncanny phenomenon since it was predicted in 1969 by the American scientist John Wheeler. Possibly the first observed that the classical non-decreasing theory of entropy is violated for a black hole when it accretes or swallows surrounding hot Mass containing entropy [1,2,3,4,7]. One observer outside the black hole observes the total entropy of the combination gets decreased instead of being increased or remaining the same as before.

It was Barden et al. in the year 1973 who presented fundamental four black hole physics laws conventionally termed as four laws of black hole thermodynamics [1,2,3,7]. Formally these are (a) Zeorth law, (b) First law, (c) Second law and finally (d) Third law.

Zeroth Law presents us ever constant and uniform surface gravity on the event horizon surface of any black hole.

The first law states when matter accretion takes place within a black hole, the way accreted matter would be used up and distributed to change the entropy, electrical charge, angular momentum as well as finally total mass change due to change in immovable Mass scattered at the outskirt of the black Hole.

Classical Second Law states that for a black hole, entropy (which is due to the geometric structure of black Hole), as well as the event horizon surface area, never decreases. But also, we know 'quantum effect' invalidates the second law of non-decreasing entropy. With quantum effect, the antiparticle of the particle-antiparticle pair formed near the event horizon rushes away from the event horizon by absorbing energy from the black Hole, thereby decreasing the Mass and entropy of the Black Hole [4].

Generalized Second law: According to Hawking's area theorem, entropy 'S' is directly proportional to the surface area A of the event horizon of the Black Hole. So there is an apparent violation of this theorem since the entropy decreases while the surface has to increase with matter accretion. This decrease in entropy is solely due to the 'quantum effect', which causes a decrease of Mass due to 'black hole evaporation. This thereby also contradicts the Second Law, which talks about the non-decreasing entropy of the Black Hole. Thus we have to introduce the concept of generalized entropy, and the second law of black hole thermodynamics is also thereby revised as the generalized second law of black hole thermodynamics. In this version of 'generalized entropy,' the total entropy is comprised of two parts such as a contribution of the geometric structure of the black Hole to the entropy and contribution of thermal radiation and scattered matter at the outskirts or border of the black Hole to the entropy This has so far been well accepted as the best method to prevent the violation of second law of black hole thermodynamics and Hawking's area theorem. With this, the increase in the area of the black hole due to some form of matter accretion or other keeps desired resemblance with the proportionate increase in entropy.

Even if we strictly object that the black hole outskirt is a part of the Black Hole itself, then we have to consider that the thermal radiation entropy just outside the black Hole is also not a part of the black hole entropy. Thus there is a definite violation of the Second Law.

There is one way to still validate the Second Law. In [1,2], we presented the concept of Reverse Entropy within the event horizon of any black hole. The behaviour of this Reverse Entropy is just opposite to that of the conventional entropy. Matter association and contraction of the universe make this entropy increase instead of decrease, unlike 'Forward entropy' of simply called entropy. We are aware that the entropy increases with matter dissociation and universe expansion.

The universe is an ever contracting one inside the event horizon. The abnormally huge gravity of the singularity forces matter association towards it. There is thus definitely an increase of this reverse entropy when the particle of particle-antiparticle pair created due to quantum effect falls inside the event horizon. It is also true that the forward entropy, conventionally called entropy (due to the geometric structure of the black Hole), is bound to decrease due to the quantum effect. Thus the total entropy is neither increasing nor decreasing, thus validating the non-decreasing entropy and thereby Second Law of Black Hole Thermodynamics.

The third law of black hole thermodynamics, according to Bardeen et al. (1973), is: Blackhole temperature can never be reduced to zero by any procedure, by a finite operation sequence.

Another version presented by Plank states that as temperature equals zero, the corresponding entropy tends to near zero, which is an absolute constant.

II. ENTROPY

Unavailable thermal energy per unit temperature in a system for doing some useful and fruitful work is the definition of entropy in thermodynamics.

A measure of the molecular randomness or disorder of any system is the entropy since the work is only available from the ordered motion of the molecules.

The more is the system's information content. The lesser is the entropy. This is because of the fact that the ordering or association is a measure of the total amount of information in the system.

According to information theory, the total number of different classes or categories in a dataset, i.e. the total amount of disorder or variation in the dataset, is the measure of entropy.

It is worth mentioning that both the Thermodynamic definition of entropy and the Information-Theoretic definition of entropy are conceptually identical or the same.

In this case, all the records of a dataset belong to the identical Class of category. The entropy of that dataset is as low as zero.

On the other hand, in this case, if different records are uniformly distributed among the different classes, and the number of Classes is very large, then the entropy of the dataset is maximum.

