Application of The Mixed Hybrid Finite Element Approximation to Water Transfers in Variably Saturated Porous Media with Time-Varying Infiltration

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Abstract:

In this study, a two-dimensional model for simulating water in homogeneous-heterogeneous, saturated-unsaturated porous media is presented. The model is based on the mixed-hybrid finite element method and the h-based form of the Richards equation is solved **[16]**. The time-varying infiltration is approximated by a number of piecewise linear elements of different lengths and slopes depending on the nature of the variation in infiltration rate. The unsaturated flow model (MHNS_2D) is applied to a variety of rigorous problem including transient flow in dry conditions and time-varying boundary conditions. It produces accurate predicted suction distribution in dry conditions compare to the analytical solution of J.R. Philip. It is also shown to provide good global water mass balance accuracy in simulations of vertical infiltration in spite of solving the h-based form of the Richards equation. MHNS_2D is used also to predict the water table fluctuation of an unconfined aquifer in response to time varying infiltration.

Keywords: unsaturated porous media; numerical model; mixed-hybrid finite element; time-varying infiltration.

1. INTRODUCTION

Transfer of water solutes under transient unsaturated conditions plays an important role in many branches of agriculture and environmental engineering. The unsaturated zone is prone to contamination from agriculture where many chemicals such as fertilizers, pesticides, as well as those naturally present in irrigation waters, are frequently applied to the field. When water is applied to the soil surface, either by rain or irrigation, it may transport chemical contaminants through the unsaturated zone to the underlying groundwater aquifer. Also the recharge of ground water by the natural precipitation, irrigation or artificial recharge may result into the rise of the water table closer to the ground surface. This causes many environmental disturbances as water logging, pollution of groundwater, increase in soil salinity. The latter is generally observed in irrigated land in arid and semi-arid zones. Sometimes, deep percolation of water induces an inescapable rise of the water table. Therefore, because of vulnerability of the unsaturated zone to contamination and its direct link to acquifers, a clear understanding of water transfers processes, vector of chemical transport, important for both agricultural and environmental engineers. An effective way to develop such an understanding is by means of computer simulation using numerical models. They play a significant role in the analysis of the movement of water and contaminants in porous media. That's why they are also often employed in studies addressing the water and solute transfer in porous media. Numerical models of water flow in variably saturated porous media are commonly based upon the solution of the well-known Richards equation. Alternative forms of the Richards equation are derived by selection of the primary unknown are either pressure or suction head (h) or water or moisture content (θ). Nevertheless, serious mass balance and convergence problem can appear due to its non-linear nature.

Numerical models solving the h-based form have been reported to produce significant global mass balance error **[1]-[2]-[14]-[20]**. The θ-based formulations have demonstrated reliable mass balance accuracy [5]. But application of the the θ-based formulations is restricted to unsaturated flow conditions, because of the discoutinuity of the water content. [2] proposed a general mass conservation scheme, which solves the mixed form. This numerical strategy can be modified by introducing the primary variable switching technique which represents a fast and robust strategy for unsaturated problems with dry initial conditions (Diersch and Perrochet, 199).

The objective of this paper is to develop an alternative numerical model that is able to produce accurate simulations of transient flow in 2-D unsaturated-saturated porous media. This model is based on h-based of Richard's equation, and solved by mixed-hybrid finite element method. It has been quite successful for solving the flow equation in saturated porous media [10]. It provides separate approximation of the pressure head and the Darcy's velocity. This property allows for a precise determination of flow lines and propagation of contaminants. We have also introduced in the model, a scheme of time-varying infiltration. The time-varying infiltration is approximated by a number of piecewise linear elements of different lengths and slopes depending on the nature of the variation in infiltration rate.

The accuracy of the proposed model is evaluated by comparing its results with analytical solution for unsaturated flow. We will also present the results of water table fluctuation characteristics due to change in the rate of infiltration.

