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Original Article

Passage of Time for an Astronaut Rotating Around a Schwarzschild Black Hole - An Application of General Theory of Relativity

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Abstract *- The General Theory of Relativity (GTR) is a fundamental framework that describes the gravitational interaction between matter and spacetime, formulated by Albert Einstein in 1916. Within the scope of GTR, the Schwarzschild Black Hole is a notable solution, representing a non-rotating, uncharged, spherically symmetric black hole. Time dilation, a concept intrinsic to GTR, manifests as a relativistic effect, where time progresses at different rates in different regions for different observers. The time elapsed in the astronaut clock is the proper time, and the time passed in any other frame of reference is the improper time. Time dilation is a function of the distance of the astronaut to the radius of the black hole factor (DTRF). This paper calculates the time dilation factor for DTRF 1.1 to 7. It is observed that the DTRF increases as the time dilation factor increases. For instance, for DTRF 1.1, the time dilation factor is 0.9258, and when the DTRF is 7, the time dilation factor is 0.3015. It is also observed that the time dilation factor increases rapidly and non-linearly for DTRF 1.1 to 4 and increases approximately linearly thereafter.*

Keywords *- General theory of relativity, Schwarzschild black hole, Time dilation, Gravitational potential, Non-linear and linear.*

1. Introduction

There have been two major perspectives on the concepts behind black holes, and they find their roots in two different yet complementary frameworks: Newtonian gravity and Einstein's theory of general relativity. The Newtonian explanation of the black hole describes the celestial body as a body with a radius and strong gravity. In Newtonian physics, the concept of a black hole arises from a simple yet profound consideration of escape velocity. [1] According to Newton's laws of motion and gravitation, escape velocity is the minimum speed an object must reach to break free from the gravitational pull of a celestial body, such as a planet or a star. If an object's kinetic energy exceeds the gravitational potential energy at its location, it can escape to infinity [3]. Consider a planet or star with a given mass and radius. Using Newtonian physics, one can show that the escape velocity (v_{escape}) from the surface of any astrophysical object is given by equation 1[5]:

$$V_{escape} = \sqrt{\frac{2GM}{R}} \tag{1}$$

Where G is the gravitational constant, M is the mass of the celestial body, and R is its radius. The escape velocity is directly proportional to the square root of the mass and inversely proportional to the square root of the radius. This equation implies that a more massive and compact object will have a higher escape velocity at its surface.

As we delve deeper into this concept, a remarkable realization emerges: if a celestial object's escape velocity exceeds the speed of light (c = 299,792,458 m/s), then nothing, not even light, can escape from its surface. [3] This critical escape velocity, known as the speed of light, represents an important cosmic speed limit beyond which conventional physics, known at the time, cannot explain the behaviour of objects. In this context, an "invisible" region emerges around an object with a sufficiently high escape velocity, creating what is effectively an "event horizon" in classical terms. [6] The immense gravitational force traps any object or particle within this region and is unable to reach escape velocity, effectively becoming trapped within this dark and unseen domain. This early intuition for the existence of massive, compact objects with escape velocities surpassing the speed of light laid the foundation for the eventual conceptualization of black holes in the framework of general relativity. Through the development of Einstein's theory of general relativity, black holes evolved from being an interesting consequence of Newtonian physics to becoming an intricate and profound prediction about the nature of spacetime and gravity [1].

Einstein's field equations [8] represent a profound achievement in the realm of theoretical physics, introducing a revolutionary way to understand gravity. These equations, formulated by Albert Einstein as part of his theory of general relativity in 1916, describe the intricate interplay between the geometry of spacetime and the distribution of mass and energy within it. In essence, the field equations reveal that the presence of mass and energy warps the fabric of spacetime, resulting in the phenomenon we perceive as gravity [9]. Unlike Newtonian gravity, where gravitational forces are described as instantaneous action at a distance, general relativity paints a picture of gravity as the curvature of spacetime itself, with massive objects like stars and planets influencing the curvature of the surrounding spacetime. Mathematically, the field equations are a set of ten non-linear partial differential equations. These equations form a system that ties together the various components of the metric tensor, a mathematical construct that describes the geometric properties of spacetime. Solving these equations yields the metric tensor for a given distribution of mass and energy, which, in turn, allows us to understand the curvature of spacetime and its impact on the motion of objects within it [8].

An everlasting question that has been contained in the minds of many Theoretical Physicist (a thought experiment): "If an astronaut spends 1 day around a black hole, what would be the effective time dilation for that person as observed by another person who is in rest w.r.t the astronaut"? That is the question sought to be answered in this research paper.

2. Methodology

This section discusses the methodology used to conduct the research.

2.1. Aim of the Study

To calculate the time dilation near a Schwarzschild black hole.

2.2. Research Design

A thought experiment: An astronaut revolving around a Schwarzschild Black hole. The time dilation is calculated by keeping the astronaut at different distances from the center of the Black Hole.