According to information theory, we can mathematically formulate the entropy as below:

D = Dataset
M = Total number of a different class of records in Dataset D
N = Total number of records in Dataset D
I = I th Class
i = no. of records in i th Class

The probability of the 'i'th Class is:

$$p_i = \frac{i}{n} \ldots\ldots\ldots\ldots (1)$$

Suppose that M different record categories have probabilities as below:

p1, p2, p3 ……………pm.

The entropy of the dataset is defined as:

$$H(D) = -\sum_{i=1,2,\ldots\ldots m} pi * log pi \ldots\ldots\ldots\ldots (2)$$

[We assume

$$0 \log 0 = 0 \ldots\ldots\ldots\ldots (3)$$]

When the universe was just formed, i.e. during the 'Big Bang', there was one and only one category whereby the entropy of the universe was zero

With the advance of time, the same 'Big Bang' universe is continuously broken down into different terrestrial bodies, and the number of different classes or categories or variations are continuously increasing. This explains the continuous incremental increase of entropy of our universe.

As already told earlier, it is obvious that both the thermodynamic definition of entropy and the definition of entropy from the angle of view of information theory are conceptually equivalent as both of them measure the amount of some form of disorder or other.

III. THERMODYNAMICS OF BLACK HOLE

Black Hole behaves just as hot body and radiates. In the year 1975, the very basic concept of Black Hole thermodynamics [1,2,3,7] was established and presented by Bekenstein and Hawking. If we assume the analogy between hot body radiation and that of a stationary black hole, then according to them:

$$S_H = \frac{A_H}{4l_{Pl}^2} \quad \dots\dots (4)$$

Where

$$A_H = 4\pi r_s^2 \quad \dots\dots (5)$$

is the surface area of the event horizon of the Black Hole.

This clearly indicates that the entropy due to the geometric structure of the black hole is directly proportional to the Black Hole's event horizon surface area.

IV. HAWKING RADIATION

Everywhere in this universe, a particle-antiparticle pair is formed and instantaneously combined together. But when the particle-antiparticle formation is near a black hole, the situation is something completely different. Due to the huge gravity of the Black Hole, the particle is dragged down towards the black hole, while the antiparticle rushes away from the black hole due to the same huge gravity. (gravity applies a repulsive force to antiparticle). This is valid for both stationary and rotating black holes. While in the case of any rotating black hole, the dragged particle rotates with the black Hole falling inside it, in the case of a stationary black Hole, it is simply dragged down inside the Black Hole. Anyway, there is no chance of the particle-antiparticle pair getting combined together. Energy is absorbed by both particles and antiparticles. All the antiparticles surrounding the black Hole, after absorbing energy from the black Hole and rushing away from it, create radiation which is called Hawking Radiation. This radiation is created near a black hole due to the quantum effect by the creation of particle-antiparticle pair, and thus, it is also called quantum radiation of a black hole. This radiation is basically thermal in nature. So it is also sometimes called the thermal radiation of a black hole. This thermal radiation creates some chaotic disorder entropy at the outskirt or periphery of the black Hole.

V. FOUR LAWS OF BLACK HOLE THERMODYNAMICS

Below we are formally presenting the Four Laws governing Black Hole Thermodynamics [1,2,3,7].

Only three parameters namely as below:

Mass = M

Angular Momentum = 'J'

Electrical Charge = 'Q'

Describes the stable state of the black Hole at equilibrium when it completes its relaxation process.

A. Zeroth Law

Everywhere on the surface of the event horizon, the surface gravity κ, angular velocity Ω and the electric potential φ of a black hole at equilibrium are ever constant. This implies if the different parts of the same black Hole are at different temperatures, then the black Hole can never attain the state of its equilibrium.

B. First Law

In the case due to mass accretion, mass change of a black hole when it moves from one stable state to another is given by:

$$dM = \theta\, dS_H + \Omega\, dJ + \varphi\, dQ + \delta q \quad \dots\dots\dots\dots (6)$$

dM = Amount of accreted Mass in the black Hole. This causes the change in entropy, angular momentum, electrical charge and finally, the physical change in the total Mass by matter scattering at the periphery.

dS_H = Change in entropy S_H (which corresponds to geometric structure) of the Black Hole

dJ = Change in angular momentum J in case of a rotating black hole.

dQ = Change in electrical charge Q in the case of an electrically charged black hole.

δq = Effective increase in total Mass due to increase in scattered Mass at the outskirt of the black Hole.

A nice analogy between general thermodynamics of physics and black hole thermodynamics of astrophysics exists as below:

Ɵ acts the role of the temperature of the black hole while Ω and φ represent the angular velocity and electric potential of that black Hole.

