

# Comparison of Footing Widths of Proportionally-Sized Reinforced Concrete Retaining Walls Under Extreme Loading

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**Abstract:** *Even today, the retaining walls and their different types are still used and investigated. Conventional design methods and construction principles have been converting to the mechanically stabilized earth walls. This study performs to reveal footing width variation by changing the dimensions of different reinforced concrete (RC) cantilever walls within proportionality limits. These walls are differentiated as T-shaped RC cantilever walls, stem-stepped RC walls and counter-fort walls. In loading laterally walls, the M-O method was used for calculating total active lateral pressure with/without earthquake motion and hydraulic pressure. At first, the stability analysis was applied to the initial dimensions of walls in the scope of experienced design parameters. After checking wall safety against sliding, overturning and bearing capacity failure, reinforcement process of wall components was conducted. Cross sections of concrete wall components and their steel reinforcements were checked by using TS500 specification. It is determined that the extreme loading required the greatest footing widths and otherwise, the smallest widths revealed without earthquake and hydraulic pressure. When considered all loading cases, it can be said that stem-stepped cantilever walls provided the most suitable wall dimensions satisfying conditions regarding stability analysis and reinforcement. This process is followed by counter-type cantilever wall as second one and finally by T-shaped cantilever wall.*

**Keyword:** *Retaining Walls, Stability and Reinforced Concrete Walls, Proportionally-Sized Retaining Walls, Extreme Loading*

## I. INTRODUCTION

Retaining walls are usually used for constructing roadway in rugged terrain with steep slopes, reducing the grades of roads

and land alongside the road, stabilizing hillsides, trenches, soil slopes, controlling erosion, urban locations where the availability of land to accommodate earth batters is restricted, constructing bridge abutments and wings. Before constructing any retaining wall, some design procedures should be carried out such as factors influencing wall type, its structural system, and loading type and associated calculation method.

In order to select suitable retaining walls from aspect of construction material such as wood, steel, concrete externally or internally stabilized structures, and conventionally or mechanically construction. Some factors may be considered. These may be cost, available clearance to boundary fence, safety, foundation conditions, maintenance, suitability for use adjacent to footways and pavements, compatibility with adjacent wall types, and appearance [12]. Structural systems of earth-retaining walls are generally classified into two groups: externally and internally stabilized systems. In this study, externally stabilized system including T-shaped cantilever walls, stem-stepped cantilever walls, counter-fort walls is taken into account. While designing a retaining wall, the loading types such as soil pressures, hydrostatic pressure, surcharge and earthquake must be assigned. In general, retaining walls are designed by lateral earth pressures based on Rankine or Coulomb's theories. Mononobe and Matsuo, and Okabe (M-O) proposed a method to determine lateral earth pressure of granular cohesionless soils during earthquake. The method was a modified version of Coulomb theory in which earthquake forces are applied to the failure mass by pseudo-static method [11]. In stabilization of a wall, the general procedures are; check for overturning about its toe, check for sliding along the base, check for bearing capacity failure, settlement, and overall stability.

## II. LITERATURE and AIM of the STUDY

*Wu et al. (2001)* studied rigid retaining walls experience significant displacements during earthquakes. In this study, several investigations had developed 1-D and 2-D models to predict displacements. A newly-developed 2-D model considering strain dependant soil stiffness and material damping, sliding and rocking motions, and practical field water conditions behind the wall as per Eurocode (1994) had been presented. Lastly, a comparison of prediction and performance of a centrifuge model had been shown in good agreement. This model was proposed to the practicing engineers [3]. *Taylor et al. (2007)*, presented a paper regarding performance based design philosophy and implementation in the design of gravity retaining structures subject to seismic loading. A comparison in that study was provided of some of the methods such as simple pseudo-static limit equilibrium (PS-LE) based tools and fully dynamic numerical analyses capable of modelling non-linear soil behaviour. Results of a simple application of some of these PS-LE methods to a selected design problem were indicated significant variability between methods dealt with [6]. *Visone et al. (2009)*, studied the Performance-Based Design philosophy regarding the response of the construction, both in terms of stresses in the structural elements and displacements in the soils to by predict the performances of the system when it was subjected to the dynamic actions related to the expected earthquake motion. Besides that, they illustrated the damage parameters and some acceptability limit values used for the retaining walls with the application of the hierarchical strength criteria and the preferential sequence yielded.