2. FLOW EQUATION

The mathematical model used to describe fluid flow in partially saturated rigid porous media is obtained by combining the mass conservation equation with the generalized Darcy equation in the following form:

$$\left(\rho S_w(\alpha+\phi\beta)+\phi\rho\frac{\partial S_w}{\partial p}\right)\frac{\partial p}{\partial t}+\frac{\partial}{\partial x_i}(\rho q)=0 \qquad (1)$$

Where ρ is the masse density ($M\,L^{-3}$); (S_w(h) degree of water saturation ($0 < S_W \le 1$, $S_w = 1$ if the medium is saturated) ($L^3\,L^{-3}$); x_i ($i = 1,2$) the spatial coordinates (L); α is the coefficient of skeleton compressibility ($M^{-1}LT^2$); β is the fluid compressibility ($M^{-1}LT^2$); ϕ is the porosity ($L^3\,L^{-3}$), p(=ρg h) is the pressure ($ML^{-1}\,T^{-2}$): q Darcy's velocity (LT^{-1}) and h pressure head (L) linked to H (L) piezometric head by the classical relation H = h + z where z is the vertical coordinate positive upward ($h \ge 0$ in saturated medium and $h < 0$ in unsaturated medium). If $\alpha = 0$ then the porosity is not a function of the pressure, if $\beta = 0$, then the mass density is not a function of the pressure. If the mass density is constant then the equation (1) leads to:

$$\left(\phi\frac{\partial S_w}{\partial \rho}\right)\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_1}(q)=0 \qquad (2)$$

By the use of the volumetric water define as $\theta = \varphi S_w$ ($L^3\,L^{-3}$), equation (2) becomes:

$$\frac{\partial\theta}{\partial t}+\frac{\partial}{\partial x_i}(q)=0 \qquad (3)$$

The water flow in saturated porous media can be represented by the generalized Darcy's law given by:

$$\vec{q}=-K(h)\vec{\nabla}H \qquad (4)$$

in which K(h) is hydraulic conductivity in unsaturated domain (LT^{-1}) with $K(h,x,z) = K_s = (x,z)\,K_r(h,x,z)$, $K_s(x,z)$ is the hydraulic conductivity at saturation and $K_r(h,x,z)$ the relative hydraulic conductivity of the soil.

The mass conservation law can be written as it follows:

$$C(h)\frac{\partial H}{\partial t}+\nabla.\vec{q}=f \qquad (5)$$

C(h)(=dθ / dh) is the capillary capacity (L^{-1}), *h* is the pressure (L), *f* represents the volumetric flow rate via sources or sinks per unit volume of the medium (T^{-1}) Combining equations (4) and (5) leads to the h-based form of the Richards' equation given as:

$$C(h)\frac{\partial h}{\partial t}-\frac{\partial}{\partial x_i}\left[K(h)\left(\frac{\partial h}{\partial x_j}+u_j\right)\right]=f \qquad (6)$$

Where u_j is the unit vector in the direction of the x_2 coordinate.

Richards's equation can be written in different form by using either the pressure head (h) as a principal variable (h -based form) or the water content (θ) as a principal variable (θ-based form).

If equation (6) is applied to a planar flow in a vertical cross section then x_1 = x represents the horizontal coordinate and x_2 = z the vertical coordinate, the latter taken to be positive upward. Any initial condition in terms of pressure head or water content can be invoked. Dirichlet (pressure head) or Neumann (flux) boundary conditions at the top or bottom of the profile must be associated to the partial differential equations.

3. MIXED HYBRID APPROXIMATION

We would like to approximate the piezometric head H (L) in the domain Ω. This domain will be discretize by triangular elements E. We are looking for a way which permits an approximation of the piezometric head H (L) and it's associated velocity field $\vec{q}$ (L / T).

3.1 The mixed formulation

The mixed formulation gives a separate approximation of the piezometric head $H[L]$ and the Darcy's velocity $\vec{q}[LT^{-1}]$ **[15]**.

Let us consider a triangular element E With edges F_i ,i = 1,2,3. In the mixed-hybrid formulation of the mixed approximation, H and $\vec{q}$ are approximated over each element E by:

- The mean value of H over the element E: denoted by H_E
- The mean value of H over edge $F_i, i = 1,2,3$: denoted by $TH_{E,i}\ i = 1,2,3$ (the notation TH_{Ei} stands for ''trace of H on edge *i* of element E and represents the piezometric head on the edge)
- Approximation of $\vec{q} = -K(h)\vec{\nabla}H$ over element E denoted by $\vec{q}_E$ having the following properties over E [3].
- $\nabla.\vec{q}_E$ is constant over element E.
- The scalar product $\vec{q}_E.\vec{n}_{Ei}$ is constant over edge F_i, of E where $\vec{n}_{Ei}$ is the outer vector normal to F_i.
- $\vec{q}_E$ is determined by the knowledge of its flux Q_{Ei} through the edge F_i , I = 1,2,3.

