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Original Article

An Approach of Damped Electrical and Mechanical Resonators

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Abstract - *The Dinesh Verma Transform analyzes the damped mechanical and electrical oscillators in this paper. This paper introduces the Dinesh Verma Transform as a new mathematical approach for analyzing the damped mechanical and electrical oscillators. The Dinesh Verma Transform provides a new mathematical tool for obtaining the responses of damped mechanical and electrical oscillators and reveals that it is also effective and simple, like other integral transforms and approaches.*

Keywords - *Damped Mechanical and Electrical Oscillators, Response, Dinesh Verma Transform.*

I. INTRODUCTION

The Dinesh Verma Transform (DVT) has been applied in different areas of science, engineering, and technology [1], [2], [3] [4], [5], [6], [7]. The Dinesh Verma Transform (DVT) is implemented in various fields and fruitfully solves linear differential equations. Via Dinesh Verma Transform (DVT), Ordinary linear differential equations with constant coefficient and variable coefficient and simultaneous differential equations can be easily resolved without finding complementary solutions. It also becomes a very effective tool for analyzing differential equations, Simultaneous differential equations, Integral equations, etc.

Mostly, the damped mechanical oscillator and the electrical oscillator are Studied by different integral transforms or Proceed towards, for example, the Convolution theorem approach [1, 2, 3], Laplace Transform [4, 5, 6], Mohand Transform [7, 8], Matrix method [9, 10, 11, 12,21,22], Residue theorem approach [13], Gupta transform [14], etc. The paper Studies the damped mechanical oscillator and the electrical oscillator (LCR circuit) by the new integral transform known as Dinesh Verma Transform. The author has proposed the 'Dinesh Verma Transform' in recent years, and it has been applied in studying initial value problems in most science and engineering disciplines [15, 16, 17, 18, 19, 20]. The paper's objective is to show the relevance of the Dinesh Verma Transform to find out the damped mechanical and electrical oscillators; it is also helpful and easy. The Dinesh Verma Transform of g(y), denoted by D{g(y)}, is defined [15, 16, 17, 18, 23, 24,25] as $D\{g(y)\} = r^5 \int_0^\infty e^{-ry} g(y)dy$, shows that the integral is convergent, where r may be a real or complex parameter. The Dinesh Verma Transform of some of the derivatives of a function is

$$D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$$

$$D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$$

$$D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0)$$ And so on.

$$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp},$$

$$D\{tf'(t)\} = \frac{5}{p}\left[p\bar{f}(p) - p^5 f(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5 f(0)\right]$$ and

$$D\{tf''(t)\} = \frac{5}{p}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right] - \frac{d}{dp}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right]$$ And so on.

DINESH VERMA TRANSFORM OF ELEMENTARY FUNCTIONS

According to the definition of Dinesh Verma transform (DVT),

$$D\{t^n\} = p^5 \int_0^\infty e^{-pt} t^n dt$$

$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad , z = pt$$

$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z} (z)^n dz$$

Applying the definition of gamma function,

$$D\{y^n\} = \frac{p^5}{p^{n+1}} \Gamma(n+1)$$

$$= \frac{1}{p^{n-4}} n!$$

$$= \frac{n!}{p^{n-4}}$$

Hence, $D\{t^n\} = \frac{n!}{p^{n-4}}$

Dinesh Verma Transform (DVT) of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}$, $where\ n = 0,1,2, ..$
- $D\{e^{at}\} = \frac{p^5}{p-a}$,
- $D\{sinat\} = \frac{ap^5}{p^2+a^2}$,
- $D\{cosat\} = \frac{p^6}{p^2+a^2}$,
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2}$,
- $D\{coshat\} = \frac{p^6}{p^2-a^2}$.
- $D\{\delta(t)\} = p^4$

- **T**he Inverse Dinesh Verma Transform (DVT) of some of the functions is given by
- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}$, $where\ n = 0,1,2, ..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}$,
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat$,
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat,$
- $D^{-1}\{p^4\} = \delta(t)$

II. MATERIAL AND METHOD

A. Mechanical (Damped) Oscillator

The differential equation of the damped mechanical oscillator [8, 9, 14] is given by

$\ddot{x}(t) + 2b\dot{x}(t) + \sigma^2 x(t) = 0$ …. (1), where $2b = \frac{r}{m}$ represents the damping constant per unit mass, $\sigma = \sqrt{\frac{K}{m}}$ represents the natural frequency of the oscillator. For a lightly damped oscillator, $b < \sigma$.

