Bending Analysis of Simply Supported and Clamped Circular Plate

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ABSTRACT : *The aim of study is static bending analysis of an isotropic circular plate using analytical method i.e. Classical Plate Theory and Finite Element software ANSYS. Circular plate analysis is done in cylindrical coordinate system by using Classical Plate Theory. The axisymmetric bending of circular plate is considered in the present study. The diameter of circular plate, material properties like modulus of elasticity (E), poissons ratio (μ) and intensity of loading is assumed at the initial stage of research work. Both simply supported and clamped boundary conditions subjected to uniformly distributed load and center concentrated / point load have been considered in the present study. In this research work, the effect of varying thickness of the plate on its deflection and bending stress is studied. So, the key point of research work is thickness variation of plate. Modeling and analysis of circular plate is done in ANSYS APDL.4noded shell 181 element is used for modeling of circular plate. Once deflection is obtained by using CPT, bending moments and bending stresses are easily calculated by usual relations. Analytical results of CPT are validated with ANSYS results.*

Keywords*-Isotropic circular plate, Classical Plate Theory (CPT), ANSYS APDL, 4noded Shell 181.*

I. INTRODUCTION

Plates are extensively used in many engineering applications like roof and floor of building, deck slab of bridge, foundation of footing, water tanks, turbine disks etc. Plates used in such applications are normally subjected to lateral loads, causing bending of the plate. Bending of plates or plate bending refers to the deflection of a plate perpendicular to the plane of plate under the action of external forces and moments. Hence bending analysis of plate is of utmost importance. The geometry of the plate normally defined by middle plane which is plane equidistance from the top and bottom faces of the plate. The flexural properties of plate largely depend on its thickness rather than its two dimensions (length and width). The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. The stresses in the plate can be calculated from these deflections. Once the stresses are known, failure theories can be used to determine whether a plate will fail under a given load [1].

Vanam et al.[2] done static analysis of an isotropic rectangular plate using finite element analysis (FEA).The aim of study was static bending analysis of an isotropic rectangular plate with various boundary conditions and various types of load applications. In this study finite element analysis has been carried out for an isotropic rectangular plate by considering the master element as a four noded quadrilateral element. Numerical analysis has been carried out by developing programming in mathematical software MATLAB and the results obtained from MATLAB are giving good agreement with the results obtained by classical method - exact solutions. Later, for the same structure, analysis has been carried out using finite element analysis software ANSYS.

Kavade et al.[3] analyzed circular plate with a hole by FEM. In this research work they analyzed a plate with a circular hole subjected to a uniform stress, the effect of an initial stretching of a rectangular plate with a cylindrical hole on the stress and displacement distributions around the hole, which are caused by the additional loading, was studied using the finite element method. It is assumed that the initial stresses are caused by the uniformly stretching forces acting on the 2 opposite ends. It is also assumed that the cylindrical hole contained by the thick plate is between these ends and goes in parallel with them. They analyzed a plate with a circular hole subjected to a uniform stress and observed the variation in the results obtained through various meshes.

Wang et al.[4] studied three-dimensional solution of axisymmetric bending of functionally graded circular plates. They investigated the axisymmetric bending of transversely isotropic and functionally graded circular plates subject to arbitrarily transverse loads using the direct displacement method based on three-dimensional theory. Kirstein and Woolley[5] studied symmetrical bending of thin circular elastic plates on equally spaced point supports. Solution for a thin circular elastic plate supported at points equally spaced along a concentric support circle and subjected to a transverse load which is symmetrically distributed over a concentric circular area is obtained. Sukla [6] studied the FEM analysis on circular stiffened plates using ANSYS. He analyzed the structural behavior of a stiffened plate under static uniform loads. Analysis of a circular plate with radial stiffeners was done. The linear and nonlinear behavior of the beams is studied under static and dynamic loading .The simulation analysis is completed with a numerical analysis of the system using the ANSYS program.

Khobragade and Deshmukh [7] studied thermal deformation in a thin circular plate due to a partially distributed heat supply. In this research work, they developed an integral transform to determine temperature distribution in a thin circular plate, subjected to a partially distributed and axisymmetric heat supply on the curved surface and studied thermal deformation. Meghare and Jadhav [8] presented a simple higher order theory for bending analysis of steel beams. A new shear deformation theory for the bending analysis of thick isotropic beams made up of steel is studied. The theory presented herein is built upon the elementary theory of beams. The transverse shear stress can be obtained directly from the constitutive relations satisfying the shear stress free surface conditions on the top and bottom surfaces of the beam, hence the theory does not require shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The simply supported thick isotropic beams are considered for the detailed numerical studies. Results of displacements and stresses are compared with those of other refined theories and exact theory to show the efficiency of proposed theory.

