Reliability-based Evaluation of Bending Moment Magnifiers for the Design of Reinforced Concrete Short Columns under Biaxial Bending

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Abstract

In this paper, the moment magnifiers that have been provided in the BS 8110 for the design of reinforced concrete short rectangular columns under biaxial bending was examined using a developed FORTRAN reliability-based program. First Order Reliability Method (FORM) was employed in the analysis. The reliability-based evaluation of the bending moment of coefficient (υ) of 0.58 yielded satisfactory safety index of 2.74 under the ultimate loading and specified geometric properties. It was noted that a better safety index was achieved when the moment magnifier, characteristic concrete grade, diameter of steel reinforcement, concrete cover, and depth of column are systematically choosing through reliability-based analysis. It was found among the findings that there is an increase of safety index from 2.74 to 3.98 when the moment magnifier increases from 0.58 to 3.98 respectively for the same loading, materials and geometric properties of the column. It is therefore concluded that the reliability-based approach for the design of columns under biaxial bending moment is quite suitable.

Keywords*:* *Reliability-based, reinforced concrete, short columns, moment magnifiers, biaxial bending*

I. INTRODUCTION

Design and analysis of reinforced concrete columns in biaxial bending had received extensive attention [1], [2] and [3], but uncertainties that exist in design planning, and construction of engineering structures [4] were neglected. These uncertainties result from the random nature of loading and structural resistance, human error, negligence, and workmanship as well as the prediction of failure events and modeling. Engineering structures are designed based on codes, which are known to be satisfactory by engineering judgment and previous experience with similar structures [5] rather than understanding the uncertainties that influence the strength and loads of the structure. This had been employed through the probabilistic concepts that were proven to be fairly consistent [6], [7], [8], [9], [10] and [11].

Lack of understanding or under-estimation of uncertainties sometimes leads to the collapse of structures. For instance, in [12] states that over 230 lives were lost and several others wounded due to the collapse of building across the country between 1976 and 1995. According to [13] over 260 lives were also lost between 1976 and 2000. According to [14] over 200 lives were lost in the incidence of collapse of buildings in the country, which include the Garki four-storey uncompleted building in August 2010, the Ebute Meta building in July 2013, the synagogue building in September 2014, the Jos school building in September 2014, the Lekki building in March 2016. However, this is a clear indication that there must have been under-estimation of some uncertainties.

A design code [15] for reinforced concrete elements is based on the limit state philosophy. The limit state concept safeguards engineering structures against failure through partial safety factors. These factors are applied to both the loading and material properties with the view to achieve low probability of failure. This approach is semi-probabilistic; uncertainties in individual design variables were not properly accounted for, and it is not clear how far the design is to failure. To properly accommodate randomness and uncertainties and at the same time maintain a known and uniform level of safety there is need for code review using a reliability-based approach [8], [11], [16], and [17].

According to [18], failure of any structural element is considered as a system failure if system performance is considered as a series system. However, it has been observed that failure of columns may result to the total collapse of a structure. Columns are used primarily to support axial compression loads [17] and [18]. The majority of compression members carry a portion of their loads in bending [19]. Collapse of structures occurs when structural systems can no longer carry gravity loads. Columns are the most critical elements in collapse analysis of reinforced concrete buildings [20], it therefore need exceptional attention.

II. STRUCTURAL RELIABILITY AND RELIABILITY METHODS

Having accepted the dichotomy of structural behaviour as failure and no failure, one can proceed to consider the methods that can be used to determine the probability of each state. A reliability method, in the narrowest sense, is a method to evaluate the reliability of a system [21]. The probability of failure, P_f, is equal to the probability that the undesired performance will occur. Mathematically, this can be expressed, depending on the basic design variables mentioned above, as follows:

$$P_f = P(R < Q) = P(R - Q < 0) = P(R = Q < 1) \qquad (1)$$

As previously mentioned, the state of the structure can be described using various random parameters (variables) X_1, X_2, ….., X_n, which are load and resistance parameters, such as dead load, live load, compressive strength, and yield strength. In this case, the limit state function showing the conditions of failure can be expressed as a function of the vector of random variables, x. The failure domain is denoted by F = {x; g(x) < 0}, the safe domain S = {x; g(x) > 0}, and the limit surface, the boundary of F by G = {x; g(x) = 0} [22]. This is illustrated in Figure 1 in the case of a 2-dimensional state space.

