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The Vector Hypercomplex Numbers and the Matter Waves

Vlad L. Negulescu

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Abstract *- In various articles[1, 2, 3], the author developed a mathematical model showing interesting applications in physics. The following article analyzes the coordinates transformation produced by a vectorial pseudo-rotation. The invariant quantities after a pseudo-rotation and the Doppler effect for light are also presented. Still, the paper's main purpose is to deduce the matter wave equation as the natural consequence of intrinsic properties of the "physical world" and its H-number representation. Ever since de Broglie formulated his hypothesis, the physics of matter waves was playing an important role in the field of the measurement technique[4]. In the last decade, matter waves optics enabled numerous applications ranging from basic science to navigation or detection systems technologies.*

Keywords *- VH- numbers representation, Geometrized unit system, Vector pseudo-rotation, Generalized action invariant, Wave associated with a particle, Phase velocity, Group velocity.*

1. Introduction

An ideal particle is associated[3] with a Vector-Hyper-Complex number, or VH-number, which can be formally written as:

$$p = t + iz + +k\mathbf{u}(x + iy) \qquad (1.1)$$

Where $t + iz$ is the scalar part of this number, and $k\mathbf{u}(x + iy)$ represents the vector part. Its geometrical correspondence is a point in an eight-dimensional space. The symbols 1, i, j and k are fundamental units of H-numbers defined in the reference paper[1]. The symbol **u** signifies an arbitrary unit vector in the Euclidean three-dimensional space. Table 1 shows the multiplication rules of the fundamental units.

Table1. Units' Multiplication Table

x	**1**	**i**	**j**	**k**
1	1	i	j	k
i	i	-1	-k	j
j	j	-k	-1	i
k	k	j	i	1

The four parameters of the particle's representation are time (t), mass (z), the magnitude of momentum (y) and the magnitude of space(x). The space and momentum are vectors with directions given by the unit vector, **u**. The geometrized system of units[5] enables the expression of all these parameters using a common unit, meter, as shown below in Table 2.

Table 2. The conversion of the international system of units (SI) to a geometrized system of units (GU); c is the velocity of light and G gravitational constant; both expressed in SI

	GU		SI		Conversion
	symbol	unit	symbol	unit	SI↔GU
Length	x	m	l	m	1↔1
Time	t	m	t	s	$c \leftrightarrow c^{-1}$
Velocity	v	none	v	ms^{-1}	$c^{-1} \leftrightarrow c$
Mass	z	m	m	Kg	$Gc^{-2} \leftrightarrow G^{-1}c^{2}$
Momentum	y	m	p	$Kgms^{-1}$	$Gc^{-3} \leftrightarrow G^{-1}c^{3}$
Force	F	none	F	N	$Gc^{-4} \leftrightarrow G^{-1}c^{4}$

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According to reference papers[2,3] the eight-dimensional space to which belongs a VH-number is the so called "physical world". The four-dimensional continuous Space-Time (ST), or the Minkowski space modified[3], is a subset of the physical world. The following expression represents a point in ST space:

$$p = t + \mathrm{k}\mathbf{u}\mathrm{x} \qquad (1.2)$$

2. Multiplication with a pseudo-rotor

A vector-hypercomplex number or VH number can be alternately written as follows:

$$p = \tau + \mathrm{k}\mathbf{u}\chi = \tau + \mathbf{k\chi} \qquad (2.1)$$

Where: **u** represents an arbitrary unit vector and:

$$\tau = t + iz \;\;, \chi = x + iy \text{ and } \boldsymbol{\chi} = \mathbf{u}\chi \qquad (2.2)$$

The significations of the parameters t, z, y and x remain the usual ones, i.e., time, mass, momentum and space magnitude.

We consider a coordinates transformation using the multiplication with a vector pseudo-rotor (see reference paper[2], paragraph 2.2):

$r = e^{\mathrm{k}\mathbf{v}\gamma} = cosh\gamma + \mathrm{k}\mathbf{v}\sinh\gamma$, where **v** denotes a unit vector, γ is the magnitude of an antireal argument, and e is the Euler's number.