Manzoor et al.[9] focuses on a numerical model developed for Hollow tube and concrete filled steel tube (CFST) columns under monotonic loading and. The study was conducted using MSC NASTRAN, PATRAN finite element software. Three dimensional nonlinear finite element models developed to study the force transfer between steel tube and concrete core. 8 Noded Hexagonal elements are considered for finite element analysis. Analysis is done for 3 different grades of concrete M20, M30 & M40. 1%, 2%, 3% Epoxy is used as infill in M20, M30, M40 .For different lengths of specimens that is 250mm, 350mm, 450mm with the different thickness 3.2mm,4mm and 4.2mm and different diameter that is 33.4mm,48.3mm,60.3mm are to be analyzed. Analysis was run for both Hollow tubes and Concrete filled steel tube (CFST). Result of Analytical solution was compared with Experimental results and Design code such as EUROCODE 4, ACI CODE.

In this paper classical plate theory is used for bending analysis of isotropic circular plate. Analytical results obtained from CPT are validated with the results of ANSYS software. The bending or static analysis of thin circular plate includes the use of cylindrical coordinate system in which normal and shear stresses are functions of two coordinates viz. r and θ. This theory does not take in to account the shear effects.

2. METHODOLOGY

Classical Plate Theory is used for calculating flexural parameters of an isotropic circular plate. Following are the various cases of the plate which are considered for present study:-

1) Simply supported circular plate subjected to Uniformly Distributed Load.
2) Clamped circular plate subjected to Uniformly Distributed Load.
3) Simply supported circular plate subjected to Centre concentrated or Point Load.
4) Clamped circular plate subjected to Centre concentrated or Point Load.

Following are the standard values which are assumed at the initial stage of research work:-

Plate Diameter = 200mm

Poisson's ratio of steel (μ) = 0.3

Young's modulus of steel (E) = 200 GPa

Distributed Load = 275 kPa

Centre Concentrated Load = 275 kN.

Thickness of the plate is varied throughout for various boundary and loading conditions as 10mm, 20mm, 50mmm, 100mm, 200mm and 500mm. Deflections and bending stresses are calculated for various thicknesses of the plate.

2.1 Classical Plate Theory

Classical Plate Theory is the thin plate theory based on Love-Kirchhoff's hypothesis which makes assumptions similar to those made by the Bernoulli-Navier hypothesis used in the theory of thin or shallow beams. It is also called as small deflection theory.

2.2 Assumptions in Classical Plate Theory

The following fundamental assumptions are made in the classical small deflection theory of thin homogenous elastic plates.

1. Straight line initially normal to the middle surface to the plate remains straight and normal to the deformed middle surface of the plate and unchanged in length.

2. Displacement w is assumed to be very small. This means the slope of the deflected surface is small and hence square of the slope would be negligible in comparison with unity.
3. The normal stresses σ_x and σ_y in plane shear stress τ_{xy} are assumed to be zero at middle surface of the plate. i.e. $(w << h)$.
4. Stress σ_z i.e. transverse normal stress is small as compared to other stress components and may be neglected in stress strain relationship.
   $\sigma_z << (\sigma_x, \sigma_y, \tau_{xy})$
5. The mid plane remains unstrained after bending.

The above assumptions, known as Kirchhoff's hypothesis, reduces the three dimensional plate problem to two dimensions. Hence only normal stresses and shear stresses would exists in the plate. In cylindrical coordinate system these stresses are the functions of r and θ.

2.3Governing Differential equation for symmetrical bending of circular plate:

$$\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr} = \frac{Q}{D}$$

In other form it can be written as

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D}$$

3. INDENTATIONS AND EQUATIONS

3.1 Simply supported circular plate subjected to uniformly distributed load

The deflection, moment and transverse shear are to be finite at the center of the plate (r = 0). Therefore deflection is maximum at the center of the plate i.e. at r = 0 and it is given by

$$\text{Deflection } (w)_{\max} = \frac{qa^4}{64D}\frac{(5+\mu)}{(1+\mu)}$$

Maximum bending stress will occur at center of the plate i.e. at r = 0 and it is given as follows

$$(\sigma_r)_{\max} = \frac{3}{8}q\eta^2(3+\mu)$$

$$(\sigma_\theta)_{\max} = \frac{3}{8}q\eta^2(3+\mu)$$

Where [$\eta = \frac{a}{h}$ =Aspect Ratio]