2.3. Hypothesis

Null hypothesis: There is no change in time flow near a Schwarzschild black hole.

Alternate hypothesis: There is a change in time flow, that is, time dilation near a Schwarzschild black hole.

2.4. Tools Used

Google Collab, Python [11]

2.5. Data Collection Procedure

The time dilation formula for the Schwarzschild solution is used. The time dilation is the improper time, which is the measurement of the passage of time of the astronaut as seen by a person who is at rest w.r.t the astronaut. Whereas the passage of time in the astronaut's clock is the proper time. The improper time [12] is the product of the proper time and time dilation factor. The time dilation factor depends on the distance of the astronaut from the centre of the black hole. For convenience, the ratio of the distance between the astronaut and the radius of the black hole is defined as DTRF. The DTRF varies from 1.1 to 7, and the corresponding time dilation (improper time) is calculated.

3. Results

The Schwarzschild Equation plays a vital role in the field of gravitational physics, providing deep insights into the nature of black holes and their underlying geometry. This equation arises as a fundamental solution to Einstein's field equations in the context of general relativity, describing the gravitational field around a spherically symmetric, non-rotating mass. The radius of a Schwarzschild black hole is given by equation 2 [5].

$$R = \frac{2GM}{c^2} \qquad (2)$$

R represents the Schwarzschild radius, "G" is the gravitational constant, "M" stands for the mass of the black hole, and c is the speed of light. Solving Einstein's field equation equations for a spherically symmetric, non-rotating mass distribution yields the Schwarzschild metric. This metric describes the spacetime around a spherically symmetric mass, such as a non-rotating black hole or a massive celestial body. It is characterized by key terms like "r," representing the radial distance from the center, and "t," denoting time. Once the line element (ds^2) for the Schwarzschild Black Hole is derived, its constituent terms become crucial in understanding the structure of spacetime in its vicinity.

$$ds^2 = -(1 - \frac{2GM}{c^2 r}) d(ct)^2 + (1 - \frac{2GM}{c^2 r})^{-1} dr^2 + r^2(d\theta^2 + sin^2(\theta)\, d\varphi^2) \qquad (3)$$

Equation 2 is the Schwarzschild equation, which describes the geometry of a non-rotating, spherically symmetric gravitational field. Within this equation, "r" represents the radial coordinate, "t" signifies time, and "M" stands for the mass of the central object, often associated with a black hole[14]. The term "$2GM/c^2$" relates to the Schwarzschild radius, a critical boundary within which the gravitational field becomes significant, and "c" again denotes the speed of light. "$d\theta^2$" and "$\sin^2(\theta)d\varphi^2$" encapsulate the angular coordinates in spherical polar coordinates, completing the Schwarzschild metric. This equation reveals that gravitational effects grow stronger as an object approaches the event horizon (the Schwarzschild radius), leading to time dilation. Essentially, it implies that time passes more slowly for an observer near a massive object, such as a black hole, when compared to an observer at a distant point in spacetime. This effect is encapsulated in the "$d(ct)^2$" term within the Schwarzschild metric, highlighting how the curvature of spacetime profoundly influences the passage of time near massive celestial bodies.

An astronaut revolves around a Schwarzschild Black Hole of radius $r_s = 2GM/c^2$, where M is the mass of the Black Hole. The astronaut can revolve in a variable distance of DTRF (distance of the astronaut to the radius of the black hole factor) ranging from 1.1 to 7. The time dilation factor is calculated using the formula below; refer to Figure 1.

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Code 1: Calculation of time dilation factor for DTRF from 1.1 to 7

```
import matplotlib.pyplot as plt
import numpy as np
r = np.arange(1.1, 7, 0.1)
# Normalization for simple calculation
rs = 1
#distance to the radius factor
#dtrf = r/rs ^
#change of dtrf is very 1.1-4 increases with almost a constant slope
#make 2 tables: 1-1-4 and 4-8 and put the sr number, then r/rs, time-dilation factor

def f(r):
y = np.sqrt(1-rs/r)
return y

for i in r:
print(format(f(i), '.4f')) # cut all digits after four digits after decimal
```

Table 1. For DTRF 1.1 to 4

Sr No	DTRF(r/r_s)	Time Dilation Factor	Sr No	DTRF(r/r_s)	Time Dilation Factor
1	1.1	0.3015	16	2.6	0.7845
2	1.2	0.4082	17	2.7	0.7935
3	1.3	0.4804	18	2.8	0.8018
4	1.4	0.5345	19	2.9	0.8094
5	1.5	0.5774	20	3.0	0.8165
6	1.6	0.6124	21	3.1	0.8231
7	1.7	0.6417	22	3.2	0.8292
8	1.8	0.6667	23	3.3	0.8348
9	1.9	0.6882	24	3.4	0.8402
10	2.0	0.7071	25	3.5	0.8452
11	2.1	0.7237	26	3.6	0.8498
12	2.2	0.7385	27	3.7	0.8542
13	2.3	0.7518	28	3.8	0.8584
14	2.4	0.7638	29	3.9	0.8623
15	2.5	0.7746	30	4.0	0.8660