We may write the dot or scalar product of these two unit vectors as follows:

$$\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u} = cos\,\Phi \qquad (2.3)$$

If on the number p is applied a pseudo-rotation defined by r, then this fully acts only on the scalar part and the vector part, which is parallel with **v**. The vector part, perpendicular to **v**, remains unchanged (see reference[1], paragraph 3.6, and reference[2] paragraph 3.1). The unit vector **u** can be written as:

$$\mathbf{u} = \mathbf{v}cos\phi + (\mathbf{u} - \mathbf{v}cos\phi) = \mathbf{v}cos\phi + \mathbf{v}_p \qquad (2.4)$$

It is easy to see that:

$$\mathbf{v} \bullet \mathbf{v}_p = 0 \qquad \mathbf{v}_p^2 = sin^2\phi \qquad (2.5)$$

Processing further, it obtains:

$$\tau' = \tau cosh\gamma + \chi cos\,\Phi\,sinh\,\gamma \qquad (2.6)$$

$$\boldsymbol{\chi}' = \chi\mathbf{v}_p + \mathbf{v}(\chi cosh\gamma cos\Phi + \tau\sinh\gamma)$$

Using relations (2.2), we finally get:

$$t' = tcosh\gamma + xcos\Phi\sinh\gamma$$

$$z' = zcosh\gamma + ycos\Phi sinh\gamma \qquad (2.7)$$

$$\mathbf{x}' = x\mathbf{v}_p + \mathbf{v}(xcosh\gamma cos\Phi + \mathrm{t}\sinh\gamma)$$

$$\mathbf{y}' = y\mathbf{v}_p + \mathbf{v}(ycosh\gamma cos\Phi + \mathrm{z}\sinh\gamma) \quad |$$

Processing the equations (2.7), we deduce the expressions of the following three invariants after pseudo-rotation:

$$t'^2 - \mathbf{x}'^2 = t^2 - \mathbf{x}^2$$

$$z'^2 - \mathbf{y}'^2 = z^2 - \mathbf{y}^2 \qquad (2.8)$$

$$t'z' - \mathbf{x}' \bullet \mathbf{y}' = tz - \mathbf{x} \bullet \mathbf{y} = \mathcal{A}$$

These relations are valid in both time-like and space-like zone of ST. The invariant expressions shown in (2.8) are the space-time invariant, the mass-momentum invariant and the generalized action invariant.

3. The invariant expressions in the time-like, respectively in the space-like zones (see reference[3])

As was shown in reference paper[3], a material particle located on the time axis in ST space is said to be at "space rest" or s-rest. The reference frame attached to the particle will be called a s-rest frame. In the Special Relativity (SR), this frame is known as *the proper or commoving frame*. If such a particle is the initial one, then the associated VH-number is purely complex:

$$p = t_0 + iz_0 \qquad (3.1)$$

Starting from it, the space-time invariant takes the expression:

$$t^2 - \mathbf{x}^2 = t_0^2 \qquad (3.2)$$

Converting to SI, it obtains:

$$c^2t^2 - \mathbf{l}^2 = c^2t_0^2$$

The mass-momentum invariant becomes:

$$z^2 - \mathbf{y}^2 = z_0^2 \qquad (3.3)$$

Converting to SI, it obtains the well-known energy-momentum relation for the subluminal particles:

$$E^2 - (pc)^2 = c^4m_0^2$$

The formula of the generalized action is:
$\mathcal{A} = tz - \mathbf{x} \bullet \mathbf{y} = t_0z_0$

In SI, the formula becomes:

$$A = tE - \mathbf{l} \bullet \mathbf{p} = t_0m_0c^2$$

We may consider the space-like zone an initial particle at the time-rest or t-rest state. Its VH-number representation is:
$$p = k\mathbf{u}(x_0 + iy_0)\;. \qquad (3.4)$$