Therefore maximum stresses in r and θ are same .Hence

$$(\sigma_r)_{\max} = (\sigma_\theta)_{\max}$$

3.2Clamped circular plate subjected to uniformly distributed load

Deflection is maximum at the center of the plate i.e. at r = 0 and it is given by

$$\text{Deflection } (w)_{\max} = \frac{qa^4}{64D}$$

Bending stresses in the plate are to be found out from following equations

$$(\sigma_r)_{\max} = \frac{-3}{4}q\eta^2$$

$$(\sigma_\theta)_{\max} = \frac{-3}{4}q\eta^2$$

Therefore maximum stresses in r and θ are same. Hence

$$(\sigma_r)_{\max} = (\sigma_\theta)_{\max}$$

3.3 Simply supported circular plate subjected to Centre concentrated / Point Load

For simply supported plate subjected to center point load, maximum deflection is at the center of the plate and is given by following equation

$$\text{Deflection } (w)_{\max} = \frac{q}{16\pi D}\left(\frac{3+\mu}{1+\mu}\right)a^2$$

Bending stresses are to be find out using following equations

$$(\sigma_r) = \frac{3q}{2\pi h^2}\left[\log\left(\frac{r}{a}\right)(1+\mu)\right]$$

$$(\sigma_\theta) = \frac{3q}{2\pi h^2}\left[\log\left(\frac{r}{a}\right)(1+\mu)+(1-\mu)\right]$$

3.4 Clamped circular plate subjected to Centre concentrated /Point Load

The deflection, moment and transverse shear are to be finite at the center of the plate (r = 0). Therefore deflection is maximum at the center of the plate i.e. at r = 0 and it is given by

$$\text{Deflection } (w)_{max} = \frac{qa^2}{16\pi D}$$

At r =a

$$(\sigma_r)_{max} = -\frac{3q}{2\pi h^2} \qquad (\sigma_\theta)_{max} = -\frac{3\mu q}{2\pi h^2}$$

3.5 ANSYS Solution

Thin circular plate for various boundary conditions and loading conditions is analyzed by using ANSYS APDL.ANSYS is one of the finite element analysis based software which is numerical technique based. The plate geometry is modeled in ANSYS APDL. Shell 181 4 noded element is used for circular plate analysis. This element is 2D element with six degrees of freedom. Deflections and bending stresses are calculated for various cases considered for present research work. Here meshing is done by mapped mesh with quad element. ANSYS results are validated with the analytical solutions (i.e.CPT results).

4. ILLUSTRATIVE EXAMPLE

The analytical results obtained from Classical Plate Theory are compared with ANSYS software solution for validation. Deflections and bending stresses are calculated to study the flexural behavior of an isotropic circular plate.

Analytical solution for simply supported circular plate subjected to UDL with 10 mm thickness is done as follows:-

Deflection of a simply supported circular plate is given as follows

$$\text{Deflection } (w)_{max} = \frac{qa^4}{64D}\frac{(5+\mu)}{(1+\mu)}$$

Flexural rigidity is given by following equation

$$D = \frac{Eh^3}{12(1-\mu^2)} = \frac{200\times10^9\times(0.010)^3}{12(1-0.3^2)}$$

$$= 18315.02$$

$$\text{Deflection } (w)_{max} = \frac{275\times10^3\times(0.1)^4\times5.3}{64\times18315.02\times1.3}$$

$$= 9.56 \times 10^{-5}\text{m} = 0.0956 \text{ mm.}$$

Bending Stresses in the plate are found out by following equations

$$(\sigma_r)_{max} = \frac{3}{8}q\eta^2(3+\mu)$$

$$= \frac{3}{8}\times275\times10^3\times\left(\frac{0.1}{0.010}\right)^2\times3.3$$

$$= 34031250 \text{ N/m}^2$$

$$= 34.03 \text{ N/mm}^2$$

$$(\sigma_\theta)_{max} = \frac{3}{8}q\eta^2(3+\mu)$$

$$= 34.03 \text{ MPa}$$

Therefore maximum stresses in r and θ direction are same.

$$(\sigma_r)_{max} = (\sigma_\theta)_{max}$$

$$= 34.03 \text{ MPa}$$

Similarly, deflections and bending stresses are calculated by analytical equations for various boundary conditions, loading conditions and for various thicknesses of the circular plate. Then results of analytical equations are compared with ANSYS results.