C. Second Law

The classical process never allows any black hole A and its entropy to decrease in any way. This means:

$$\delta S_H \geq 0 \; \ldots\ldots\ldots\ldots (7)$$

This non-decreasing nature of S_H simply concludes that it is completely impossible for anybody to extract any information about the black hole inside a structure.

a) Generalized Second Law

According to the well-known Hawking's Area Theorem [3,4], entropy S_H is directly proportional to the black hole surface area A.

$$S_H = k\,A \ldots\ldots\ldots\ldots (8)$$

[k is the proportionality constant]

Now the applicability of the Hawking area theorem is violated by quantum effects creating quantum evaporation [3,4]. This quantum evaporation reduces the Mass as well as the black hole area. Thus, S_H also decrease. This clearly violates the non-decreasing S_H given by equation (7).

Now we are also aware that the nature of Hawking Radiation [4] is thermal, which is called thermal or quantum radiation of the Black Hole. As the black hole evaporates due to radiation, there is an accompanying increase in entropy of the matter just at the outskirt of the core black hole. In this case, the black Hole is globally visualized along with its surrounding space. Thus the total entropy of the entire system is equal to the summation of

(a) The core black hole entropy S_H

(b) the thermally radiating matter's entropy at the outskirt of the black Hole symbolizes S_M.

$$S = S_H + S_M \ldots\ldots\ldots\ldots (9)$$

This can never decrease.

This emerges the idea of so-called 'Generalized Entropy' given by the below formula:

$$S = S_H + S_M \geq 0 \ldots\ldots\ldots\ldots (10)$$

The corresponding Second Law is formally termed in the black Hole thermodynamics as **'Generalized Second Law',** also sometimes called **'Revised Second Law'.**

Thus as the area of the black hole 'A' decreases with quantum effect, S_H decreases, and there is a corresponding rise of S_M as a result of thermal radiation, thereby making the total area A and S ('generalized entropy') always non decreasing.

Thus the Second Law is only valid if the sum of the black hole entropy S_H plus the entropy of the matter outside the black Hole (due to thermal radiation) S_M is ≥ 0

The most interesting feature of this equation is that it combines two qualitatively completely different quantities, namely the core black hole entropy (a geometric quantity) with the entropy of thermal radiation at the outskirt of the black Hole (an information-theoretic quantity).

Also, there is always a chance that the thermal radiation at the outskirt of the black Hole is insufficient (for some giant or supergiant black holes) to increase the value of S_M to the desired extent, such as to invalidate the Second Law as well as Revised Second Law.

D. Third Law

We can present the third law in different versions, although they are conceptually identical.

Version 1: At the temperature $\Theta \rightarrow 0$, the entropy of a system approaches an absolute constant zero.

Version 2: Even for a maximally idealized procedure, it is quite impossible to decrease the black hole temperature to zero by any finite sequence of a different number of operations [Bardeen et al. (1973)].

VI. INFORMATION PARADOX

According to Information Theory, 'Information' can neither be created nor destroyed. Similarly, energy can neither be created nor destroyed.

Entropy is the amount of unusable information in the system or equivalently the amount of unusable energy in the system.

When is a Black Hole first created, whence the unusable information or energy is entrapped in the Black Hole?

Similarly, when a Black Hole is completely evaporated due to quantum effect thermal radiation, where the entrapped information or energy goes?

The above two are called Information Paradox.

The answer lies in the Hawking Radiation or quantum effect radiation of black Hole.

When a Black Hole radiates, the entrapped unusable information / thermal energy is released in the form of thermal radiation.

VII. MAXIMUM AND MINIMUM MASS CONVERSION INTO THERMAL RADIATION DUE TO QUANTUM EFFECT

Suppose there are two Schwarzschild Black holes of identical Mass M colliding and merging together so that matter accretion takes place in either of the black Hole.

Also, suppose the initial entropy of the two black holes were **S,** and the final entropy of the resultant black Hole after merging is s'. Also, suppose the Mass of the resultant black Hole is **M'.**

According to Generalized Second Law:

$$S' \geq 2S$$

Since entropy is directly proportional to the area of the event horizon of the Black Hole:

$$A' \geq 2A$$

Let the event horizon radius of the initial two black holes be **Rs** and the event horizon radius of the resultant black Hole after merging **Rs'**

