A Simple Higher Order Theory for Bending Analysis of Steel Beams

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ABSTRACT : *A new shear deformation theory for the bending analysis of thick isotropic beams made up of steel is presented in this paper. The theory presented herein is built upon the elementary theory of beams. The transverse shear stress can be obtained directly from the constitutive relations satisfying the shear stress free surface conditions on the top and bottom surfaces of the beam, hence the theory does not require shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The simply supported thick isotropic beams are considered for the detailed numerical studies. Results of displacements and stresses are compared with those of other refined theories and exact theory to show the efficiency of proposed theory.*

Keywords *- Shear deformation, refined theory, , shear correction factor, Four variables.*

1.INTRODUCTION

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis disregards the effects of shear deformation and stress concentration. The theory is suitable for slender beams and is based on the assumptions that the transverse normal to neutral axis remains so during bending and after bending, which means transverse shear strain is zero. Thus this theory underestimates the deflection in case of thick beams where shear strain is significant.

The first-order shear deformation theory (FSDT) is an improvement over the elementary theory of beam. It is based on the hypothesis that the normal to the mid-surface before deformation remain straight but not necessarily normal to the mid-surface after deformation. This is known as first order shear deformation theory because the thickness wise displacement field for the in axial displacement is linear or of the first order. Timoshenko developed the FSDT for the displacement and stress variations for the thick prismatic bars. But the deficiencies in ETB and FSDT still exist. Therefore, higher order shear deformation theories are developed to obtain the improved results for the thick beams. In these theories the displacement field is expanded up to the third power of thickness coordinate of beams to have the parabolic variation of transverse shear stresses. The numbers of displacement variables are more in these theories. The higher order theory is developed by Reddy to get the parabolic shear stress distribution through the thickness of beam and to satisfy the shear stress free surface conditions on the top and bottom surfaces of the beam to avoid the need of shear correction factors.

Timoshenko [1] proposed a hypothesis for the development of first order shear deformation theory which states that the plane section which is perpendicular to the neutral axis before bending remains plane but not necessarily perpendicular to the neutral axis after bending. In this theory the transverse shear strain distribution over the cross-section of the beam is assumed to be constant through the thickness and thus require shear correction factor. Levinson [2] presented parabolic shear deformation theories assuming a higher order variation of in-plane displacement in terms of thickness coordinates. Sayyad and Ghugal [3] proposed a new hyperbolic shear deformation theory is developed for the static flexure of thick isotropic beam, considering hyperbolic functions in terms of thickness co-ordinate associated with transverse shear deformation effect. Rotation of normal is taken as combined effect of shear slope and bending slope at the neutral axis. The most important feature of the theory is that the transverse shear stress can be obtained directly from the constitutive relations satisfying the shear stress free surface conditions on the top and bottom of the beam. Hence the theory obviates the need of shear correction factor.
Comprehensive reviews of higher order shear deformation theories have been given by Ghugal and Shimpi [4]. The theories are reviewed for the both isotropic and anisotropic laminated beams. Ghugal and Sharma [5] have developed a variationally consistent refined hyperbolic shear deformation theory for flexure and free vibration of thick isotropic beam. Recently Ghugal and Nakhate [6] has developed trigonometric shear deformation theory for the static flexure of thick isotropic beam and obtained the general solution of thick isotropic beam with various support and loading conditions. Sayyad, Ghugal and Borkar [8] analyzed the single layer fibrous composite beam using several displacement based shear deformation theories. A

static flexural analysis is carried out for simply supported fibrous composite beams subjected to different mechanical loadings. The results obtained using all the theories are compared with those obtained by exact elasticity solution for a sinusoidal load and then their validity is checked for other loading conditions.