The initial conditions [8, 9, 14] are as follows: If the time is measured from the instant when the oscillator crosses its mean position, then at t = 0, x (0) = 0. Also, at the instant t = 0^+, it is assumed that the velocity of the oscillator is maximum, i.e., $\dot{x}(0^+) = u_0$.

The Dinesh Verma Transform [15, 16, 17,] of (1) provides

$p^2\bar{x}(p) - p^6 x(0) - p^5\dot{x}(0) + 2a\{p\bar{x}(p) - p^3 x(0)\} + \omega^2\bar{x}(p) = 0$…. (2)

Here $\bar{x}(p)$ denotes the Dinesh Verma Transform of x(t).

Applying boundary conditions $x(0) = 0$ and $\dot{x}(0) = u$ and simplifying (2), we get

$$\bar{x}(p) = \frac{(u_0)p^5}{p^2 + 2b\,p + \sigma^2}$$

Or

$$\bar{x}(q) = \frac{(u_0)p^5}{(p + b_1)\,(p + b_2)}$$

$$where\ \alpha_1 = b + i\sqrt{\sigma^2 - b^2}\ and$$

$$\alpha_2 = b - i\sqrt{\sigma^2 - b^2} \text{ such that } \alpha_1 - \alpha_2 = 2i\sqrt{\sigma^2 - b^2}$$

Or

$$\bar{x}(p) = \frac{(u_0)p^5}{(\alpha_2-\alpha_1)(p+\alpha_1)} - \frac{(u_0)p^5}{(\alpha_2-\alpha_1)(p+\alpha_2)}$$

Applying inverse Dinesh Verma Transform [15] and simplifying, we get

$x(t) = (u_0)\frac{[e^{-\alpha_1 t} - e^{-\alpha_2 t}]}{(\alpha_2-\alpha_1)}$ …….. (3)

Or

$$x(t) = u_0 e^{-bt}\,\frac{[e^{i\sqrt{\sigma^2-b^2}t} - e^{-i\sqrt{\sigma^2-b^2}t}]}{2i\sqrt{\sigma^2-b^2}}$$

Or

$x(t) = \frac{u_0 e^{-bt}}{\sqrt{\sigma^2-b^2}} \sin\sqrt{\sigma^2 - b^2}\,t$…. (4)

When $r = 0, b = \frac{r}{2m} = 0,$ then equation (5) reduces to

$x(t) = \frac{u_0}{\sigma} \sin \sigma t$….. (5)

Equation (4) verifies a lightly damped oscillator. It also shows that the oscillator's behavior is oscillatory, with the amplitude of oscillations diminished with time exponentially. Where the damping force is equivalent to zero, the amplitude of oscillations is constant [8, 9, 12,].

For an overdamped oscillator [10], $a > \omega,$ therefore, replacing $\sqrt{\sigma^2 - b^2}$by $i\sqrt{b^2 - \sigma^2}$ in (4), the displacement of an overdamped oscillator is given by

$$x(t) = \frac{v_0 e^{-bt}}{i\sqrt{a^2-\sigma^2}} \sin i\sqrt{a^2 - \sigma^2}\,t$$

Or

$x(t) = \frac{v_0 e^{-at}}{\sqrt{a^2-\sigma^2}} \sinh\sqrt{a^2 - \sigma^2}\,t$ .... (6)

This equation (6) responds heavily damped oscillator and reveals that the motion of a heavily damped oscillator is non-oscillatory.

B. Electrical (Damped) Oscillator

The differential equation of the damped electrical oscillator (LRC circuit) [8, 14] is given by $\ddot{Q}(\mathrm{t}) + 2b\dot{\mathrm{Q}}(\mathrm{t}) + \omega^2\mathrm{Q}(\mathrm{t}) = 0$ …. (7), where $\sigma = \sqrt{\frac{1}{\mathrm{LC}}}$ represents the angular frequency of the electrical oscillator, $2b = \frac{\mathrm{R}}{\mathrm{L}}$ represents the damping coefficient. Q(t) is the instantaneous charge.