Table 2. For DTRF 4.1 to 7

Sr No	DTRF (r/r_s)	Time Dilation Factor	Sr No	DTRF (r/r_s)	Time Dilation Factor
1	4.1	0.8695	16	5.6	0.9063
2	4.2	0.8729	17	5.7	0.9081
3	4.3	0.8760	18	5.8	0.9097
4	4.4	0.8790	19	5.9	0.9113
5	4.5	0.8819	20	6.0	0.9129
6	4.6	0.8847	21	6.1	0.9144
7	4.7	0.887.	22	6.2	0.9158
8	4.8	0.8898	23	6.3	0.9172
9	4.9	0.8921	24	6.4	0.9186
10	5.0	0.8944	25	6.5	0.9199
11	5.1	0.8966	26	6.6	0.9211
12	5.2	0.8987	27	6.7	0.9224
13	5.3	0.9007	28	6.8	0.9235
14	5.4	0.9027	29	6.9	0.9247
15	5.5	0.9045	30	7.0	0.9258

The data in Tables 1 and 2 is plotted and is shown in Figure 1. A few additional lines of codes are added to Code no 1 to plot Figure 1. The added lines of code are mentioned below:

Code 2: Code to Plot the time dilation factor vs DTRF(r/r_s)

```
plt.xlabel("r/rs")
plt.ylabel("time-dilation factor")
plt.title("Time Dilation")

plt.scatter(r, f(r), s=5)
plt.plot(r, f(r))
plt.show()
```

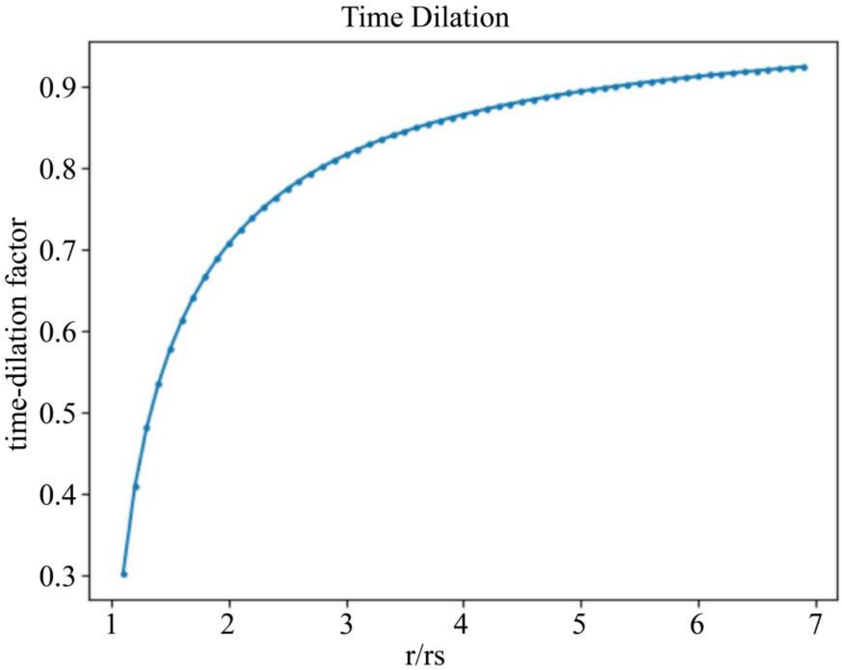

Fig. 1 Plot of Time dilation factor for DTRF 1.1 to 7

4. Discussion

Time elapsed for an observer observing the astronaut revolving around the Schwarzschild black hole is called improper time. The improper time is defined as the proper time times the dilation factor. We see from Figure 1 that the time dilation factor has a non-linear increase for the DTRF < 4, and for DTRF>4, it shows a linear nature. If the passage of time for the astronaut is one day in his clock, then it will be different distances at which the astronaut revolves around the black hole from the center of the black hole. For instance, when one $r = 1.1r_s$, the improper time elapsed is 0.3015 days (refer to Table 1). And as the distance of the astronaut increases, the improper time increases. When the astronaut is at a distance $r = 7r_s$, the improper elapsed time is 0.9258.

5. Conclusion

The DTRF, the ratio of the astronaut's distance to the black hole's radius, determines the time dilation factor. As is shown in this paper, the proper time, that is, the passage of time in the astronaut's frame of reference, is a function of DTRF. The improper time, which is the flow of the astronaut's time as seen from another frame of reference, is the product of the time elapsed in the astronaut's frame and the time dilation factor. The time dilation factor decreases rapidly as the astronaut moves away from the black hole. The exact details are provided in Tables 1 and 2.

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