Ghugal and Dahake [9] introduced a trigonometric shear deformation theory for flexure of thick or deep beams, taking into account transverse shear deformation effects. The number of variables in the present theory is same as that in the first order shear deformation theory. The sinusoidal function is used in displacement field in terms of thickness coordinate to represent the shear deformation effects. The fixed isotropic beams subjected to parabolic loads are examined using the present theory. Sayyad [10] presented, a refined shear deformation theory is developed for the static flexure and free vibration analysis of thick isotropic beams, considering sinusoidal, hyperbolic and exponential functions in terms of thickness co-ordinate associated with transverse shear deformation effect. Rotation of normal is taken as combined effect of shear slope and bending slope at the neutral axis. Vinay, Raju, Adil Dar and Manzoor[11]focuses on a numerical model developed for Hollow tube and concrete filled steel tube (CFST) columns under monotonic loading and. The study was conducted using MSC NASTRAN.PATRAN finite element software. Three dimensional nonlinear finite element models developed to study the force transfer between steel tube and concrete core. 8 Noded Hexagonal elements are considered for finite element analysis. Analysis are done for 3 different rates of concrete M20, M30 & M40. 1%, 2%, 3% Epoxy used as infill in M20, M30, M40 .For different lengths of specimens that is 250mm, 350mm, 450mm with the different thickness 3.2mm,4mm and 4.2mm and different Diameter that is 33.4mm,48.3mm,60.3mm are to be analysed. Analysis was run for both Hollow tubes and Concrete filled steel tube (CFST). Result of Analytical solution was compared with Experimental results and Design code such as EUROCODE 4, ACI CODE, BS5400 CODE. Kumar and Ravish[12] introduced the use of fiber reinforced polymer (FRP) reinforcements in concrete structures has increased rapidly in the last 10 years due to their excellent corrosion resistance, high tensile strength, and good non-magnetization properties. Fiber-reinforced polymer (FRP) application is a very advanced method for the purpose of repair and strengthens structures which are weak during their life span. FRP repair systems provide an economical and effective method for repairing repair systems and as a material.

In this paper a displacement based higher order shear deformation theory (HOSDT) is used for the bending analysis of thick isotropic beams which includes effect of transverse shear deformation and rotary inertia. The displacement field of the theory contains one variable of beam. The theory is shown to be simple and more effective for the bending analysis of isotropic beams.

2. Theoretical formulation

2.1. Isotropic Beam under Consideration

Consider a beam made up of isotropic material. The plate occupies a region $0 \le x \le a,\ -h/2 \le z \le h/2$, where, ‘a’ is length and ‘h’ is total thickness of beam.

2.2. Assumptions Made In the Present Theory

Theoretical formulation of present theory is based on the following assumptions.

1. The displacements are small in comparison with the beam thickness and therefore strains involved are infinitesimal.
2. The displacements u in x-direction consists of extension, bending, and shear components
3. The transverse displacement includes two components i.e. bending (w_b) and shear (w_s)
4. The beam is subjected to transverse load only.
5. The body forces are neglected.

2.3. The Displacement Field

Based upon the before mentioned assumptions, the displacement field of the proposed beam theory is given as below:

$$u(x) = u_0(x) - z\frac{dw_b(x)}{dx} - f(z)\frac{dw_s(x)}{dx} \quad (1)$$

$$w(x) = w_b(x) + w_s(x) \quad (2)$$

2.4. Strain-Displacement Relationships

Normal strains ε_x and shear strains γ_{zx} are obtained within the framework of linear theory of elasticity using the displacement field given by Eq. (1).

$$\varepsilon_x = \frac{du}{dx} = \frac{du_0}{dx} - z\frac{d^2 w_b}{dx^2} - f(z)\frac{d^2 w_s}{dx^2} \quad (3)$$

$$\gamma_{zx} = \left(\frac{du}{dz} + \frac{dw}{dx}\right) = \frac{d}{dz}\left(u_0(x) - z\frac{dw_b(x)}{dx} - f(z)\frac{dw_s(x)}{dx}\right) + \frac{d}{dx}\left(w_b(x) + w_s(x)\right)$$