*Wu et al. (2010)* presented a study on retaining walls which had failed either by sliding away from the backfill or due to combined action of sliding and rocking displacements, during earthquakes. With performance based design of the retaining walls in seismic areas, a realistic model, including non-linear stiffness of soil, and geometric and material damping and coupling effects, for estimating dynamic displacement were developed. The model to calculate the displacement for several combinations of backfill and foundation soil conditions had been presented with typical design charts for preliminary design [9]. *Yazdani et al. (2013)* studied Mononobe-Okabe (M-O) method solving the equations of equilibrium and suggests seismic active and passive lateral earth pressures. Especially, in case of noncontinues backfill slopes, cohesive soils, and rising water behind the wall, they proposed an iterative method to overcome the limits of the M-O method. Based on trial and error process, the proposed method was able to cover many of the defects which regularly occur in civil engineering when M-O had no direct answer [11]. *Rajeev et al. (2014)* studied the design of retaining walls requiring the complete knowledge of the earth pressure distribution behind the wall. Several earth pressure models developed over the years to integrate the dynamic earth

pressure with the static earth pressure and to improve the design of retaining wall in seismic regions and also Mononobe Okabe (M-O) method were used to estimate the magnitude of seismic earth pressures in retaining walls and were adopted in design practices around the world (e.g., EuroCode and Australian Standards). Besides that, this study revealed the accuracy of the M-O method to predict the dynamic earth pressure in sheet pile wall. Finally, the applicability of M-O methods to compute the seismic earth pressure was discussed [13]. *Luu et al. (2014)* presented a paper about an innovative design of the reinforced concrete counterfort retaining wall as part of the Barangaroo Headland Park project in Sydney. They prepared the project on the counterfort wall required to retain up to 18.5m of fill and support the roof of the underground Future Cultural Space building. And also their design solution comprised buttresses at 6 m centres supported on a narrow 5.5 m wide base slab with prestressed ground anchors located at the top of the wall for global stability. Results obtained from their soil-structure interaction model demonstrated the active soil stress in the narrow base slab, the ground anchors prestressed to a set lock-off load and the wall built with a 1 horizontal to 120 vertical rake into the backfill [14].

*Patil et al. (2015)* carried out a study which includes reducing the stresses over the retaining face of the cantilever retaining wall, determining the most economical location of step along length and also along height of wall. In addition to this, it provided assigning cross section of the RC step depending on the stresses originated from frictional forces in step for unit width, and comparing the costs of three different retaining walls [15]. *Bhadke et al. (2016)* presented a paper providing a case study of various types of retaining structures behaviour under seismic condition and effect of earth pressure on their stability. In order to improve the stability of the structure, a study the effect of various loadings in details has revealed [16].

It is aimed that *the proportional dimensions required by stabilizing gravity and flexible type retaining walls with varying depths are compared by supposing to keep the design parameters constant for all wall types*. For this aim, extreme loading condition for each wall are performed by considering hydraulic pressure, earthquake and surcharge loads in addition to lateral and vertical soil pressures.

## III. METHODOLOGY

### A. Total Earth Pressures due to Extreme Loads

M-O model foreseen by Turkish Specification for Building to be Built in Sismic Zones (2007) is used in this study. Total active,  $K_{at}$ , and passive,  $K_{pt}$ , pressure coefficients can be expressed as the sum of static soil pressure, and additional dynamic soil pressure included by earthquake by the following Eq.1a and Eq.1b [7].