5. NUMERICAL RESULT AND DISCUSSION

5.1 Numerical Result

Flexural parameters like deflections and bending stresses for various cases of an isotropic circular plate are obtained from analytical method and ANSYS are compared as follows

Table1. Comparison of deflection and bending stress for simply supported circular plate subjected to UDL obtained from analytical method and ANSYS

Thk (mm)	Max. Deflection (mm)		Max. Bending Stress (MPa)	
	Analytical	ANSYS	Analytical	ANSYS
10	0.0956	0.0945	34.03	32.70
20	0.01196	0.0122	8.507	8.19
50	0.000765	0.000963	1.3612	1.31
100	9.56×10^{-4}	0.201×10^{-3}	0.3403	0.3277
200	1.19×10^{-5}	0.654×10^{-4}	0.0851	0.08204
500	7.65×10^{-7}	0.222×10^{-4}	0.01361	0.0131

Table2. Comparison of deflection and bending stress for clamped circular plate subjected to UDL obtained from analytical method and ANSYS

Thk (mm)	Max. Deflection (mm)		Max. Bending Stress (MPa)	
	Analytical	ANSYS	Analytical	ANSYS
10	2.987	3.132	1313.02	2570
20	0.373	0.487	328.257	642
50	2.389×10^{-2}	0.0739	52.52	103
100	2.987×10^{-3}	0.0283	13.1302	25.70
200	3.734×10^{-4}	0.130×10^{-1}	3.2826	6.43
500	2.389×10^{-5}	0.510×10^{-2}	0.5252	1.03

Table3. Comparison of deflection and bending stress for simply supported circular plate subjected to center concentrated load obtained from analytical method and ANSYS

Thk (mm)	Max. Deflection (mm)		Max. Bending Stress (MPa)	
	Analytical	ANSYS	Analytical	ANSYS
10	7.5827	7.748	1706.93	3890
20	0.9478	1.064	426.73	972
50	0.06066	0.111	68.277	156
100	0.00758	0.0329	17.069	38.9
200	9.478×10^{-4}	0.136×10^{-1}	4.267	9.73
500	6.066×10^{-5}	0.513×10^{-2}	0.6876	1.56

Table4. Comparison of deflection and bending stress for clamped circular plate subjected to center concentrated load obtained from analytical method

Thk (mm)	Max. Deflection (mm)		Max. Bending Stress (MPa)	
	Analytical	ANSYS	Analytical	ANSYS
10	0.02346	0.0238	20.62	17.10
20	0.00293	0.00337	5.156	4.34
50	1.87×10^{-4}	0.396×10^{-3}	0.825	0.7174
100	2.34×10^{-5}	0.130×10^{-3}	0.2062	0.1833
200	2.93×10^{-6}	0.565×10^{-4}	0.05156	0.0463
500	1.87×10^{-7}	0.217×10^{-4}	0.00825	0.00743

and ANSYS

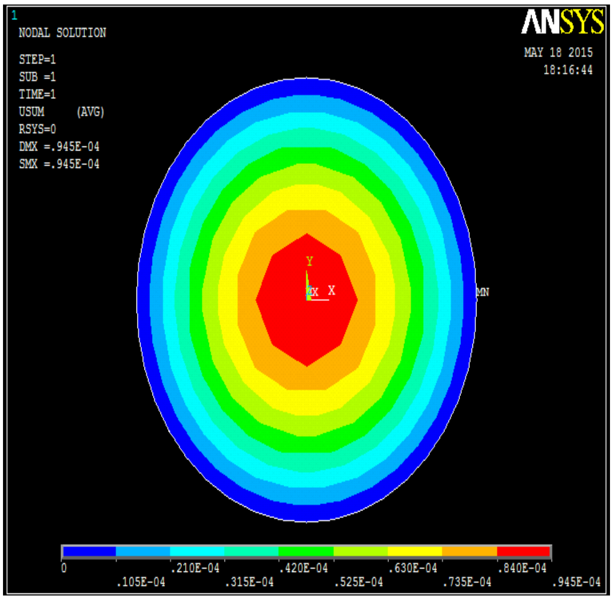

Figure1. Deflection contour for 10 mm thickness of simply supported plate with UDL of 275kPa