$$\gamma_{zx} = \left(1 - \frac{4z^2}{h^2}\right).\frac{dw_s}{d_x} \quad (4)$$

2.5. Stress-Strain Relationships

$$\sigma_x = E.\varepsilon_x = E.\left(\frac{du_0}{dx} - z\frac{d^2 w_b}{dx^2} - f(z)\frac{d^2 w_s}{dx^2}\right) \quad (5)$$

$$\tau_{xz} = G.\gamma_{zx} = \frac{E}{2(1+\mu)}.\gamma_{zx} \tag{6}$$

3. Governing Equations and Boundary Conditions

The variationally consistent governing equation of equilibrium and boundary conditions associated with the present theory can be derived using the principle of virtual work. The analytical form of principle of virtual work can be written as-

$$\int_0^L q(x)dx = \int_0^L \int_{-h/2}^{h/2} (\sigma_x.\delta\varepsilon_x + \tau_{zx}.\delta\gamma_{zx})dx.dz \tag{7}$$

Separating above equation for further calculation,

$$\int_0^L \int_{-h/2}^{h/2} \tau_{zx}.\delta\gamma_{zx}.dx.dz = \int_0^L \int_{-h/2}^{h/2} \frac{E}{2(1+\mu)}.\left(1-\frac{4z^2}{h^2}\right).\frac{dw_s}{d_x}.\left(1-\frac{4z^2}{h^2}\right).\frac{d\delta w_s}{d_x}dx.dz \tag{8}$$

Where δ be the arbitrary variations. Integrating above equation by parts and collecting the coefficients of $\delta u_0, \delta w_b$ and δw_s to obtain governing equation of equilibrium and boundary conditions associated with the present theory.

The governing equations of equilibrium are as follows:

$$A_0 \frac{\partial^2 u_0}{\partial x^2} - B_0 \frac{\partial^3 w_b}{\partial x^4} - C_0 \frac{\partial^3 w_s}{\partial x^3} = 0 \tag{9a}$$

$$-B_0 \frac{\partial^3 u_0}{\partial x^3} + D_0 \frac{\partial^4 w_b}{\partial x^4} + E_0 \frac{\partial^4 w_s}{\partial x^4} = q \tag{9b}$$

$$-C_0 \frac{\partial^3 u_0}{\partial x^3} + E_0 \frac{\partial^4 w_b}{\partial x^4} + F_0 \frac{\partial^4 w_s}{\partial x^4} - A_{00} \frac{\partial^2 w_s}{\partial x^2} = q \tag{9c}$$

Where, $A_0 = \int_{-h/2}^{h/2} E.dz \quad B_0 = -\int_{-h/2}^{h/2} E.z.dz$

$$C_0 = -\int_{-h/2}^{h/2} E.f(z).dz \quad D_0 = \int_{-h/2}^{h/2} E.z^2.dz$$

$$E_0 = \int_{-h/2}^{h/2} E.z.f(z).dz \quad F_0 = \int_{-h/2}^{h/2} E.f^2(z).dz$$

$$A_{00} = \int_{-h/2}^{h/2} G.(1-f'(z))^2.dz \tag{10}$$

4. Illustrative Examples

A simply supported isotropic beam occupying the region given by the Eq. (1) is considered. The beam is subjected to uniformly distributed transverse load, $q(x)$ on surface $z = -h/2$ acting in the downward z-direction as given below:

$$q(x) = \sum_{m=1,3,5}^{\infty} q_m \sin\frac{m\pi x}{L} \tag{11}$$

Uniformly distributed load

$$q_m = \frac{4q_0}{m\pi^2} \quad \text{For } m = 1,3,5,...,$$

Sinusoidal load

$$q_m = q_0 \tag{12}$$

The following solution form is assumed for unknown displacement variable $\delta w_b, \delta w_s$ satisfying the boundary conditions for simply supported beam exactly.

$$\begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \begin{Bmatrix} w_{bm} \sin\frac{m\pi x}{L} \\ w_{sm} \sin\frac{m\pi x}{L} \end{Bmatrix} \tag{13}$$

Where w_{bm} and are arbitrary constants, which can be calculated by substituting the above solutions in governing differential equations resulting in to following equations(shown in Matrix form).Substitution of this form of solution and transverse load q(x) into governing equations leads to following equations.