$$K_{at} = \frac{(1 \mp C_V) \cos^2(\varphi - \lambda - \alpha)}{\cos \lambda \cos^2 \alpha \cos(\delta + \alpha + \lambda)} \left[ 1 + \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi - \lambda - i)}{\cos(\delta + \alpha + \lambda) \cos(i - \alpha)}} \right]^{-2}$$

$$K_{pt} = \frac{(1 \mp C_V) \cos^2(\varphi - \lambda + \alpha)}{\cos \lambda \cos^2 \alpha \cos(\delta - \alpha + \lambda)} \left[ 1 + \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi - \lambda + i)}{\cos(\delta - \alpha + \lambda) \cos(i - \alpha)}} \right]^{-2}$$

where  $\varphi$  is the soil friction angle,  $\delta$  is the wall friction angle,  $\alpha$  is the wall inclination with respect to vertical, and  $i$  is the ground inclination with respect to horizontal on both sides of the wall. The angle  $\lambda$  in Eq.1a, and Eq.1b is defined by Eq.2a and Eq.2b:

$$\lambda = \arctan \left[ \frac{C_h}{(1 + C_V)} \right] \{ \text{dry soil} \} \quad (\text{Eq.2a})$$

$$\lambda = \arctan \left[ \frac{\gamma_s C_h}{\gamma_b (1 + C_V)} \right] \{ \text{submerged soil} \} \quad (\text{Eq.2b})$$

where  $\gamma_b$  is submerged unit volume weight of soil, and  $\gamma_s$  is saturated unit volume weight of soil. Equivalent lateral seismic coefficients,  $C_h$ , for vertical free cantilever and soil anchors, dealt with in Eq.2a and Eq.2b are written as in Eq.3a and Eq.3b;

$$C_h = 0.2(I + 1)A_0 \quad \{ \text{cantilever} \} \quad (\text{Eq.3a})$$

$$C_h = 0.3(I + 1)A_0 \quad \{ \text{anchor} \} \quad (\text{Eq.3b})$$

where  $I$  is building importance factor, and  $A_0$ , effective ground acceleration coefficient. Equivalent vertical seismic coefficient,  $C_V$ , appearing in Eq.1a-b is defined by Eq.4:

$$c_V = 2c_h/3 \quad (\text{Eq.4})$$

Dynamic active pressure coefficient,  $K_{ad}$ , and dynamic passive pressure coefficient,  $K_{pd}$ , included by earthquake shall be determined by Eq.5a and Eq.5b:

$$K_{ad} = K_{at} - K_{as} \quad (\text{Eq.5a})$$

$$K_{pd} = K_{pt} - K_{ps} \quad (\text{Eq.5b})$$

Variation of dynamic active and passive soil pressures along the depth of soil,  $z$ , during earthquake are defined as Eq.6a and Eq.6b:

$$p_{ad}(z) = 3K_{ad}(1 - z/H)p_V(z) \quad (\text{Eq.6a})$$

$$p_{pd}(z) = 3K_{pd}(1 - z/H)p_V(z) \quad (\text{Eq.6b})$$

By integrating Eq.6a and Eq.6b, the resultant  $P_{ad}$  and  $P_{pd}$  of dynamic active and passive soil pressures are obtained respectively as seen in Eq.7a and Eq.7b:

$$P_{ad} = 0.5\gamma K_{ad} H^2 \quad \{ z_{cd} = H/2 \} \quad (\text{Eq.7a})$$

$$P_{pd} = 0.5\gamma K_{pd} H^2 \quad (\text{Eq.7b})$$

where  $\gamma$  is the unit weight of soil, and  $H$  is the vertical height of the wall. Variation of dynamic active and passive soil pressures  $\{q_{ad}(z), q_{pd}(z)\}$  along the depth of soil,  $z$ , in case of uniformly distributed external loads during earthquake are defined as Eq.8a and Eq.8b:

$$q_{ad}(z) = 2q_0 K_{ad} \left( 1 - \frac{z}{H} \right) \frac{\cos \alpha}{\cos(\alpha - i)} \quad (\text{Eq.8a})$$

$$q_{pd}(z) = 2q_0 K_{pd} \left( 1 - \frac{z}{H} \right) \frac{\cos(\alpha)}{\cos(\alpha - i)} \quad (\text{Eq.8b})$$

By integrating Eq.8a, and Eq.8b, the resultant of active and passive soil pressures included in addition to static soil pressure by contribution of earthquake are obtained respectively as seen in Eq.9a and Eq.9b:

$$Q_{ad} = q_0 K_{ad} H \frac{\cos \alpha}{\cos(\alpha - i)} \quad \{ z_{cd} = H/3 \} \quad (\text{Eq.9a})$$

$$Q_{pd} = q_0 K_{pd} H \frac{\cos \alpha}{\cos(\alpha - i)} \quad (\text{Eq.9b})$$

where  $Q_{ad}$  and  $Q_{pd}$  are respectively the resultant of active and passive soil pressures.