Thus

$$A' = 4\pi R_s'^2 = 16\pi G^2 M'^2/c^4 \;\ldots\ldots\ldots.\ldots.(11)$$

Also

$$A = 4\pi R_s^2 = 16\pi G^2 M^2/c^4 \;\ldots\ldots\ldots.\ldots.(12)$$

[Event horizon radius R_S of any black hole of mass M is

$$Rs = \frac{2GM}{c^2} \ldots\ldots\ldots\ldots..(13)$$]

Thus from equations (11) and (12)

$$M'^2 \geq 2\,M^2$$

And thus:

$$M' \geq \sqrt{2}\,M$$

Thus the Mass of the resultant black Hole will be greater than or equal to $\sqrt{2}$ times the Mass of the individual black holes. This also implies that the Mass of the resultant black hole will be at least equal to

$$\sqrt{2}\,M$$

It is equivalent to saying that each original black Hole of original mass M will now have a reduced mass of at least equal to

$$M/\sqrt{2}$$

The total mass of the combination is lying in the closed interval

$$2M \geq M' \geq \sqrt{2}\,M$$

We assumed that the two merged Black Holes are Schwarzschild Black Hole (i.e. Stationary Non-Rotating and Uncharged Black holes). In such a case, the maximum quantum effect of thermal radiation (Hawking Radiation) in terms of equivalent Mass is given by the following:

$$\frac{(2M - \sqrt{2}\,M)}{2} = M\,(1 - \frac{1}{\sqrt{2}})$$

While the minimum amount of conversion of Mass into thermal radiation is zero, in this case, the resultant merged Mass is equal to 2M.

VIII. VALIDATION AND VIOLATION OF SECOND LAW OF BLACK HOLE THERMODYNAMICS

Let us denote

M = Mass of the Black Hole

R = Amount of Quantum Effect Thermal Radiation

$$\partial M/\partial t$$

= rate of increase of mass of the black hole due to surrounding matter accretion.

$$\left(\frac{dM}{dt}\right) critical$$

= Critical Rate of change of Mass of the black Hole due to surrounding mass accretion.

$\partial R/\partial t$ = rate of increase of quantum effect, i.e. Hawking Radiation due to matter accretion.

$\partial S_H/\partial t$ = rate of increase of geometric structure entropy of the Black Hole.

$(\partial S_H/\partial t)\ mass\ accretion$ = rate of increase of geometric structure entropy due to mass accretion.

$(\partial S_H/\partial t)\ quantume\ effect$ = rate of decrease of geometric structure entropy due to quantum effect. This is a negative quantity.

$\partial S_M/\partial t$ = rate of increase of quantum effect entropy of the Black Hole.

In the case if:

$$\left(\frac{\partial S_H}{\partial t}\right) mass\ acretion \; + \; \left(\frac{\partial S_H}{\partial t}\right) quantum\ effect \;\; \geq 0$$

There is a deposit of accreted Mass in the black Hole; The S_H part of the entropy is itself increasing or at least non decreasing even with quantum evaporation.

Here we don't need Revised Second Law to validate non-decreasing entropy.

On the other hand, if:

$$\left(\frac{\partial S_H}{\partial t}\right) mass\ acretion \ + \left(\frac{\partial S_H}{\partial t}\right) quantum\ effect \ < 0$$

Then there is a loss of Mass due to quantum evaporation. Here S_H part of the entropy is decreasing. We need Revised Second Law to validate non-decreasing entropy.

Case – 1

Black Hole is of insignificant Mass (e.g. Primordial Black Holes formed during the creation of our universe during 'Big-Bang).

For those Black Holes: Rate of increase of Mass of the black Hole due to surrounding matter accretion

$$\partial M/\partial t = 0$$

This is only because of the fact that, for those black holes, there is no matter accretion from the surroundings due to their small mass and gravity. For those black holes evidently:

$$\left(\frac{\partial S_H}{\partial t}\right) mass\ acretion \ < \ \left(\frac{\partial S_H}{\partial t}\right) quantum\ effect$$

In this case, due to uniform quantum effect radiation, the black Hole will ultimately evaporate to vanish itself.

Since the rate of quantum effect radiation

$$\partial R/\partial t > 0$$

Which makes:

$$\left(\frac{\partial S_M}{\partial t}\right) quantum\ radiation \ > 0$$

The black Hole will continuously lose its own Mass due to thermal radiation. Eventually, its area will decrease. So the part of the entropy due to the geometric structure of the black Hole, i.e. $S_{H,}$ will continuously get decreased. Thus the original Second Law of Black Hole Thermodynamics gets violated for these types of small black holes. The Revised Second Law only validates non-decreasing entropy.

Case – II

Black Holes are of significant Mass, such that there is matter accretion and the rate of matter accretion is non zero i.e.

