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Original Article

Thermobiological Mathematical Model for the Study of Temperature Response After Cooling Effects

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Abstract *- An thermobiological mathematical model is developed to study the recovery of skin tissue to the ambient temperature after the exposure of skin to cold temperature till the skin reaches externally applied temperature. This model is based on Pennes heat balance equation. The Laplace transform method and boundary condition of core temperature are used. The study shows the linear dependence of steady-state temperature on blood perfusion rate. It can be concluded that the skin starts to recover to the ambient temperature above 800 seconds. The recovery profile or temperature profile of skin is not constant but depends on various physical parameters of skin tissue. Therefore, the temperature profile is studied for different parameters like tissue thermal conductivity, perfusion rate of blood, and metabolic heat generation. It is found that skin temperature reaches ambient temperature soon if thermal conductivity & metabolic heat generation is high and blood perfusion is low. This study also finds the steady state temperature of the tissue, i.e., when too much time is passed, the temperature is calculated. It is found that the value of steady state temperature is different for different values of blood perfusion rate.*

Keywords *- Ambient temperature, Blood perfusion, Laplace transform method, Metabolic heat generation, Perfusion rate. Thermal conductivity.*

1. Introduction

Skin is the largest and most important organ of the body; it protects from harmful external conditions, provides sensations, and regulates body temperature. There are three main layers of the skin, dermis, epidermis, and hypodermis [4, 11, 15]. The thermal properties of skin play a significant role in obtaining the body's overall thermal balance. Energy reaches the skin through some transport mechanism: conduction, convection, or radiation. The degree to which this energy is transferred to raise the skin temperature is obtained by the behaviour of the various thermal properties of the skin. For example, radiation reaching the skin or transmitted through the skin layers. Reflection occurs not only at the skin surface but also from within due to the scattering of a portion of the transmitted radiation [1, 6, 8]. The internal scattering and consequent internal reflections are due to inhomogeneities in the skin, which are on the order of the wavelength of the incident radiation [3, 12, 18]. The net effect of this internal scattering is that the radiation or optical properties of the skin must be considered as a function not only of the surface conditions but also of the chemical composition and homogeneity of the various skin layers. Consequently, changes in the optical properties of the skin will mirror changes in the skin architecture. Similarly, the transfer of energy within the skin is controlled by the thermal conductivity, examining a central portion of the skin and assuming that in this portion, the skin is homogeneous and that heat flows steadily in one direction.

Hypothermia is an abnormally low temperature of the body. It is a medical emergency due to cold laser therapy [7, 9, 15].In this condition, the body loses heat faster than it can produce heat. In this pathological condition, tissues may be damaged [2, 5, 16]. Many cooling devices have been developed, and several methods have been integrated into laser systems to protect the skin layers, erythema reducing pain, and the efficacy of laser. There are two types of cooling, contact and non-contact cooling. Laser dermatologic surgery is very common nowadays. Many methods are available to minimize the damage caused by the temperature-induced epidermal process. For many years, cooling of the patient's body has been required for surgical procedures [10, 14, 19]. It is required to decrease the metabolic rate of the tissues due to less amount of consumption of oxygen. Dangerous damage is possible to any tissue due to less oxygen deficiency. The surgeon can have more time for any further complications during surgery.

In the cooling conditions, the skin temperature decreases and increases after removing the cooling effects. However, these parameters never return to their original initial values during the cooling period [11, 13, 17].In infrared thermography, the steady-state skin surface temperature is often used as a parameter for pathological conditions; for example, the temperature difference between the two supraorbital areas can indicate an obstruction in one of the arteriae carotisinternae. Sometimes the thermogram is not abnormal, although the angiograms show internal carotid stenosis [16, 18]. During the surface cooling process of the skin, the blood flow decreases, which is an important parameter in the application of vasoconstriction. The fundamental function of the skin blood flow is the metabolic process of the tissues and skin cells. Also, maintain the heat transfer between the outer and body core temperatures. One important factor of surface cooling is to insulate the body core from the outer temperature, which only can be done by decreasing the magnitude of the SBF. In this study, it has been decided to use a mathematical model of the heat flow in the skin to achieve a better understanding of the temperature profile and to find a way of improving the existing methods.

1.1. Formulation of the problem

In this work, a mathematical model has been obtained, which is dimensional, involving a uniform slab of tissue [4]. Here the body core temperature has been considered at 37 ^{0}C (figure. 1), and the temperature in the cooling environment for the skin surface has been considered to be 20 ^{0}C [8].

The mathematical differential equation used for the temperature is as below,

$$\lambda\frac{\partial^2 T}{\partial x^2} + \dot{m}c_B(T_c - T') + Q_0 = \rho c\frac{\partial T}{\partial t} \tag{1}$$

where T is the temperature in tissue, and it varies with distance x and time t.

m = blood perfusion

c_b= specific heat of blood

ρ = density of the tissue

h = thermal conductivity

T_c= core temperature

T_a = temperature of the capillary bed

Q_0 = metabolic heat generation

c = specific heat of the tissue

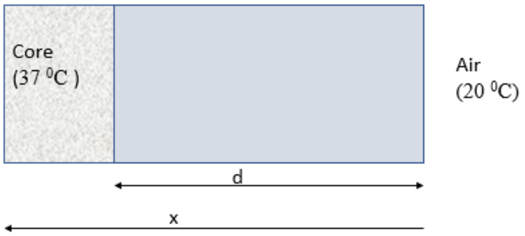

Fig. 1 One-dimensional model of skin tissue

The above equation represents the linear heat flow in a semi-infinite solid that generates heat.