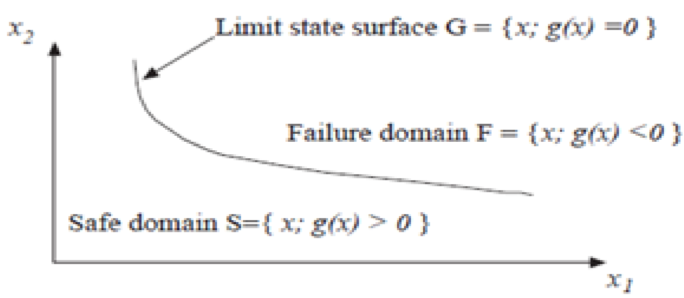


Figure 1: Failure Domain, Limit State Surface, and Safe Domain

Then the problem is to compute the probability of failure given as,

$$P_f = \int_{g(X)\le 0} f_x(x)dx, \qquad (2)$$

where f_x (x) represents the joint probability density function (PDF) of the random vector and P_f is valid for continuous random variables only. Even though equation (2) seems simple, evaluating this integral is very difficult in most cases. The integration requires special numerical techniques and the accuracy of these techniques may not be adequate [23]. Therefore, some other procedures, which will be explained in the following sections, need to be used to evaluate the integral in equation (2).

The calculation of the probability of failure was not a simple task until the concept of the reliability index, first proposed by Freudenthal (1956), was introduced. The proposal for a reliability index is given by [24]. Later, [25] introduced a new reliability index definition that is the shortest distance from the origin of reduced variables to the limit state surface ($g(x)$ = 0). It has a very important characteristic that is invariant with respect to different choices of the limit state function for a given failure domain. According to this definition, the reliability index β is calculated as follows:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \qquad (3)$$

where g(R,Q) = R – Q; R and Q are uncorrelated, normally distributed random variables.

The reliability index is obtained by minimizing equation (3) through an optimization procedure over the failure domain F corresponding to G(**X**)=0 using FORM5 [26]. FORM5 is a program written in FORTRAN that can give a solution to the minimization problem by transforming correlated and non-normal variables [26], and then calculating the probability of failure, Pf using the equation:

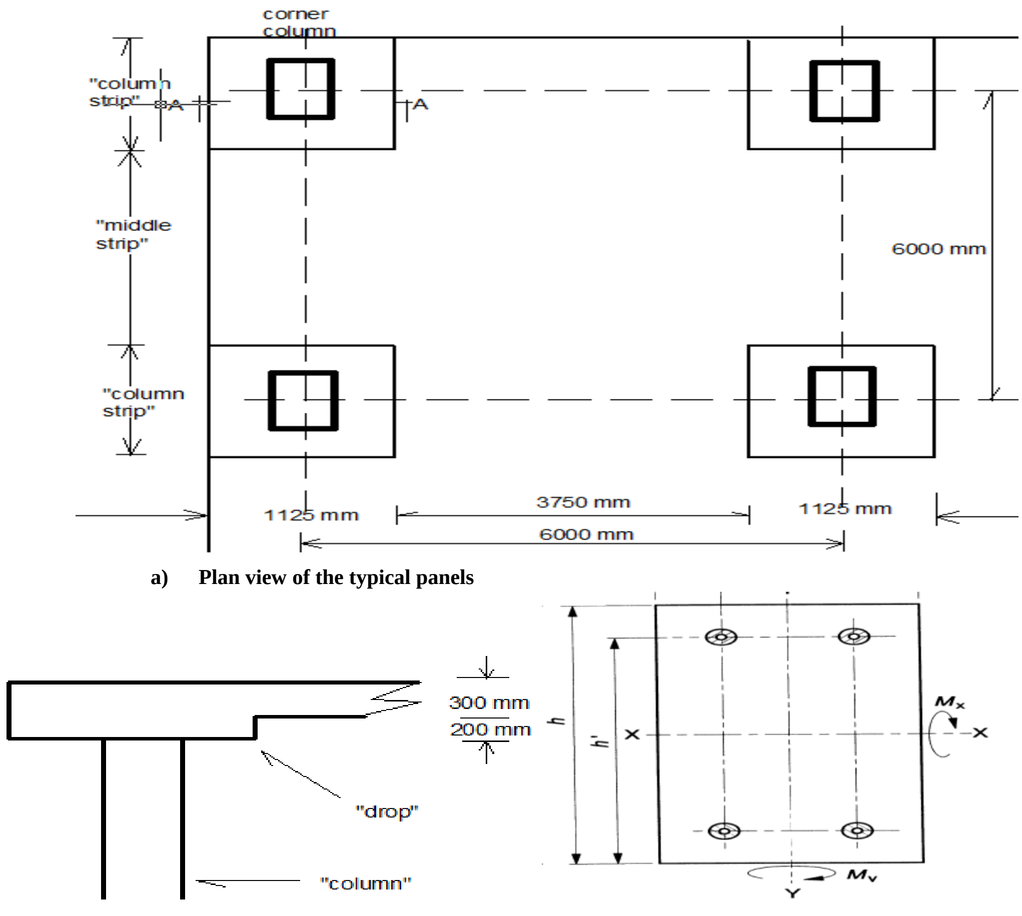
$$P_f = \Phi(-\beta) \qquad (4)$$

The reliability index can therefore be obtained from [27]:

$$\beta = \Phi^{-1}(P_f) \quad (5)$$

III. STRUCTURAL MODEL AND LIMIT STATE FUNCTIONS

The structural model used in study is a supermarket comprising a ground floor shopping complex hall. The arrangement consists of flat slab arrangement of columns that were designed to resist biaxial bending (as shown in Fig. 2). The panel is monolithic with the frame on all the sides, so a design for two-way spanning flat slabs as supported by a generally rectangular arrangement of columns[28]. It was assumed that the imposed load for shopping hall and offices is 5.0 kN/m^2, column dimension is 300 mm x 400 mm, concrete strength is 25 N/mm^2 and concrete cover is 40 mm.



a) Plan view of the typical panels

b) Section A-A of the corner column c) Cross-section of biaxial bent column

Figure 2: Structural Configuration of the Corner Column

The biaxial bending moment magnifiers provided in [15] as checked through first order reliability method (FORM5). The statistical parameters of the stochastic model used in the reliability analysis are presented in Table 1.

The limit state functions used in the analysis is given as;

$$G(X) = \upsilon - M_f \quad (6)$$

$$M_f = \left((\gamma\text{-}1)/\left(\frac{h'}{b'}\right)\left(\frac{M_x}{M_y}\right)\right) \quad (7)$$

$$M_x = \lambda_{sx}\left(\psi_Q + \alpha\psi_G\right)Q_k l_x^2 \quad (8)$$

$$M_x = \lambda_{sy}\left(\psi_Q + \alpha\psi_G\right)Q_k l_x^2 \quad (9)$$

$$h' = h - \left(c - \frac{\theta}{2}\right) \ \&$$

$$b' = b - (c - \frac{\theta}{2}) \quad (10)$$

where, M_f, υ, M_x, M_y, f_{cu}, b, h , θ , c are the moment magnifier, moment magnifier from [15], moment along short span, moment of long span, concrete grade, width of column, depth of the column, diameter of reinforcement, concrete cover respectively. While γ, λ_{sx}, λ_{sx}, ψ_Q, ψ_G, α, are the ratio of the increased moment in single axis to the moment along shorter span, bending moment coefficient along short span, bending moment coefficient along long span, safety factor for imposed load, safety factor for dead load and live-to-dead-load ratio respectively.