The space-time invariant takes the form:

$$\mathbf{x}^2 - t^2 = x_0^2 \quad \text{(GU)} \qquad (3.5)$$

$$\mathbf{l}^2 - c^2t^2 = \mathbf{l}_0^2 \quad \text{(SI)}$$

Using the second equation of (2.8), it obtains the mass-momentum and energy-momentum relations for the hypothetical tachyons (see reference[3], equations 2.22 and 2.23):

$$\mathbf{y}^2 - z^2 = y_0^2 \quad \text{(GU)} \qquad (3.6)$$

$$(\mathbf{p}c)^2 - E^2 = (p_0c)^2 \quad \text{(SI)}$$

The corresponding expressions for the generalized action are as follows:

$$\mathcal{A} = \mathbf{x} \bullet \mathbf{y} - tz = x_0y_0 \quad \text{(GU)} \qquad (3.7)$$

$$A = \mathbf{l} \bullet \mathbf{p} - tE = l_0p_0 \quad \text{(SI)}$$

4. Light particles and the Doppler Effect

The following VH-number represents a light particle:

$$p = (t + iz)(1 + k\mathbf{u}) \qquad (4.1)$$

If the time parameter is 0, then p becomes:

$$p = iz + jz\mathbf{u} = \mathrm{iz} + \mathrm{j}\mathbf{y} \qquad (4.2)$$

The equation of the associated wave of this photon is according to the reference[1]:

$$Y = Acos(\omega t - \mathbf{xK}) \qquad (4.3)$$

The angular frequency is: $\omega = \frac{2\pi z}{H}$. The vector wave number is $\mathbf{K} = 2\pi\frac{\mathbf{y}}{H}$ And **x** represents the position vector of a point in space. The constant H signifies the Plank's constant, expressed in GU system i: e. H=1.646113x10^{-69} m^2.

After a pseudo-rotation is applied to (4.2), the new parameters of the light particle become (see 2.7):

$$z' = z(cosh\gamma + cos\Phi sinh\gamma) \qquad (4.4)$$

$$\mathbf{y}' = z\left[\mathbf{v}_p + \mathbf{v}(cosh\gamma cos\Phi + \sinh\gamma)\right]$$

The associated wave equation of the transformed H-number is:

$$Y = Acos(\omega' t - \mathbf{xK}') \qquad (4.5)$$

The new angular frequency and the new wave number are: $\omega' = \frac{2\pi z'}{H}$ and respectively $\mathbf{K}' = \frac{2\pi\mathbf{y}'}{\mathrm{H}}$.

Let us now consider a light particle coming from a light source, p_s. If the light particle is seen by an observer traveling away from the light source (p_o), then we may write the following relation:

$$p_s = p_o e^{\boldsymbol{kv\gamma}} \qquad (4.6)$$

Using the first equation of system (4.4), it obtains:

$$z_s = z_o(cosh\gamma + cos\Phi sinh\gamma) = z_o cosh\gamma(1 + cos\phi tanh\gamma)$$

Finally, Plank's relation (see reference[1], equation 4.4) permits to write of the frequency of the light wave seen by the observer:

$$f_O = \frac{f_S}{cosh\,\gamma(1 + cos\,\Phi\,tanh\,\gamma)} \qquad (4.7)$$

The formula above represents the relativistic Doppler effect for light.

5. Material wave

6. Deduction of the material wave equation starting from a particle at space-rest.

A particle at ***space-rest*** can be written as:

$$p = t_0 + iz_0 = \frac{1}{2}(t_0 + iz_0)[(1 + k\mathbf{u}) + (1 - k\mathbf{u})] = \frac{1}{2}(p_{light1} + p_{light2}) \qquad (5.1)$$

After a pseudo-rotation by r, it obtains:

$$z_{light1} = z_0(cosh\gamma + cos\Phi sinh\gamma) \qquad (5.2)$$

$$y_{light1} = z_0\left[\mathbf{v}_p + \mathbf{v}(cosh\gamma cos\Phi + \sinh\gamma)\right]$$