### B.Stability Analysis

The stability of a retaining wall can be performed by analyses regarding *overturning about its toe, sliding along its base, bearing capacity failure of the base, settlement, and overall stability*. First three stability items as seen in Eq.10ab and Eq.11ab are considered in the scope of this study.

$$FS_{\text{OVERTURNING}} = \frac{\sum M_R}{\sum M_O} \quad (\text{Eq.10a})$$

$$FS_{\text{SLIDING}} = \frac{\sum F_R}{\sum F_d} \quad (\text{Eq.10b})$$

$$FS_{\text{BEARING CAPACITY}} = \frac{q_u}{q_{\text{max}}} \quad (\text{Eq.11a})$$

$$\left\{ \begin{matrix} q_{\text{max}} \\ q_{\text{teo}} \end{matrix} \right. = \left( 1 + \frac{6e}{B} \right); \left\{ \begin{matrix} q_{\text{min}} \\ q_{\text{heel}} \end{matrix} \right. = \left( 1 - \frac{6e}{B} \right) \quad (\text{Eq.11b})$$

where  $FS_{\text{OVERTURNING}}$  is safety against overturning,  $FS_{\text{SLIDING}}$  is safety factor against sliding,  $FS_{\text{BEARING CAPACITY}}$  is safety factor against bearing capacity failure,  $q_u$  is the ultimate soil-bearing capacity,  $q_{\text{max}}$  is maximum pressure at toe,  $q_{\text{min}}$  is minimum pressure at heel and  $e$  is the eccentricity value which is greater than 1/6 ratio of footing width  $B$ .

### IV. REINFORCEMENT of RETAINING WALLS

Each reinforced concrete wall was separately evaluated based on active soil pressures within its proportionality limits from

aspect of overturning, sliding, and bearing capacity stability. The resisting moments were calculated using gravity loads and the horizontal resisting forces were calculated using passive soil pressure in exception of passive hydraulic pressure. Foundation friction coefficient was determined based on foundation material type interfaced with concrete. TS500 Turkish Standard on Requirements for Design and Construction in Reinforced Concrete Structures [2] was used for determining the thickness & reinforcement required to resist the bending moment and shear force in the retaining wall components (stem, toe and heel). By pre-determining structurally the values of bending moment and shear force to be calculated for each of retaining wall components subjected to the total earth pressures due to extreme loads, the reinforcement in components can be calculated as expressed in Eq. 12ab, and Eq.13abc according to the TS500 rules:

$$M_b = A_s f_{yd} (1 - 0.59 \rho f_{yd} / f_{cd}) \quad (\text{Eq.12a})$$

$$A_s = 0.85 f_{cd} a b_w / f_{yd} \quad (\text{Eq.12b})$$

$$\max V_d = 0.22 f_{cd} A_c \quad (\text{Eq.13a})$$

$$V_{cr} = 0.65 f_{ctd} A_c (1 + \gamma N_d / A_c) \quad (\text{Eq.13b})$$

$$A_{sw} = s V_d / (f_{ywd} d) \quad (\text{Eq.13c})$$

where  $M_b$  is design bending moment,  $A_s$  is area of steel in tension zone,  $f_{yd}$  and  $f_{cd}$  are characteristic yield strength of reinforcement, and characteristics cube compressive strength of concrete,  $a$  is the total depth of the beam,  $b_w$  is the width of beam for rectangular beam,  $\rho$  is reinforcement ratio,  $V_d$  is design shear force,  $A_c$  is area of concrete section,  $V_{cr}$  is the cracking load regardless of shear reinforcement,  $\gamma$  is constant for compression (0.07) and tension (-0.3),  $N_d$  is design shear force,  $A_{sw}$  is total cross-sectional area of stirrup legs,  $s$  is the stirrup spacing,  $f_{ywd}$  is strength of stirrup legs, and,  $d$  is effective depth.