1.2. Solution of the problem

Heat production is due to metabolic processes (Q_0). By using,$T' = T$in equation (1), and solving, we get, [6],

$$\left|\lambda\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = h[T(0,t) - T_a]. \tag{2}$$

T_a is ambient temperature, and h is heat transfer coefficient.

By assuming,

$$T(x,t) = 37 \tag{3}$$

$$\theta = T - T_0 \tag{4}$$

$$\beta = \frac{\dot{m}c_B}{\rho c} \tag{5}$$

$$\gamma = \frac{Q_0}{\rho c} + \beta\theta_c \tag{6}$$

which renders equation (1) much simpler:

$$\frac{\partial\theta}{\partial t} = a\frac{\partial^2\theta}{\partial x^2} - \beta\theta + \gamma \tag{7}$$

with the boundary conditions:

$$\left|\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = h'[\theta(0,t) - \theta_a]. \tag{8}$$

$$\theta(d,t) = 17. \tag{9}$$

In which $a = \lambda/\rho c$ is the thermal diffusivity,$h' = h/\lambda(m^{-1})$. Equation (7) is obtained and gets the solution of it by applying the Laplace transformation method, and then the inverse Laplace transform method has applied to get the solution of it,

$$\theta(x,t) = \theta(x,0) + A\frac{\sinh\alpha(d-x)}{\alpha\cosh\alpha d + h'\sinh\alpha d} - 2aA\sum_{n=1}^{\infty}\frac{\exp(-\mu_n t)p_n^2\sin p_n\left[1-\left(\frac{x}{d}\right)\right]}{\mu_n d[p_n^2 + h'd(1+h'a)]\sin p_n}\theta(x,t) \tag{10}$$

$$A = \frac{\alpha[\theta_c + \left(\frac{\gamma}{\beta}\right)(\cosh\alpha d - 1]}{\sinh\alpha d} + h'\theta_a \tag{11}$$

$$\mu_n = \beta + \frac{ap_n^2}{d^2},\ \beta \neq 0 \tag{12}$$

$$\tan p_n = -\frac{p_n}{h'd} \tag{13}$$

$$\alpha = \frac{1}{m}\sqrt{\frac{\beta}{a}} \tag{14}$$

when t=0, the second term of equation (10) can be written as

$$\frac{\sinh \alpha(d-x)}{\alpha \cosh \alpha d + h'^{\sinh \alpha d}} = a\sum_{n=1}^{\infty} \frac{p_n^2 \sin p_n[1-\left(\frac{x}{d}\right)]}{\mu_n d[p_n^2+h'd(1+h'a)] \sin p_n} \quad (15)$$

and equation (10) may be represented by t The below equation,

$$\theta(x,t) = \theta(x,0) + 2aA\sum_{n=1}^{\infty} \frac{[1-\exp(-\mu_n t)]p_n^2 \sin p_n[1-\left(\frac{x}{d}\right)]}{\mu_n d[p_n^2+h'd(1+h'a)] \sin p_n} \quad (16)$$

by taking x = 0 [14, 20], for surface temperature, we get,

$$\theta(0,t) = 2aA\sum_{n=1}^{\infty} \frac{[1-\exp(-\mu_n t)]p_n^2}{\mu_n d[p_n^2+h'd(1+h'a)]} \quad (17)$$

by using equation no. (10), and putting t→ ∞

$$\theta(0,\infty) = A\frac{\sinh \alpha d}{\alpha \cosh \alpha d + h' \sinh \alpha d} \quad (18)$$

when Q_0= 0 and T_a= To, [6] equations (11) and (18) we get,

$$\theta(0,\infty) = \frac{\theta_c}{1+\frac{h' \sinh \alpha d}{\alpha \cosh \alpha d}} \quad (19)$$

$$T_m = \theta(0,\infty) + T_0 \quad (20)$$

2. Results and Discussion

Fig. 1 shows the skin temperature with time with a variation in the thermal conductivity of the tissue. It is seen that temperature recovery is inversely proportional to thermal conductivity [8, 11]. Fig. 2 shows the skin's temperature with time with various blood perfusion rates, i.e., blood flow rate into tissue. The plot shows a proportional relation between recovery temperature and perfusion rate [10]. Fig. 3 is a plot of recovery temperature with metabolic heat generation. This figure gives an important result the very high value can bring changes in recovery. Values of the same order will have overlapping recovery profiles [9].