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} u_m \\ w_{bm} \\ w_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_m \\ q_m \end{Bmatrix} \tag{14}$$

Where elements of stiffness matrix $[K]$ are as follows:

$$K_{11} = A_0 \frac{m^2\pi^2 x^2}{L^2}$$

$$K_{13} = K_{31} = -C_0 \frac{m^3\pi^3 x^3}{L^3}$$

$$K_{22} = D_0 \frac{m^4\pi^4 x^4}{L^4} \tag{15}$$

$$K_{23} = K_{32} = E_0 \frac{m^4\pi^4 x^4}{L^4} \quad K_{33} = -F_0 \frac{m^4\pi^4 x^4}{L^4}$$

Obtaining u_m, w_{bm} and w_{sm} from the equations 13 and 14 one can calculate all displacements and stresses.

5. Numerical Results and Discussion

Following non dimensional parameters are used to analyze the isotropic steel beam under various loading conditions.

5.1 Numerical Results

Results obtained for displacements and stresses will now be compared and discussed with the corresponding results of higher order shear deformation theory (HSDT) of Reddy, (FSDT) by Timoshenko, hyperbolic shear deformation theory by Sayyad and Ghugal and ETB theory by Bernoulli-Euler. The numerical results are presented in the following non-dimensional form and following material properties of isotropic beams are used.

$$\bar{u} = \frac{buE}{q_0 h} \quad \bar{w} = \frac{100Ewh^3}{q_0 L^4} \quad \bar{\sigma}_x = \frac{b\sigma_x}{q_0}$$

$$\bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0} \qquad (16)$$

Material Properties: E = 210 GPa

μ = 0.3

$$G = \frac{E}{2(1+\mu)}$$

Table1. Comparison of non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beam subjected to sinusoidal loads.

S	Theory	Method	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
4	Present		12.715	1.4291	9.9863	1.9062
	Sayyad and Ghugal	TSDT	12.704	1.427	9.977	1.894
	Reddy	HSDT	12.715	1.429	9.986	1.906
	Timoshenko	FSDT	12.385	1.43	9.727	1.27
	Bernoulli-Euler	ETB	12.385	1.232	9.727	_
	Ghugal	Exact	12.297	1.411	9.958	1.9
10	Present		194.33	1.2635	61.052	4.7732
	Sayyad and Ghugal	TSDT	194.31	1.263	61.04	4.745
	Reddy	HSDT	194.34	1.264	61.05	4.773
	Timoshenko	FSDT	193.51	1.264	60.79	3.183
	Bernoulli-Euler	ETB	193.51	1.232	60.79	_
	Ghugal	Exact	192.95	1.261	60.91	4.771

Table2. Comparison of non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beam subjected to uniformly distributed loads.

S	Theory	Method	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
4	Present		16.503	1.8059	12.2631	2.981
	Sayyad and Ghugal	TSDT	16.486	1.804	12.254	2.882
	Reddy	HSDT	16.504	1.806	12.263	2.908
	Timoshenko	FSDT	16	1.806	12	1.969
	Bernoulli-Euler	ETB	16	1.563	12	_
	Timoshenko and Goodier	Exact	15.8	1.785	12.2	3.00
10	Present		251.27	1.6025	75.2674	7.3605
	Sayyad and Ghugal	TSDT	251.23	1.601	75.259	7.312
	Reddy	HSDT	251.27	1.602	75.268	7.361
	Timoshenko	FSDT	250	1.602	75	4.922
	Bernoulli-Euler	ETB	250	1.563	75	_
	Timoshenko and Goodier	Exact	249.5	1.598	75.2	7.5

Table3. Comparison of non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beam subjected to sinusoidal, uniformly distributed load and linearly varying load.