Table I: Statistical Parameters of The Stochastic Model

Basic variables	Mean	Coefficient of variation	Probability distribution
Concrete grade, f_{cu} (N/mm^2)	25	0.15	Lognormal
Depth of column, h (mm)	400	0.01	Normal
Breadth, b (mm)	300	0.01	Normal
Imposed load, Q_K (N/mm^2)	5	0.37	Gumbel
Concrete cover, c (mm)	40	0.15	Normal
Diameter of steel, θ (mm)	16	0.015	Normal
Moment magnifier, υ (BS 8110)	0.58	-	Deterministic
length of column, L (mm)	32000	-	Deterministic
Short length of panel, $\mathbf{l}_x$ (mm)	6000	0.044	Normal

Source: [10], [11] and [17]

IV. RESULTS AND DISCUSSION

The magnifiers of reinforced concrete short rectangular columns provided in [15] were examined considering the uncertain variables in the design using reliability-based approach. The result of the analysis at the ultimate state of loadings and specified geometric properties recorded the value of safety index, β of 2.74. This is appreciable compared to safety index range of 2.5 to 3.0 recorded by [29] for the flexural and compression of reinforce concrete members under the load combinations. This implies that the moment magnifiers are adequate. The sensitivity analysis was conducted to ascertain the effects of some of the basic variables for both the materials and loads as employed in the design of reinforced concrete rectangular short columns under biaxial bending [15].

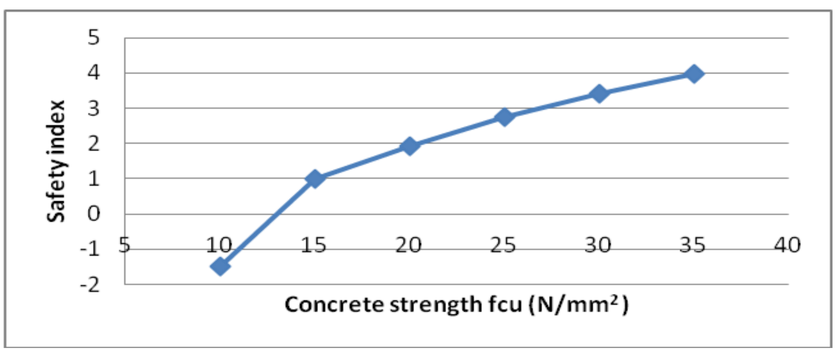


Figure 3: Variation of Safety Index with Concrete Strength

Figure 3 shows the relationship between safety index (β) and concrete grade (f_{cu}). It had been observed that, at constant bending moment magnifier a general increase in safety index (β) from -1.42 to 3.98 was noted as the concrete grade was increased from 10 N/mm^2 to 35 N/mm^2. The result revealed that below concrete grade of 20 N/mm^2 the safety index value is not adequate, but above 20 N/mm^2, the moment magnifiers for the design of reinforced concrete rectangular short columns under biaxial bending as provided in [15] is adequate as recorded by [17].

Figure 4 shows the relationship between safety index (β) and the diameter of reinforcement (ϕ) at constant bending moment magnifier of 0.58. The safety index (β) general increased from 1.4 to 4.08 when the diameter of reinforcement was increased from 12 mm to 32 mm. The result also revealed that diameter of steel reinforcement has significant effect on safety level of moment magnifiers of BS 8110 (1997) for the design of reinforced concrete short columns but the safety level cannot be quantitatively estimated without employing reliability-based approach. The result implies that reduction in the structural resistance of structural member e.g., diameter of reinforcement of column reduces the safety level [30].

Figure 5 shows the relationship between safety index (β) and the variable live loads at constant moment magnifier of 0.58. A decrease in safety index (β) was recorded from 6.28 to -1.08 as the live load drastically increases from 2 kN/m^2 to 10 kN/m^2 respectively. This decrease of safety index (β) of the magnifier for the design of rectangular short column under biaxial bending implies a decrease of the reliability of the column designed using the magnifiers of [15]. This decrease in the reliability index could be attributed to the fact that the carrying capacity of the structural element is being exceeded thereby leading to the chances of failure [31]. A maximum of 5 kN/m^2 live load was adequately sustained by the column at the specified loading and geometric properties.