The parameters of the p_{light2} result by replacing $\mathbf{v}_p$ with $-\mathbf{v}_p$ and cosΦ with – cosΦ, i.e.:

$$z_{light2} = z_0(cosh\gamma - cos\Phi sinh\gamma) \qquad (5.3)$$

$$y_{light2} = z_0\left[-\mathbf{v}_p - \mathbf{v}(cosh\gamma cos\Phi - \sinh\gamma)\right]$$

Consequently, the angular frequency and respectively wave number of the associated light waves are:

$$\omega_1 = \frac{2\pi z_0}{H}(cosh\gamma + cos\Phi sinh\gamma)$$

$$\omega_2 = \frac{2\pi z_0}{H}(cosh\gamma - cos\Phi sinh\gamma)$$

$$\mathbf{K}_1 = \frac{2\pi z_0}{H}\left[\mathbf{v}_p + \mathbf{v}(cosh\gamma cos\Phi + \sinh\gamma)\right];$$

$$\mathbf{K}_2 = \frac{2\pi z_0}{H}\left[-\mathbf{v}_p - \mathbf{v}(cosh\gamma cos\Phi - \sinh\gamma)\right]$$

We may write the associated waves equations:

$$Y_{light1} = \frac{B}{2}cos(\omega_1 t - \mathbf{K}_1\mathbf{x}) = \frac{\mathrm{B}}{2}cos\Omega_1 \qquad (5.4)$$

$$Y_{light2} = \frac{B}{2}cos(\omega_2 t - \mathbf{K}_2\mathbf{x}) = \frac{B}{2}cos\Omega_2$$

After addition, it obtains the expression of the resultant wave:

$\mathrm{Y} = \mathrm{B}\cos\frac{\Omega_1+\Omega_2}{2}\cos\frac{\Omega_1-\Omega_2}{2}$

$Y = Bcos\frac{2\pi z_0}{H}(tcosh\gamma - \mathbf{xv}sinh\gamma)cos\frac{2\pi z_0}{H}\left[tcos\Phi sinh\gamma\mathbf{x}(\mathbf{v}_p + \mathbf{v}cosh\gamma cos\Phi)\right]$

For $\mathbf{x}= x\mathbf{v}$ the above expression becomes:

$$Y = Bcos\frac{2\pi z_0}{H}(tcosh\gamma - xsinh\gamma)cos\frac{2\pi z_0}{H}[\cos\Phi(tsinh\gamma - xcosh\gamma)]$$

As shown in the reference paper[1], the value of cosΦ must be zero; otherwise, the expression above is absurd. It means that the material wave is a typical transverse one.
With all of this, it obtains the final expression of the wave equation:

$$Y = Bcos\frac{2\pi z_0}{H}(tcosh\gamma - xsinh\gamma) \qquad (5.5)$$

This equation is valid for the time-like zone.

But the mass and the momentum of a subluminal particle are expressed by:

$z = z_0 cosh\gamma$, and $y = z_0 sinh\gamma$, as we see in reference[3], equations (2.9).

Consequently, we may write an alternate form of the formula (5.5).

$$Y = Bcos\frac{2\pi}{H}(tz - xy) = Bcos\frac{2\pi}{H}\mathcal{A} \qquad (5.6)$$

Where $\mathcal{A}$ is the generalized action.
If the initial particle is at ***time-rest***, then its corresponding VH-number is expressed by the formula (3.4).
Processing it obtains:

$$p = \frac{1}{2}[(x_0 + iy_0)(k\mathbf{u} + 1) + ((x_0 + iy_0)(k\mathbf{u} - 1)] = \frac{1}{2}(p_{light1} + p_{light2})$$

The equation above is formally identical to the expression (5.1), and using the same calculation; it arrives at the wave equation (5.6), considering that the mass and the momentum of a superluminal particle are, according to reference[3], equations (2.20):

$z = y_0 sinh\gamma$, and $y = y_0 cosh\gamma$

The particle-wave equation (5.6) was also deduced in the reference[6] using the parameters of the compressed vector belonging to a particle.

The phase velocity and the group velocity of the wave expressed by the formula (5.6) are related by:

$$v_p v_g = 1 \qquad (5.7)$$

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