S	Theory	Method	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
4	Present		8.2519	0.9029	6.1315	1.454
	Sayyad and Ghugal	TSDT	8.243	0.902	6.127	1.441
	Reddy	HSDT	8.252	0.903	6.1315	1.454
	Timoshenko	FSDT	8	0.903	6	0.9845
	Bernoulli-Euler	ETB	8	0.7815	6	_
	Timoshenko and Goodier	Exact	7.9	0.8925	6.1	1.5

10	Present		125.63	0.800	37.633	3.680
	Sayyad and Ghugal	TSDT	125.61	0.800	37.628	3.656
	Reddy	HSDT	125.63	0.801	37.634	3.6806
	Timoshenko	FSDT	125	0.801	37.5	2.461
	Bernoulli-Euler	ETB	125	0.7815	37.5	_
	Timoshenko and Goodier	Exact	124.75	0.800	37.6	3.75

Table4. Non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beams for different aspect ratios and subjected to sinusoidal load.

S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
2	1.712	2.0162	2.6897	0.9477
4	12.715	1.4291	9.9863	1.9062
10	194.337	1.2635	61.0526	4.7732
25	3025.650	1.237	380.215	11.936
50	24192.787	1.2332	1520.08	23.873
100	193517.484	1.2322	6079.53	47.7464

Table5. Non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beam for different aspect ratios and subjected to uniformly distributed load.

S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
2	2.245	2.5315	3.2611	1.4152
4	16.504	1.8059	12.2631	2.9081
10	251.273	1.6015	75.2674	7.3605
25	3909.410	1.5687	469.03	18.4483
50	31256.106	1.5641	1875.32	36.911
100	250010.484	1.5629	7500.49	73.8291

Table6. Non-dimensional inplane displacement ($\bar{u}$), Transverse displacement ($\bar{w}$), inplane normal stress ($\bar{\sigma}_x$) and transverse shear stress $\bar{\tau}_{zx}$ in isotropic steel beams for different aspect ratios and subjected to linearly varying load.

S	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$
2	1.123	1.26575	1.63055	0.7076

4	8.252	0.90295	6.13155	1.45405
10	125.637	0.80075	37.6337	3.68025
25	1954.705	0.78435	234.515	9.22415
50	15628.053	0.78205	937.661	18.4555
100	125005.242	0.78145	3750.24	36.9146

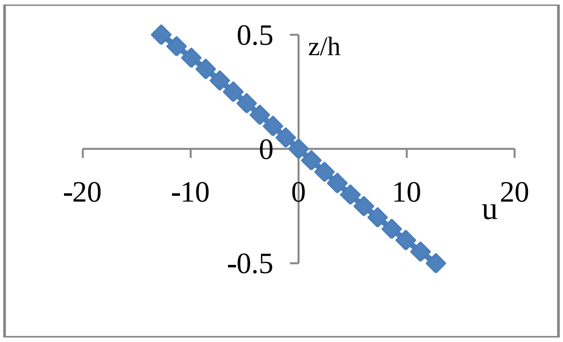

Fig.1. Through thickness variation of in-plane displacement of isotropic plates subjected to sinusoidal load for aspect ratio 4.

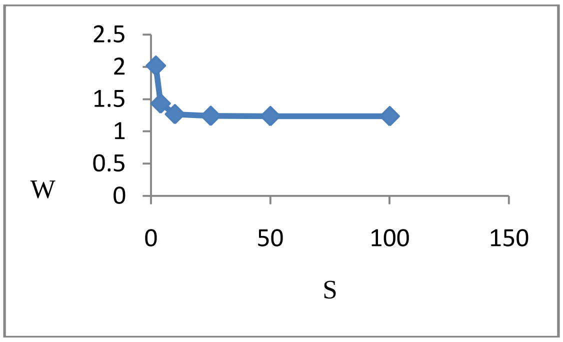

Fig. 2.Through thickness variation of transverse displacement of isotropic plates subjected to sinusoidal load for different aspect ratios.