It is defined over the whole domain by:

$$\vec{q} = \sum_{j=1}^{3} Q_{E,j}\,\vec{w}_i \qquad (7)$$

Where $\vec{w}_i$ (1, 2, 3) being the basis vectorial functions defined by:

$$\int_{Fj} \vec{w}_i.\vec{n}_{Ej} = \delta_{ij} \quad , j = 1, 2, 3 \qquad (8)$$

δ_{ij} is the Kronecker delta

The basic vectorial function $\vec{w}_i$ corresponds to a vector having its flux equal to 1 through the edge F_i and zero through the others satisfying the following condition:

$$\int_E \nabla.\vec{w}_i = \sum_{j=1}^{3} \int_{F_j} \vec{w}_i.\vec{n}_{Ej} = 1 \qquad (9)$$

This condition imply an exact local mass balance and the continuity of the normal component of the velocity vector between two adjacent elements. The mixed formulation consists on writing a numerical development of the Darcy's law and the flow equation.

3.2 Numerical development of the Darcy's law

We rewrite the Darcy's law (eq. 2) in the following form:

$$K^{-1}(h)\vec{q} = -\nabla H \qquad (10)$$

Multiplying scalarly each member of the equation with a test function $\vec{s}$, integrating over E element and using the Green formula, we obtain the following relation:

$$\int_E \left(K^{-1}(h_E).\vec{q}\right).\vec{s} = -\int_E \nabla H.\vec{s} = \int_{\partial E} H\vec{s}.\vec{n}_E$$
$$\forall\, \vec{s} \in \mathrm{H}\,(\mathrm{div}, \mathrm{E}) \qquad (11)$$

Substituting H and $\vec{q}$ by their approximations over E element in equation (11), we obtain the following elemental equation linking $\mathrm{H_E}$, $\mathrm{TH_{E,i}}\,(\mathrm{i}=1,2,3)$ and $\vec{q}_E$.

$$\int_E \left(K^{-1}(h_E).\vec{q}_E\right).\vec{s}_E = \int_E H_E \nabla.\vec{s}_E - \sum_{j=1}^{3}\int_{Fj} TH_{E,j}\,\vec{s}_E.\vec{n}_{Ej} \qquad (12)$$

Considering the three-basic function $\vec{w}_i$ of the Raviart-Thomas space, $\mathrm{H_E}$ remaining constant over E and $\mathrm{TH_{E,j}}$ also constant over F_j, we obtain from equation (12):

$$\int_E \left(K^{-1}(h_E).\vec{q}_E\right)\vec{w}_i = H_E \int_E \nabla.\vec{w}_i - \sum_{j=1}^{3} TH_{E,j}\int_{F_i} \vec{w}_i.\vec{n}_{Ej} \qquad \forall\, i=1,\ldots,3 \qquad (13)$$

Using conditions (7) and (9) into equation (13) gives:

$$\sum_{j=1}^{3} Q_{E,j}\int_E \left(\left(K_E^{-1}(h_E).\vec{w}_j\right).\vec{w}_i\right) = H_E - TH_{E,i} \qquad \forall\, i=1,\ldots,3 \qquad (14)$$

Now we define the 3 x 3 symetric matrix B_E associate to E element:

$$B_E = \left[B_{i,j}\right] \qquad (15)$$

in which

$$B_{ij} = \left[\int_E \left(\left(K_E^{-1}(h_E).\vec{w}_j\right).\vec{w}_i\right)\right]$$

K_E remains constant over E element.