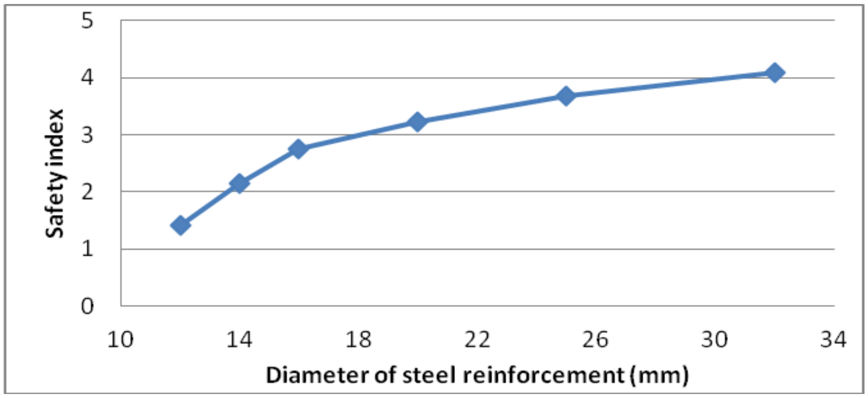


Figure 4: Variation of Safety Index with Diameter of Steel Reinforcement

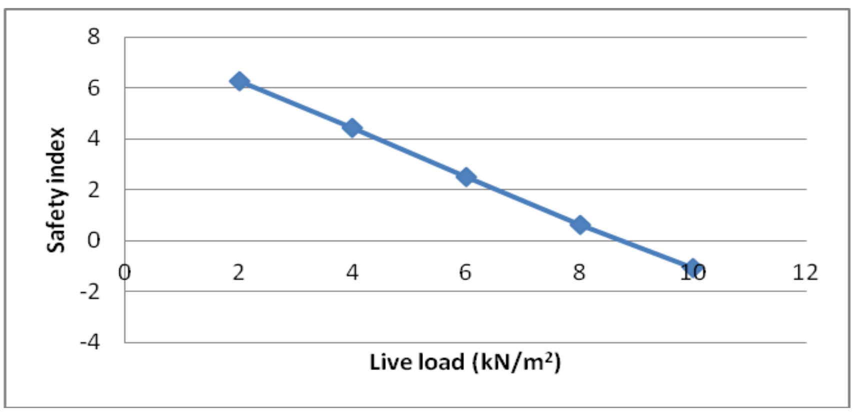


Figure 5: Variation of Safety Index with Live Load

Figure 6 shows the relationship between safety index (β) and depth of section at constant bending moment magnifier of 0.58. An increase in safety index (β) from 0.72 to 5.34 was recorded as the depth of column was increased from 300 mm to 400 mm respectively. The increase of safety index (β) of the magnifier for the design of rectangular short column under biaxial bending implies an increase of the reliability level of the column designed employing this magnifier. This increase in safety index (β) could be attributed to the increase in EI values, which increased the rigidity of the section [32].

In Figure 7 shows the relationship between safety index (β) and moment magnifiers at the specified loading and geometric properties. The safety index (β) drastically increase from 0.64 to 12.78 when the when the moment magnifier decreases from 1.0 to 0.3. If the moment magnifier decreases below 0.3, safety index tends to zero. This explained the reason behind values of moment magnifiers provided in [15] not exceeding 0.3. The implication of the result is that the high moment magnifier the better the safety index of the magnifiers provided in BS 8110 for the columns under biaxial bending and that means the high the reliability of the column, but the safety level cannot quantitatively be understood without employing reliability-based techniques.