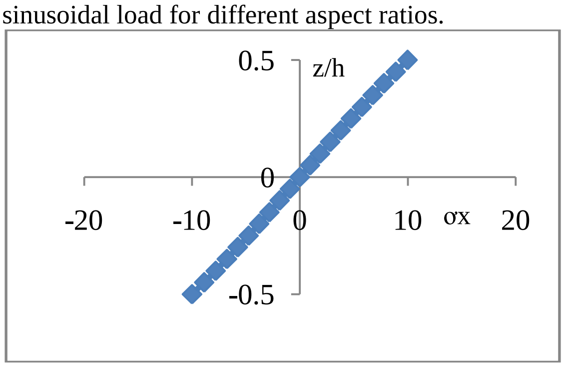

Fig.3.Through thickness variation of in-plane normal stress of isotropic plate subjected to sinusoidal load for aspect ratio 4.

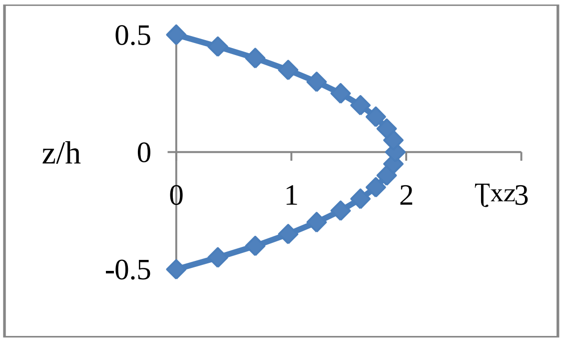

Fig.4. Through thickness variation of transverse shear stress of isotropic plate subjected to sinusoidal load for aspect ratio 4.

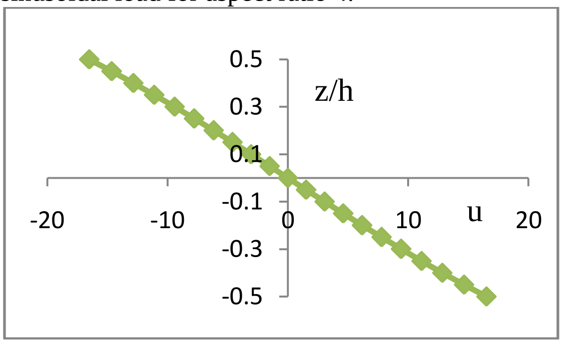

Fig.5. Through thickness variation of inplane displacement of isotropic plates subjected to uniformly distributed load for aspect ratio 4.

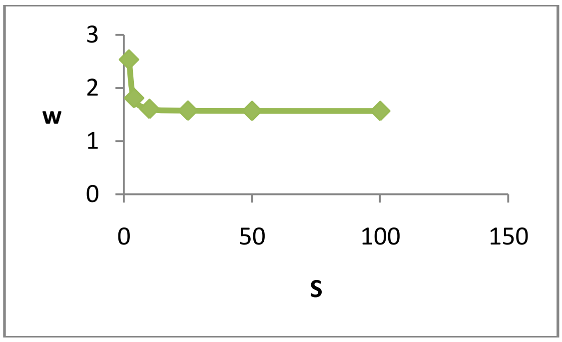

Fig. 6.Through thickness variation of transverse displacement of isotropic plates subjected to uniformly distributed load for different aspect ratios.

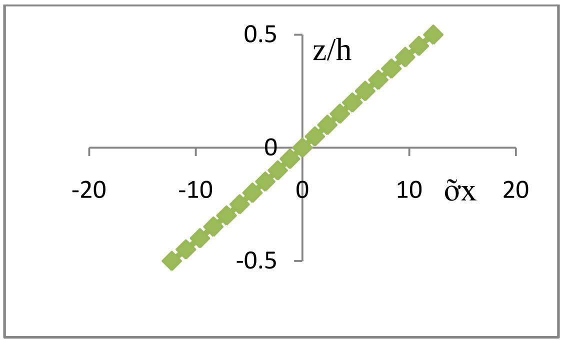

Fig.7.Through thickness variation of in plane normal stress of isotropic plate subjected to uniformly distributed load for aspect ratio 4.