By using this notation B_{ij} into equation (14), the numerical development of Darcy's law can be written as:

$$\sum_{j=1}^{3} Q_{E,j}B_{ij} = H_E - TH_{E,i} \qquad \forall\, i=1,\ldots3 \qquad (16)$$

Using a matrix notation we have the following form:

$$B_E\, Q_E = H_E\, DIV_E^T - TH_E \qquad (17a)$$

Where,

$$DIV_E^T = \begin{bmatrix}1\\1\\1\end{bmatrix}, \quad Q_E = \begin{bmatrix}Q_{E,1}\\Q_{E,2}\\Q_{E,3}\end{bmatrix},$$

$$TH_E = \begin{bmatrix}TH_{E,1}\\TH_{E,2}\\TH_{E,3}\end{bmatrix}$$

$\mathrm{DIV_E^T}$ is the transposed elemental divergence matrix.

B_E is an invertible matrix. We can write equation (17a) in the following form:

$$Q_E = B_E^{-1}\left(H_E\, DIV_E^T - TH_E\right) \qquad (17b)$$

Or in an another form,

$$Q_{E,F} = H_E a_{EF} - \sum_{F'\subset\partial E} B^{-1}_{EF,F'}\,TH_{EF'}$$
$$\text{where}\quad a_{EF} = \sum_{F\subset\partial E} B^{-1}_{EF,F'} \qquad (18)$$

∂E represent the boundaries of E element (the three E edges).

Equation (18) represents a relation among the flux on the edges $(Q_{E,F})$, the mean value of the piezometric head over E element (H_E) and the mean value of the piezometric head over each edge of E element (TH_{EF}). Knowing the mean value of the piezomzetric head over the element and its edges, the flux are then perfectly determinate.

3.3 Numerical development of the flow equation

We start discretizaton of time in intervals. Each interval has a duration Δt. The flow equation (5) can be written in the following form:

$$C(h)\frac{\partial H}{\partial t} + \nabla.\vec{q} = f \qquad (19)$$

This equation must be integrated over each element E of the domain. This is achieved by multiplying each member of equation (19) with a test function $s \in L^2(E)$ and integrating over E in order to obtain a formulation like finite volume type of the equation over E.

$$\int_E C_E(h_E)\frac{\partial H}{\partial t}s + \int_E s\nabla.\vec{q} = \int_E fs \qquad \forall s \in L^2(E) \qquad (20)$$

Substituting approximation of H and $\vec{q}$ over E element in (20), and taking into account that the approximations H_E and $\vec{q}_E$ satisfy the following conditions:

H_E is constant over E and $\nabla.\vec{q}_E = \frac{1}{|E|}\sum_{i=1}^{3} Q_{E,i}$ is constant over E, where $|E|$ represent the area of E element. We obtain from (20):

$$\int_E C_E \frac{H_E^n - H_E^{n-1}}{\Delta t} s_E + \int_E s_E \nabla.\vec{q}_E^{\,n} = \int_E f^n s_E \qquad \forall\, s_E \in \text{Raviart} - \text{Thomas space} \qquad (21)$$

in which s_E is constant over E and C_E is the capillary capacity over E and is considered to be constant over E. After dividing each member of equation (21) by s_E, we obtain a new equation in the following form:

$$|E|\, C_E \frac{H_E^n - H_E^{n-1}}{\Delta t} + \sum_{i=1}^{3} Q_{E,i}^n = F_E^n \qquad (22)$$

Where F_E^n is the approximation of $\int_E f^n$, constant over E

Using a matrix notation, the flow equation (22) can be written as:

$$|E|\, C_E \frac{H_E^n - H_E^{n-1}}{\Delta t} + DIV_E\, Q_E^n = F_E^n \qquad \forall\, E \in \Omega \qquad (23)$$

3.4. The mixed hybrid formulation

The mixed-hybrid approach consists of using the piezometric head on the edges TH_{Ei} as principal unknowns. The other unknowns $(Q_{E,F}$ and $H_E)$ are eliminated [3]; Mosé et al., 1994). The numerical development of the Darcy’s law, of the flow equation and the continuity of the fluxes between two adjacent elements $(\text{i.e. } Q_{E,F} + Q_{E,F} = 0 \text{ for every edge F, E and E}' \text{ being a}$, are the three equations used to reduce the number of unknowns to the mean value of the piezometric head of the edges.

Dirichlet conditions are achieved by an equality of the head over edges and Neumann conditions by a flux equality. The mixed approximation takes account of Neumann conditions in a very specific way by prescribed fluxes on element edges whereas the sink/source terms are averaged over the element. Initial conditions are given with the knowledge of TH_{Ei}^{0}.