## V. CASE STUDY: RC-TYPE RETAINING WALLS

### A. Retaining Wall and Soil Characteristics

The front side of the wall is considered as unloaded while there has been only backfill. Passive action of hydrostatic pressure in front of the wall is neglected. Surcharge load is taken as  $q = 80 \text{ kN/m}^2$  for all wall types. In calculation of total earth pressures due to extreme loads, M-O method is taken into account. Friction coefficient between concrete base and ground surface is 0.6. While calculating lateral earth pressure in case of earthquake motion, Eurocode-8 (1994) is used to apply loading conditions and corresponding parameters for dynamic displacements [9]. In case of no earthquake motion, safety factors are taken as 1.5 for sliding, 2 for overturning

and 1.5 for bearing capacity. Otherwise, these factors are respectively 1.1 for sliding, 1.3 for overturning and 1.5 for bearing capacity. Length of toe, top width of stem and footing thickness are respectively taken as 2.5m, 1.0m and 1.5m. Other dimensions are stated over the wall figures (Fig.2ab, Fig.3ab, and Fig.4ab). Allowable bearing capacity of ground is  $q_u = 350 \text{ kN/m}^2$ . This capacity has been increased by 50%. Unit weight of concrete is  $23 \text{ kN/m}^3$  for all walls. The considered data are taken as: height of wall is  $H = 16 \text{ m}$ , internal friction angle  $\phi = 40^\circ$ , cohesionless backfill material, bulk unit weight of soil  $\gamma = 18 \text{ kN/m}^3$ , submerged unit weight of soil  $\gamma_s = 21 \text{ kN/m}^3$ , whole backfill material submerged, wall inclination angle  $\alpha = 90^\circ$ , wall friction angle between wall and ground surface  $\delta = 0$ , backside slope of stem  $\beta = 0$ , the construction site subjected to earthquake motion and quaywall-type design, Standards to be used: EUROCODE-8 (1994)[1], DHMI (2007)[4], AIGM (2007)[7] and TS500 (2000)[2].

### B. Proportional Dimensions of RC-Type Retaining Walls

While designing any retaining wall, some of the dimensions have been assumed as proportioning to check trial sections for stability. If the stability checks yield undesirable results, the sections can be changed and re-checked.

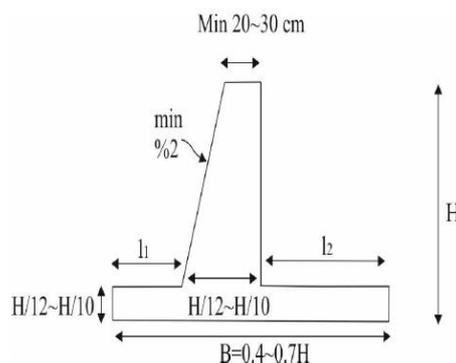


Fig.1.a. Proportionally-Sized Cantilever Wall

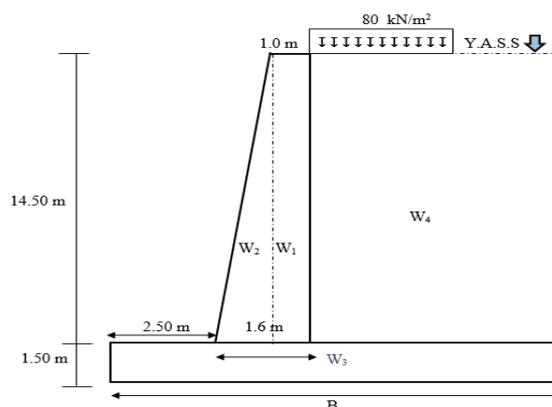


Fig.1.b. The relevant dimensions and Loading

Fig.1a and Fig.1b. show respectively the general proportions of components regarding T-shaped reinforced concrete

cantilever retaining wall, and loading scheme over its stated size.