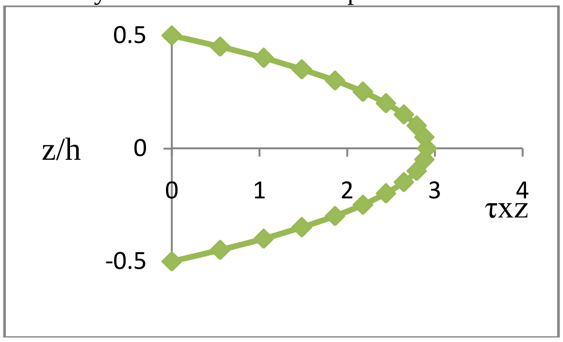

Fig.8.Through thickness variation of transverse shear stress of isotropic plate subjected to uniformly distributed load for aspect ratio 4.

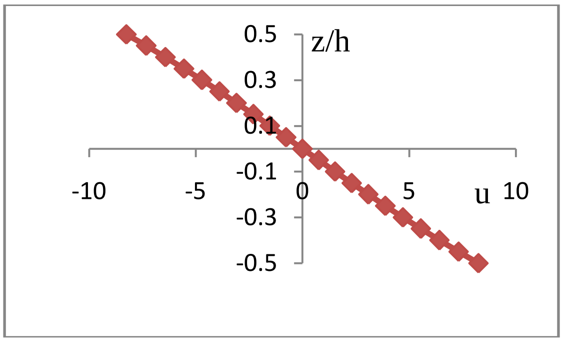

Fig.9.Through thickness variation of in-plane displacement of isotropic plates subjected to linearly varying load for aspect ratio 4.

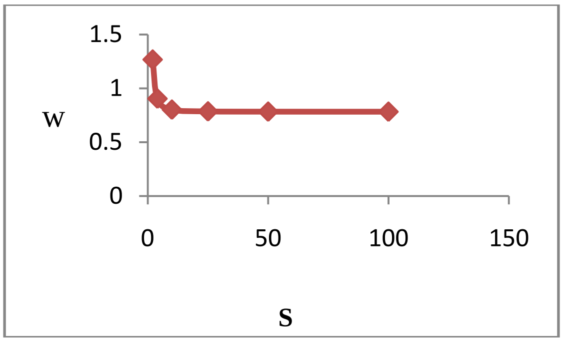

Fig. 10.Through thickness variation of transverse displacement of isotropic plates subjected to linearly varying load for different aspect ratios

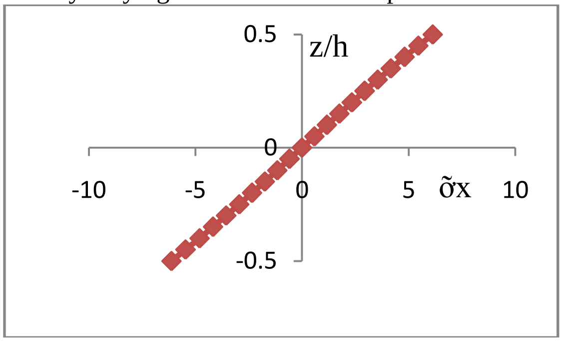

Fig. 11.Through thickness variation of in plane normal stress of isotropic plate subjected to linearly varying load for aspect ratio 4.