After solving the system of equations on trace of H, we use local equations over each element E in order to compute the flux through the edges and the mean value of the piezometric head over the element.

4. INTRODUCING TIME-VARYING INFILTRATION

We have considered that the rate of infiltration depends on time and this is more close to the reality particularly in case of natural infiltration. The infiltration can have any flux (rate of infiltration) entering the soil through its surface. The sum of the infiltration during time $I(x,t)$ from all sources of infiltration, can be represented by the relation given below **[9]-[13]**:

$$I(x,t)=\begin{cases}\sum_{i=1}^{N} q_i(t) \text{ for } x_{i1}\le x\le x_{i2} \\ 0 \qquad \text{elswhere}\end{cases} \tag{24}$$

x_{i1} and x_{i2} being the boundaries of the "i" infiltration site.

N represent the total number of infiltration zone, $q_i(t)$ is the rate of infiltration and is approximated by using a series of linear elements in the following form:

$$q_i(t)=\begin{cases} m_{ij}\,t+c_{ij} & t_j\le t\le t_{j+1} \\ c_{ik} & t>t_k \end{cases} \tag{25}$$

$$(j=1,2,\ldots\ldots\ldots,k-1)$$
$$(j=k)$$

where m_{ij} and c_{ij} are the slope and intercept of the jth linear element of the ith infiltration zone. Advantage of this type of approximation is that any complex nature of variation of infiltration rate for any number of infiltration cycle can be approximated with more accuracy.

5. EXAMPLES

To demonstrate the performance of our model, a variety of example are considered. The hydrodynamics properties are represented by Mualem (1976) and Van Genuchten's equation **[21]**:

$$\begin{cases} S=\dfrac{\theta-\theta_r}{\theta_s-\theta_r}=\dfrac{1}{\left[1+(\alpha|h|)^n\right]^{1-1/n}} & \text{with} \quad h<0 \\ S=1 & \text{with} \quad h\ge 0 \end{cases} \tag{26}$$

And

$$K=K_s\sqrt{S}\,\left[1-\left(1-S^{(n/n-1)}\right)^{(1-(1/n))}\right]^2 \quad \text{with} \quad n>1 \tag{27}$$

Where θ_s ($L^3\,L^{-3}$) and θ_r ($L^3\,L^{-3}$) are the saturated and residual water content, respectively, S is relative saturation, K_s ($L\ T^{-1}$) is the saturated hydraulic conductivity, and α (L^{-1}) and n (-) are empirical constants determining the shape of the functions.

The hydraulics properties of the soil we have used are:
$\theta_r=0.102,\ \theta_s=0.368,\ K_s=796.61$ cm/d (and we have used 8m/d in the simulations), $\alpha=0..335\ \text{cm}^{-1}$ and $n=2$.

5.1 Example 1

In this case, we compare the results of the model with those obtained by using the J.R Philip's analytical solution **[12]**. We simulate water infiltration in 30-cm column of homogenenous soil under constant surface pounding **[14]**. Specifically, the pressure head at the boundaries is such that:

$$h = -75\ cm$$

$$\text{at } z = 30 \text{ cm (top)}$$

$$h = -1000\ cm$$

$$\text{at } z = 0 \text{ (bottom)}$$

Initially, the pressure is uniform throughout the column, that is,

$$h = -1000\ \ cm \quad at \quad t = 0, \quad 0 \le z \le 30\ cm$$

This problem considers infiltration into a soil with initial water content close to residual. Results for 0,25 cm grid size and 0,1 cm grid size are compared in Figure 1 and Figure 2, respectively with the results obtained from semi analytical solution of J.R. Philip.

Figure 1 shows some discrepancy between the results of numerical solution and the analytical solution, but the trends are the same. Looking at these results, we refine the grid in order to have a good agreement between numerical and analytical solutions. Results of this operation are presented in Figure 2 for grid size equal to 0.1 cm.