Physical parameters	**Value**
Thermal conductivity of tissue	0.32 $Wm^{-1}K^{-1}$
Specific heat of blood	$3.8*10^3$ J Kg^{-1} m^3
Temperature of core	310 K
Blood perfusion rate	0.5 Kg m^3 s^{-1}
Density of tissue	$1.2*10^3$ Kg m^3
Metabolic heat generation	840 W m^{-3}
Thickness of tissue	10^{-2}
Heat transfer coefficient	12 W m^2 K^{-1}
Temperature of artery	310 K

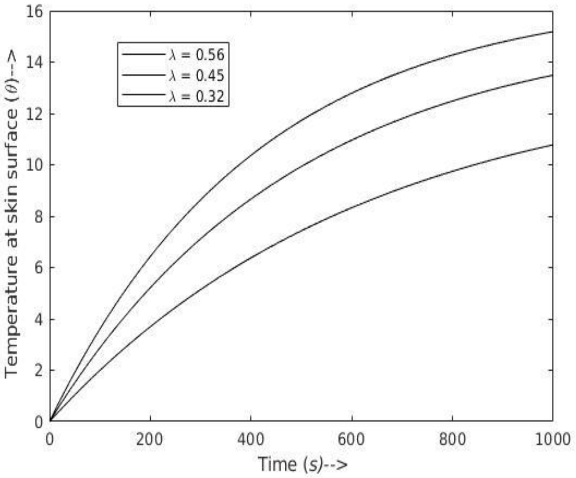

Fig. 2 Temperature at the surface of the skin with time for different values of thermal conductivity of tissue

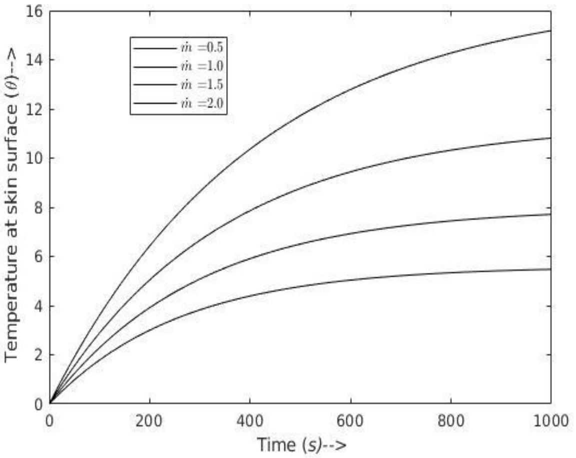

Fig. 3 Temperature at the surface of the skin over time for different values of perfusion of blood

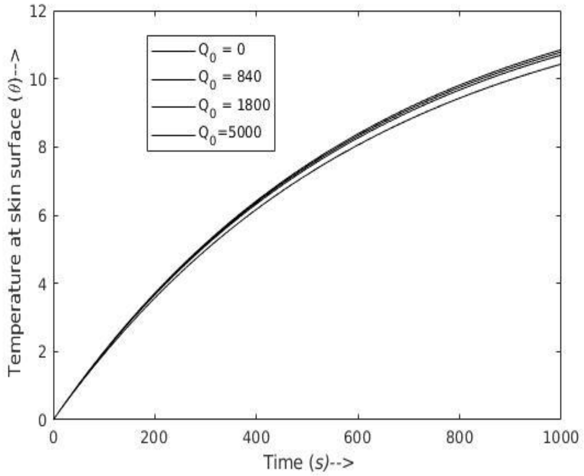

Fig. 4 Temperature on the surface of the skin with time for different values of metabolic heat generation

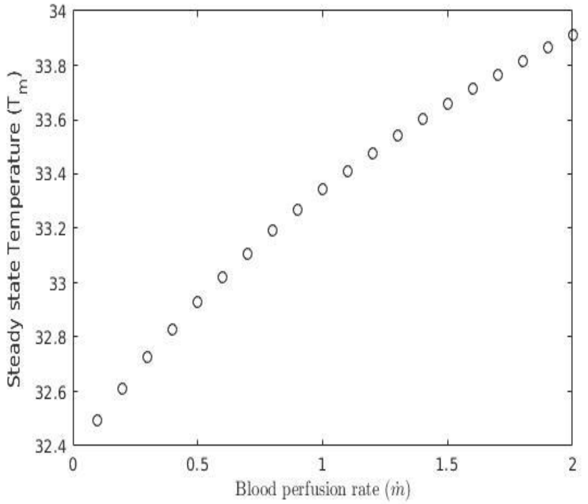

Fig. 5 Steady-state temperature of skin surface for different values of blood perfusion rate

3. Conclusion

This study investigates the thermal recovery of skin after cooling the skin up to a temperature of T_0. It can be concluded that the skin starts to recover to the ambient temperature above 800 seconds. The recovery profile or temperature profile of skin is not constant but depends on various physical parameters of skin tissue. Therefore, the temperature profile is studied for different parameters like tissue thermal conductivity, perfusion rate of blood, and metabolic heat generation. It is found that skin temperature reaches ambient temperature soon if thermal conductivity & metabolic heat generation is high and blood perfusion is low. This study also finds the steady state temperature of the tissue, i.e., when too much time is passed, the temperature is calculated. It is found that the value of steady state temperature is different for different values of blood perfusion rate.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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