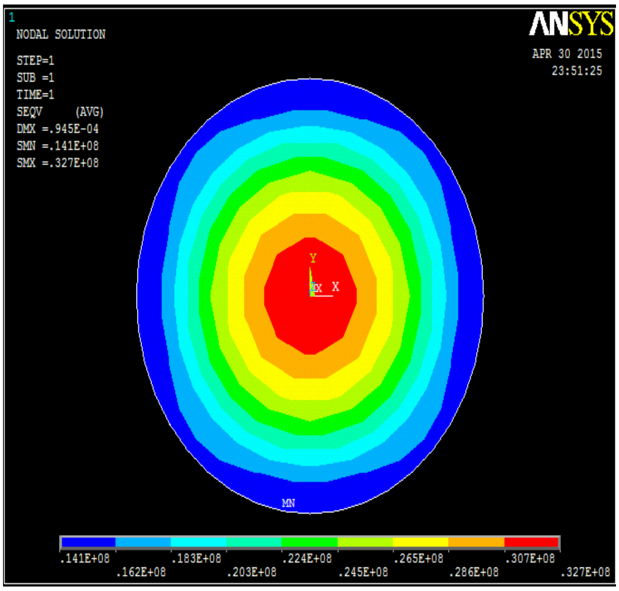

Figure2.Bending stress contour for 10 mm thickness of simply supported plate with UDL of 275kPa.

5.2 Discussion

Table 1 shows comparison of deflection and bending stress obtained from analytical method and ANSYS for simply supported circular plate subjected to UDL. From Table 1, results obtained for deflections and bending stresses from analytical equations gives good agreement with the results obtained from ANSYS. Results obtained from both the methods matches nearly about 95 to 97%. Table 2 shows comparison of deflection and bending stress obtained from analytical equations and ANSYS for clamped circular plate subjected to UDL. From Table 2, results obtained from analytical method and ANSYS for deflection and bending stress for clamped plate with UDL nearly matches about 85 to 90%.The results obtained from analytical equations and ANSYS for deflection for 200mm and 500 mm thickness of plate (Both for simply supported and clamped plate with UDL) shows greater variation because of the thickness of the plate i.e. plate become more thick for 200mm and 500mm thickness.

So From Table 1and Table 2, we can say that results obtained from analytical method for deflection and bending stress gives good agreement with the results obtained from ANSYS for both simply supported and clamped thin circular plate with UDL. Table 3 and Table 4 shows comparison of deflection and bending stresses for simply supported and clamped circular plate with center concentrated load or point load respectively. Deflection obtained from analytical solution and ANSYS for simply supported and clamped circular plate with center concentrated load are approximately equal for lower value of thicknesses and vary rapidly for higher value of thicknesses as like 200mm or 500mm. From Table 3 and Table 4, results obtained for bending stress from analytical equations and ANSYS for simply supported and clamped circular plate with center concentrated load matches nearly about 50%. Hence the flexural parameters obtained from analytical method and ANSYS with center concentrated load or point load for both the boundary conditions (simply supported and clamped) of circular plate differs greatly due to effect of stress concentration.

6. Conclusion

From the parametric study of bending analysis of simply supported and clamped isotropic circular plate by using Analytical Method i.e. CPT and ANSYS, following conclusions are drawn:

1) Deflection of thin isotropic circular plate decreases with increase in thickness. This is so, because the plate is thin where shear deformation is not considered. If the plate is thick then the effect of shear deformation is more pronounced and the results obtained may vary.

2) The results obtained from analytical method and ANSYS for deflection for 200mm and 500 mm thickness of plate (Both for simply supported and clamped plate with UDL)shows greater variation because of the thickness of the plate i.e. plate become more thick for 200mm and 500mm thickness.

3) For simply supported and clamped circular plate with UDL, bending stresses in radial and circumferential direction are equal while it is not equal in the case of concentrated or point loads due to concentration of load at a specific point.

4) Bending stresses in radial as well as circumferential direction increases with decrease in thickness. This is so because the plate is thin where shear effect is not considered. On the other hand, if the plate is thick then the effect of shear deformation is more pronounced.

5) Results for deflection and bending stress obtained from analytical method does not gives good agreement with the results obtained from ANSYS for both simply supported and clamped circular plate with center concentrated load or point load because of effect of stress concentration due to application of concentrated or point load.

6) Element used for the modeling and analysis of circular plate in ANSYS software i.e. 4 noded shell 181,shows good agreement of results with analytical method and predicts actual behavior of the circular plate according to Mindlin's Plate Theory.

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