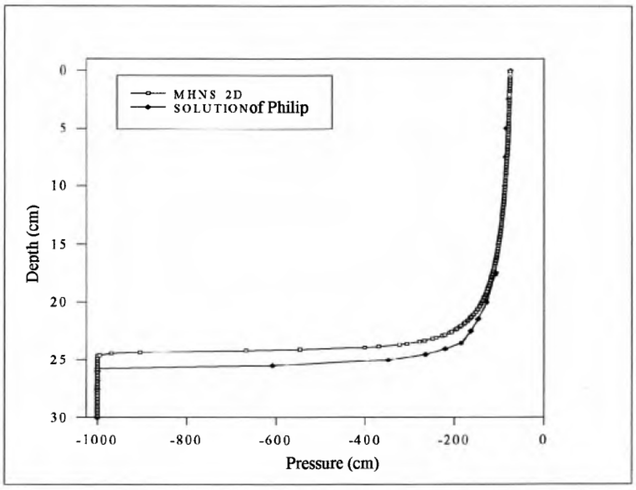

Figure 1: Comparison of pressure distribution computed by using the MHNS_2D and analytical models at
6 hours, grid size = 0.25 cm.

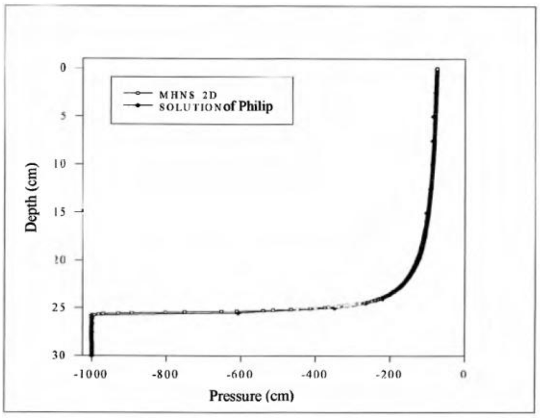

Figure 2: Comparison of pressure distribution computed by using the MHNS_2D and analytical models at
6 hours, grid size = 0.1 cm.

This test example has permitted us to verify the results of present model with the results of the analytical solution of J.R Philip. Both results are founded to be in close agreement. We obtain also a good water mass balance, 0.12 % for the case of Figure 1 and 0.06 % in the case of Figure 2. This is a good amelioration compares to those obtained when the h-based form of Richards equation is solved by traditional finite element approach **[14]**. We also don't need mass lumping to avoid overshooting or undershooting.

5.2. Example 2

In this case we have considered a vertical cross section of the soil in rectangular form with 1000 m dimension in x-direction and 10 m deep in z-direction. It receives infiltration from a canal of 4 m width and an unconfined aquifer below. The canal is located in the center. Two cycles of infiltration are considered. In this case we want to compare the aquifer response owing a constant infiltration rate and a time-varying infiltration rate. The pattern of constant infiltration rate and the time-varying infiltration are shown in Figure 3 (a) and Figure 3 (b). The infiltration operation consists of two wet periods separated by a dry one, each of 5 days duration.

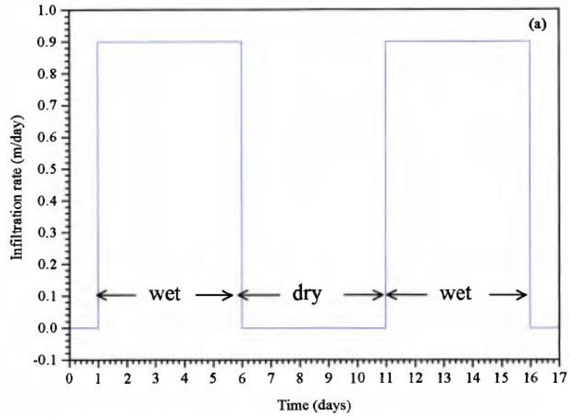

Figure 3 (a): Infiltration rate applied at the canal for constant infiltration

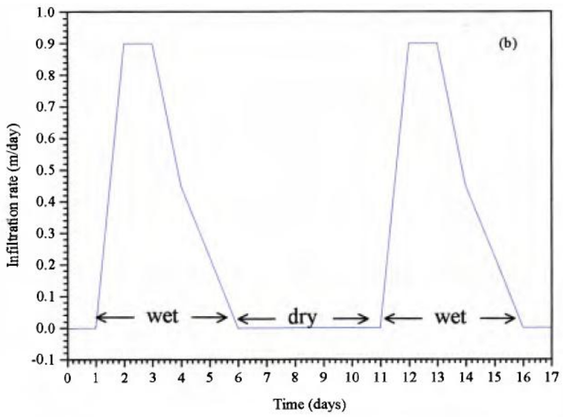

Figure 3 (b): Infiltration rate applied at the canal for time varying infiltration