The initial conditions [14, 18] as follows:

(i) At t = 0, Q (0) = 0.

(ii) Also, at the instant t = 0^+, it is assumed that the current in the circuit is maximum, i.e., $\dot{\mathrm{Q}}(0^+) = i_0$.

The Dinesh Verma Transform [15, 16, 17,] of (7) provides

$p^2\overline{\mathrm{Q}}(\mathrm{p}) - p^6\mathrm{Q}(0) - p^5\dot{\mathrm{Q}}(0) + 2b\{p\overline{\mathrm{Q}}(\mathrm{p}) - p^5\mathrm{Q}(0)\} + \omega^2\overline{\mathrm{Q}}(\mathrm{p}) = 0$…. (8)

Here $\overline{\mathrm{Q}}$(q) denotes the Dinesh Verma transform of Q(t).

Applying initial conditions [12, 19] $\mathrm{Q}(0) = 0, \dot{\mathrm{Q}}(0) = i_0$ and simplifying (8), we get

$$\overline{\mathrm{Q}}(\mathrm{p}) = \frac{(i_0)p^5}{p^2 + 2b\,p + \sigma^2}$$

Or

$$\overline{\mathrm{Q}}(\mathrm{p}) = \frac{(i_0)p^5}{(q + b_1)\,(q + b_2)}$$

$$where\ \alpha_1 = b + \mathrm{i}\sqrt{\sigma^2 - b^2}\ and$$

$\alpha_2 = b - \mathrm{i}\sqrt{\sigma^2 - b^2}$ such that $\alpha_1 - \alpha_2 = 2i\sqrt{\sigma^2 - b^2}$.

Or

$$\bar{Q}(\mathrm{q}) = \frac{(i_0)p^5}{(\alpha_2-\alpha_1)(q+\alpha_1)} - \frac{(i_0)p^5}{(\alpha_2-\alpha_1)(q+\alpha_2)}$$

Applying inverse Dinesh Verma Transform [16], we get

$$\mathrm{Q}(t) = (i_0)\frac{[e^{-\alpha_1 t} - e^{-\alpha_2 t}]}{(\alpha_2-\alpha_1)}$$

Or

$$\mathrm{Q}(t) = i_0\,\frac{[e^{-\alpha_1 t} - e^{-\alpha_2 t}]}{(\alpha_2-\alpha_1)}$$

Or

$$\mathrm{Q}(t) = i_0 e^{-bt}\,\frac{[e^{\mathrm{i}\sqrt{\sigma^2-b^2}t} - e^{-\mathrm{i}\sqrt{\sigma^2-b^2}t}]}{2\mathrm{i}\sqrt{\sigma^2-b^2}}$$

Or

$$\mathrm{Q}(t) = \frac{i_0 e^{-bt}}{\sqrt{\omega^2-b^2}}\sin\sqrt{\sigma^2 - b^2}\,t \text{…. (9)}$$

When $\mathrm{R} = 0, a = \frac{\mathrm{R}}{2\mathrm{L}} = 0$, then equation (9) reduces to

$$\mathrm{Q}(t) = \frac{i_0}{\sigma}\sin\sigma t \text{…. (10)}$$

This equation (10) responds to the damped electrical oscillator. Also, it is found that the oscillator's behavior (charge) is oscillatory, with the amplitude of oscillations decreasing with time exponentially. The decrease in amplitude, i.e., damping, depends upon resistance R in the circuit. Such damping is called resistance damping [8, 14]. If R = 0, the amplitude would remain constant. Hence in the LRC circuit, the resistance is the only dissipative element.

III. CONCLUSION

This paper concludes that the damped mechanical and electrical oscillators have been Examined by applying the new integral transform 'Dinesh Verma Transform' and representing the Dinesh Verma Transform for analyzing the theory of damped mechanical, electrical oscillators. A new and different integral transform is introduced for getting the result of damped mechanical and electrical oscillators.

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