And

$$\left(\frac{\partial M}{\partial t}\right) critical \ > \left(\frac{\partial M}{\partial t}\right) > 0$$

Such that:

$$\left(\frac{\partial S_H}{\partial t}\right) mass\ acretion \ + \left(\frac{\partial S_H}{\partial t}\right) quantum\ effect < 0$$

Here we suppose that there is a critical rate of matter accretion for some giant and/or supergiant black holes denoted by

$$\left(\frac{\partial M}{\partial t}\right) critical$$

For those black holes, the Rate of Increase of Mass lies in the open interval of zero and the critical rate. This rate is less than the critical rate, while it is greater than zero. Evidently, for those black holes, any amount of surrounding matter accretion takes place – depending on its Mass – huge or small. Quantum effect decrease of S_H is more than the mass accretion increase of $S_{H.}$ For them, the original Second Law of non-decreasing entropy is violated. We need the Revised Second Law to validate non-decreasing entropy. Those black holes eventually evaporate to ultimately vanish themselves.

This increase in the rate of quantum effect radiation, in its turn, decreases the mass of the Black Hole with quantum effect radiation and evaporation. As the quantum effect radiation rate is further increased, the total Mass (accumulated Mass from surroundings plus the original Mass of the Black Hole) is under quantum evaporation, and this only makes the black Hole ultimately evaporate and vanish.

If the rate of increase of the Mass for those black holes is too low – close to zero, then the rate of quantum effect radiation

$$\partial R/\partial t$$

dominates over

$$\partial M/\partial t$$

Then the black Hole ultimately very quickly and rapidly evaporates to vanish itself.

On the contrary, if the Rate of Increase of Mass

$$\partial M/\partial t$$

is close to the critical rate of mass increase

$$\left(\frac{\partial M}{\partial t}\right) critical$$

Then

$$\partial M/\partial t$$

dominates over

$\partial R/\partial t$

Definitely, there is a quantum radiation, but in such a case, the radiation would take much more time to make the black Hole ultimately vanish away.

Case – III

In this case, the Mass of the black Hole is substantially larger (say medium-sized to giant or supergiant black holes); due to their huge gravity, the rate of surrounding matter accretion

$\partial M/\partial t$

It is also abnormally huge.

For those black holes

$$\left(\frac{\partial M}{\partial t}\right) \geq \left(\frac{\partial M}{\partial t}\right) critical$$

Such that

$$\left(\frac{\partial S_H}{\partial t}\right) mass\ acretion \quad + \quad \left(\frac{\partial S_H}{\partial t}\right) quantum\ effect \ \geq\ 0$$

For those black holes, the increase of the S_H part of the entropy due to mass accretion is far more than the decrease of the S_H part of the entropy due to quantum evaporation. There is a deposit of Mass due to matter accretion even with the effect of quantum evaporation.

Now due to the huge value of

$\partial M/\partial t$

the rate of quantum radiation

$\partial R/\partial t$

is also heavily increased.

But in this case, since the rate of increase of Mass is huge, the rate of increase of quantum effect radiation cannot adjust, i.e. cope with the situation. The quantum effect loss of Mass is much less than the deposit of Mass due to matter accretion.

In such a situation, the Mass and thereby the area of the black Hole continuously increases. This is bound to increase the sum total or overall entropy due to the geometric structure of the black Hole, i.e. S_H. Thus for those black holes, $S_H > 0$.

Thus for those black holes, even the original (unrevised) Second Law of Black Hole Thermodynamics of non-decreasing entropy is validated.

IX. CONCLUSION

Quantum effect violates Hawking's Area Theorem and thereby Second law of Black Hole Thermodynamics of non-decreasing entropy. An intelligent revision of the Second Law - formally called Revised Second Law of Black Hole Thermodynamics, which includes into consideration the thermal radiation or Hawking Radiation entropy, is able to validate the Second Law. Although it combines two qualitatively different types of entropy, namely entropy due to the geometric structure of the black Hole and the entropy due to quantum or thermal radiation at the outskirt of the black Hole, it validates non-decreasing entropy for black holes. In this paper, we introduce a new concept of the critical rate of matter accretion for any black hole. With this, it is possible to validate the non-decreasing entropy of the original or unrevised Second Law for some giant or supergiant black holes. When the Mass of the black hole is huge, the rate of matter accretion exceeds this critical rate. There the rate of matter accretion dominates over the rate of quantum effect radiation of the black Hole. We logically established that even the entropy due to the geometric structure of those black holes is either increasing or at least non decreasing for those black holes, although there is still quantum evaporation. This establishes that the Original or Unrevised Second Law of Black Hole Thermodynamics is conditionally violated – not always.

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