Initial conditions are given firstly by considering the water table at 3 m height from the base of the aquifer and the zone above is considered as an unsaturated zone with a uniform pressure distribution of -2 m. We have done simulation without infiltration in order to obtain a steady state pressure distribution in the whole domain before the onset of the infiltration. After first simulation the water level is found at 2.6 m height from the base of the aquifer. Because of the symmetry, we have considered only one half of the domain. The domain is discretized into 6160 triangular elements with 3255 nodes and 9414 edges. Five different size of grid are used for discretization of the domain, fine grid of 0.5 m x 0.5 m size around the infiltration zone, a second large size of grid with 1 m x 0.5 m dimension size, a third large size grid of 2 m x 0.5 m dimension, a fourth large size grid of 3 m x 0.5 m dimension and the mast fifth rank, until the limit of the domain 5 m x 0.5 grid size. Numbering of nodes and elements start at the lowest left corner to the top, vertical axis is positive upward. Results of these simulations are shown below through Figure 4.

Figure 4 shows the differences in the fluctuation of zero pressure head in response to periodically applied constant infiltration rate and time varying infiltration. The difference is due to variation in the nature of infiltration rate. The magnitude of variation is maximum below the center of canal and decrease with distance away from the canal.

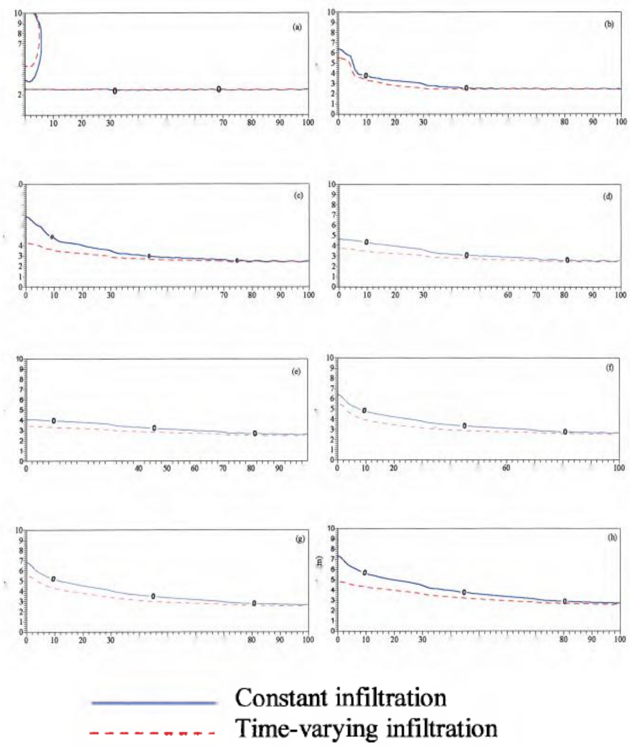

Figure 4 : Comparison between profiles of zero pressure head distribution in response to constant time varying infiltration after : (a) 2 days, (b) 3 days, (c) 5 days, (d) 7 days, (e) 10 days, (f) 12 days, (g) 13 days and (h) 15 days.

5.3 Example 3

In this example two cycles infiltration each 20 days duration separated by a dry period of 20 days as shown in Figure 5. The purpose is to compare the nature of distribution of zero pressure head due to both cycle of infiltration. This pattern of infiltration is taken from Manglik and Rai (2000) and Rai and Manglik (2000)**[9]-[13]**.