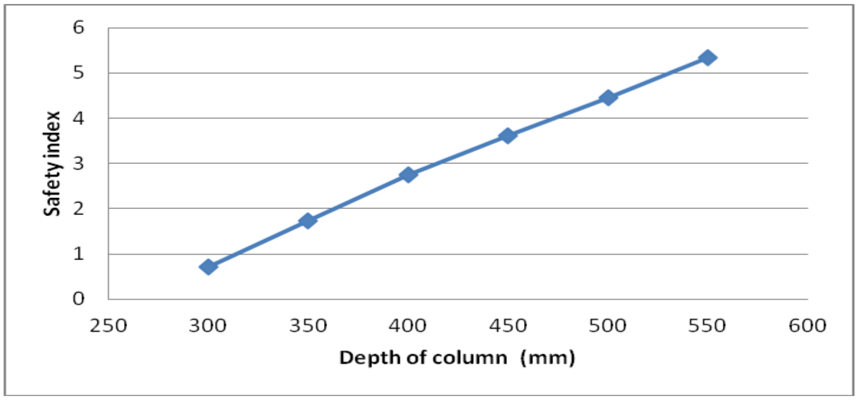


Figure 6: Variation of Safety Index with Depth of Column

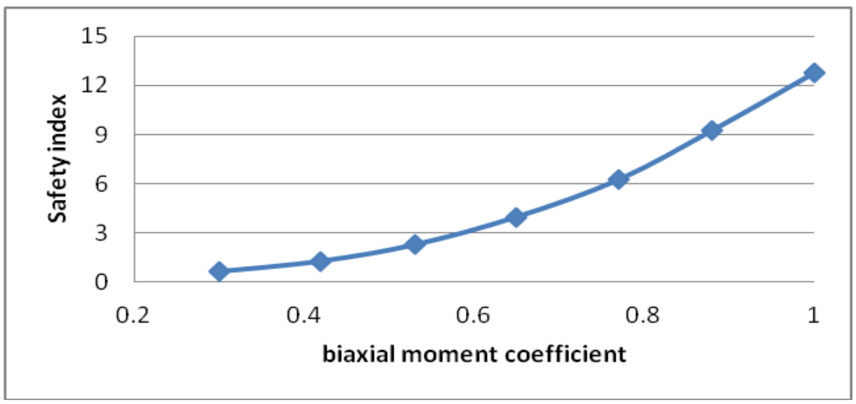


Figure 7: Variation of Safety Index with Biaxial Moment Magnifier

Figure 8 shows the relationship between safety index and concrete cover for steel reinforcement of column at constant moment magnifier of 0.58. The safety index (β) increases linearly from 0.28 to 4.38 when the concrete cover for steel reinforcement of column increases from 10 mm to 60 mm respectively. The increase of safety index (β) of the magnifier for the design of rectangular short column under biaxial bending implies that there is an increase of the reliability of the column designed employing this magnifier. This implies that concrete cover for steel reinforcement has significant effect on safety of column under biaxial bending.

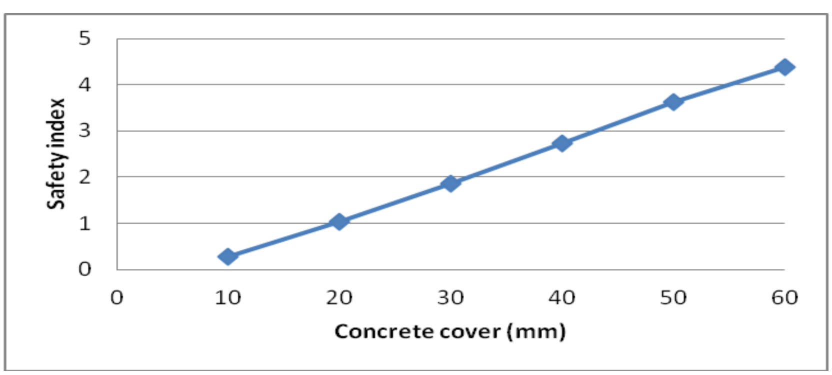


Figure 8: Variation of Safety Index with Concrete Cover

V. CONCLUSION

The adequacy of [15] design criteria for the design of reinforced concrete short rectangular columns under biaxial bending has been examined. The FORM was used through a developed FORTRAN reliability-based program to estimate the safety index or probability of failure for the values of bending moment magnifiers of [15]. It was found that satisfactory safety index (β) of 2.74 was achieved for bending moment magnifier (υ) of 0.58 under the ultimate loading and geometric properties.

The sensitivity analysis was conducted to ascertain the effects of some of the basic variables for both the materials and loads for the design of reinforced concrete rectangular short columns under biaxial bending [15]. It was found that the concrete grade, diameter of steel reinforcement, live load, depth of column, moment magnifier of BS 8110 and concrete cover for steel reinforcement has significantly influenced the safety index (β). This implies that to increase the reliability of columns the design variables should be chosen systematically through reliability-based approach.

The results from the analysis indicated that for the design of reinforced concrete short rectangular columns using the values of moment magnifiers provided in BS 8110, ignoring uncertain variables during the design process could result in a dramatic loss of structural safety. This suggestion for upgrade of the safety index value to target safety index of 3.8 implies that the design formulations of [15] need further review in order to meet the target for approved structural safety. However, this emphasized the use of reliability-based technique in collaboration with BS 8110 code for the attainment of safety and economical structures.

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