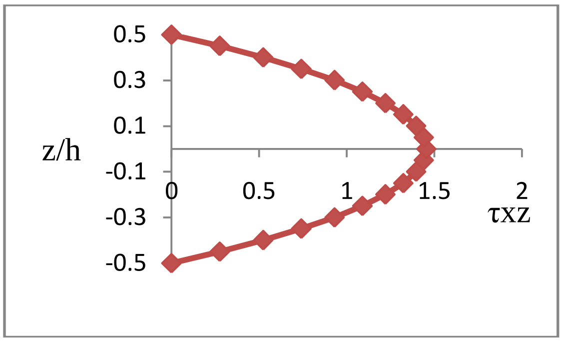

Fig. 12.Through thickness variation of transverse shear stress of isotropic plate subjected to linearly varying load for aspect ratio 4.

5.2 Discussion of Results

Table 1 shows the comparison of maximum displacements and stresses for the isotropic beam (steel) subjected to sinusoidal load. The present theory overestimates and other higher Order theories underestimate the results of in-plane displacement as compared to those of exact solution. Through thickness variation of in-plane displacement for isotropic beam subjected to sinusoidal load is shown in Fig. 2. The HSDT and FSDT overestimate the value of maximum transverse deflection for aspect ratio 4 respectively while the ETB underestimates the value of maximum transverse displacement for aspect ratios 4 and 10 respectively due to neglect of transverse shear deformation. The value of maximum normal bending stress obtained by present theory is in tune with the exact solution for all aspect ratios. Theory of Reddy overestimates the normal bending stress for aspect ratios 4 and 10 respectively compared to those of exact values. The values of normal bending stress predicted by FSDT and ETB are identical for all aspect ratios. FSDT and ETB underestimate the value of normal bending stress for aspect ratios 4 and 10 respectively as compared to exact value. The transverse shear stress satisfies the stress free boundary conditions on the top and bottom surfaces of the beam when obtained by constitutive relation. The FSDT yields lower value of transverse shear stress when obtained using constitutive relation. The maximum transverse shear obtained by present theory using equations of equilibrium is in excellent agreement with that of exact solution for the aspect ratios 4 and 10. The present theory, HSDT, FSDT and ETB gives identical values of this stress for aspect ratio 10 when obtained using equation of equilibrium.

Comparison of displacements and stresses for the isotropic beams subjected to uniformly distributed load are shown in Table 2. Through thickness variation of displacement and stresses for the isotropic beam subjected to uniformly distributed load for aspect ratio 4 are shown in Figs. 5 through 8.

The comparison of axial displacement for isotropic beam subjected to linearly varying load is shown in Table 3. The examination of Table 3 reveals that the axial displacement predicted by present theory is in excellent agreement with that of exact solution for aspect ratio 10 whereas HSDT of Reddy overestimates the same. The axial displacement predicted by FSDT and ETB are identical for both the aspect ratios. FSDT and ETB show the identical values for transverse displacement for both the aspect ratios. The axial bending stress predicted by present theory is in excellent agreement with that of exact solution whereas FSDT and ETB underestimate the same for both the aspect ratios. The through thickness variation of axial bending stress for the isotropic beam subjected to linearly varying load for aspect ratio 4 is shown in Fig. 11. The transverse shear stress predicted by present theory is in excellent agreement when obtained using constitutive relations. The through thickness variation of transverse shear stress for isotropic beam subjected to linearly varying load is shown in Figs 12.

Table 4, 5 and 6 shows the maximum values for corresponding displacements and stresses for sinusoidal load, uniformly distributed load and linearly varying load respectively.

6. Conclusions

From the study of bending analysis of thick isotropic beams by using higher order shear deformation theory (HOSDT), following conclusions are drawn:

1. The results of displacements and stresses obtained by present theory for the all loading cases are in excellent agreement with those of exact solution.
2. The present theory satisfies the shear stress free surface conditions on the top and bottom surfaces of the plate.
3. Theory is variationally consistent.
4. Theory avoid the need of shear correction factors
5. The results of transverse displacement, in-plane normal, in-plane shear and transverse shear stresses obtained are identical for isotropic beams and for all the loading cases. This shows that stresses are independent of modulus of elasticity when material is homogeneous.

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