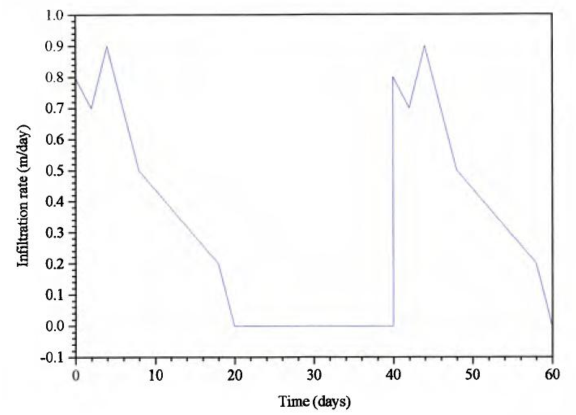

Figure 5: Time varying infiltration rate applied at the canal

Figure 6 shows some discrepancies between the distribution of zero pressure head in response to the first and the second cycle of infiltration at different times. This is principally due to the fact than the water table after 20 days of dry period has not reach its initial position as shown in Figure 7.

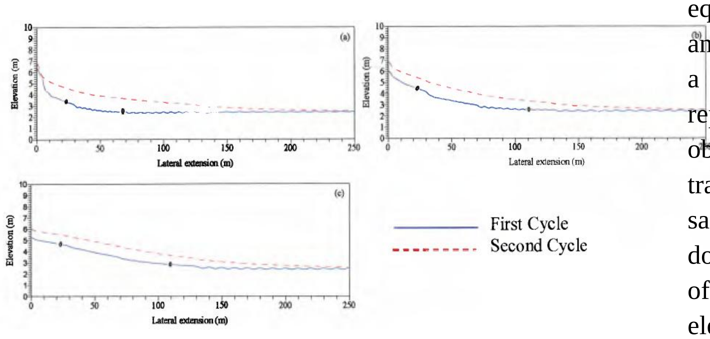

Figure 6: Comparison of profiles of zero pressure head distribution in response to time varying infiltration at: (a) 4days, (b) 10 days and (c) 20 days after the onset of both cycles of infiltration.

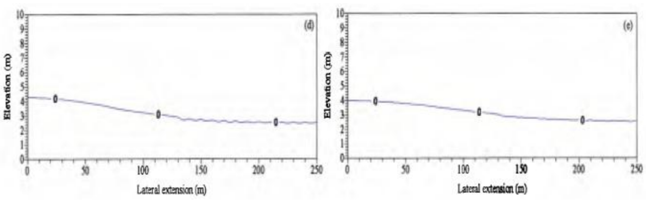

Figure 7: Distribution of zero pressure head, (d) 10 days and (e) 20 days after the onset of dry period.

The maximum growth of the water table after 10 days is 3.40 m below the infiltration site during the first cycle of infiltration and 4.2 m during the second cycle. But we note that, contrary to the case of second example, the influence of the infiltration on the water table movement is negligible after only 250 m as against 100 m for the case of the second example. This is because of the long duration of the infiltration period. These results indicate that the model is able to predict pressure head distribution in unsaturated porous media in the presence of constant or time varying infiltration. The knowledge of zero pressure head distribution in response to natural or artificial infiltration is very important to maintain the regional water balance in order to prevent environmental problems like water logging, soil salinity, etc.

CONCLUSION

In this work, we present a 2-dimensional model for the simulation of water flow in unsaturated porous media based on the mixed hybrid finite element theory. This model solves the h-based Richards equation and provides good results compared to the analytic solution of J.R. Philip **[12]**. It provides also a good water mass balance (less than 1%), which represents a good improvement compare to those obtained by Rathfelder and Abriola (1994) using traditional finite element method (9.5%) with the same temporal and spatial discretization of the flow domain. This improvement is due to the continuity of flux imposed at the interface of two adjacent elements as well as the local mass balance which is one of the properties of the mixed hybrid method. We have also introduced a scheme of approximation of time-varying infiltration (Manglik and Rai, 2000; Rai and Manglik, 2000). Results show the difference between zero pressure head distribution due to periodically applied constant and time varying infiltration. This sheme of approximation of time varying infiltration is more close to the reality particularly for natural infiltration.

Nevertheless, the weakness of the mixed hybrid approximation lies in the computational cost involved. This, because the number of unknowns is more in mixed hybrid approximation than in standard finite element or finite difference schemes. On the other hand the separate approximation of the pressure head and the Darcy's velocity in mixed hybrid approach allows for a precise determination of flow lines and propagation of contaminants. Therefore, if a good precision of pressure distribution and the velocity field is needed in any simulation case, the use of the mixed-hybrid finite element approach becomes a necessity.

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