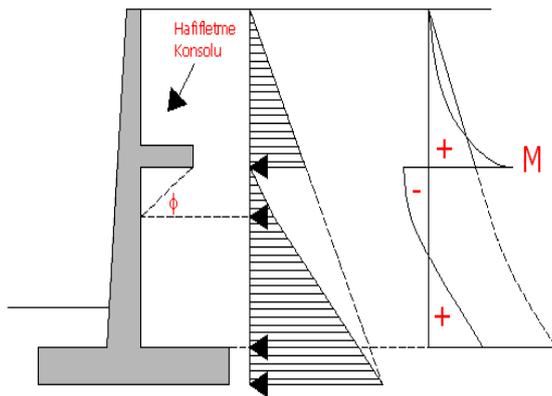


Fig.2.a. Proportionally-Sized Stem-Stepped Wall

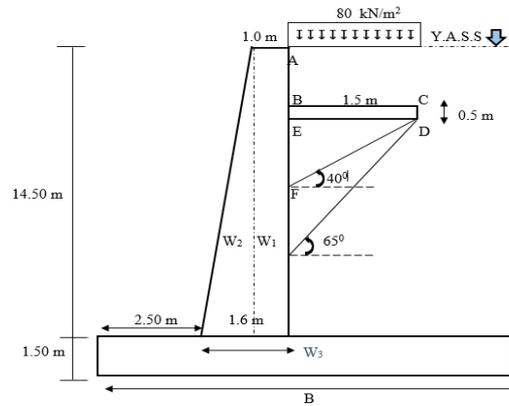


Fig.2.b. The relevant dimensions and Loading

Fig. 2a and Fig.2b. show respectively the general proportions of components regarding stem-stepped reinforced concrete

cantilever retaining wall, and loading scheme over its stated size.

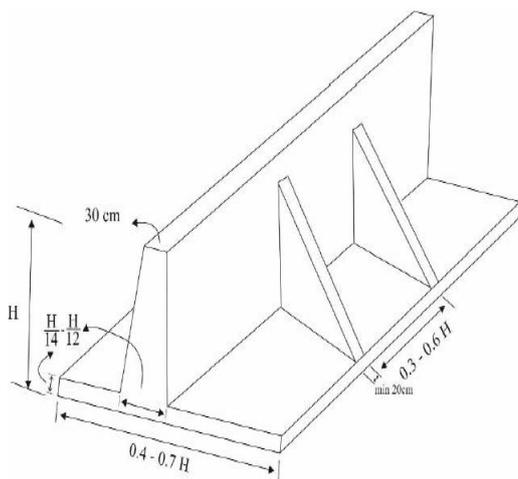


Fig.3a. Proportionally-Sized Counter-Fort Wall

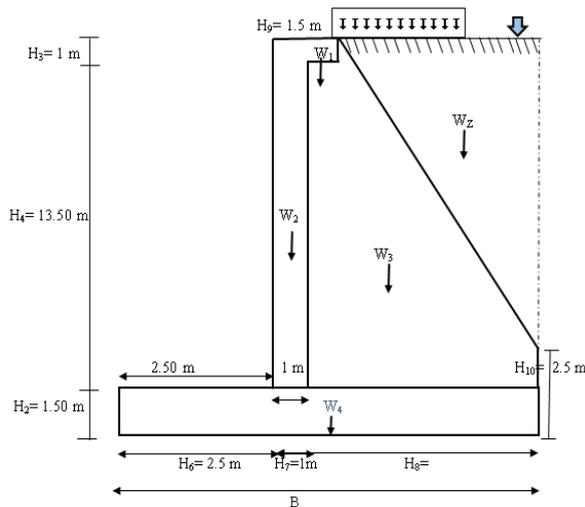


Fig.3.b. The relevant dimensions and Loading

Fig.3a and Fig.3b. show respectively the general proportions of components regarding counter-fort type reinforced concrete cantilever retaining wall, and loading scheme over its stated

size. The actual dimensions of retaining walls presented in Fig.1ab, Fig.2ab and Fig.3ab are iteratively determined within the stated proportionality limits.

## VI. CONCLUSION

This study performs to reveal footing width variation by changing the dimensions of reinforced concrete cantilevers within proportionality limits. For this aim, the M-O method was used for calculating total active lateral pressure

with/without earthquake motion and hydraulic pressure. At first, the stability analysis was applied to the initial dimensions of walls in the scope of experienced design parameters. After checking wall safety against sliding, overturning and bearing

capacity failure, reinforcement process of wall components was conducted. Cross sections of concrete wall components

and their steel reinforcements were checked by using TS500 specification. The results are shown in Fig.4.

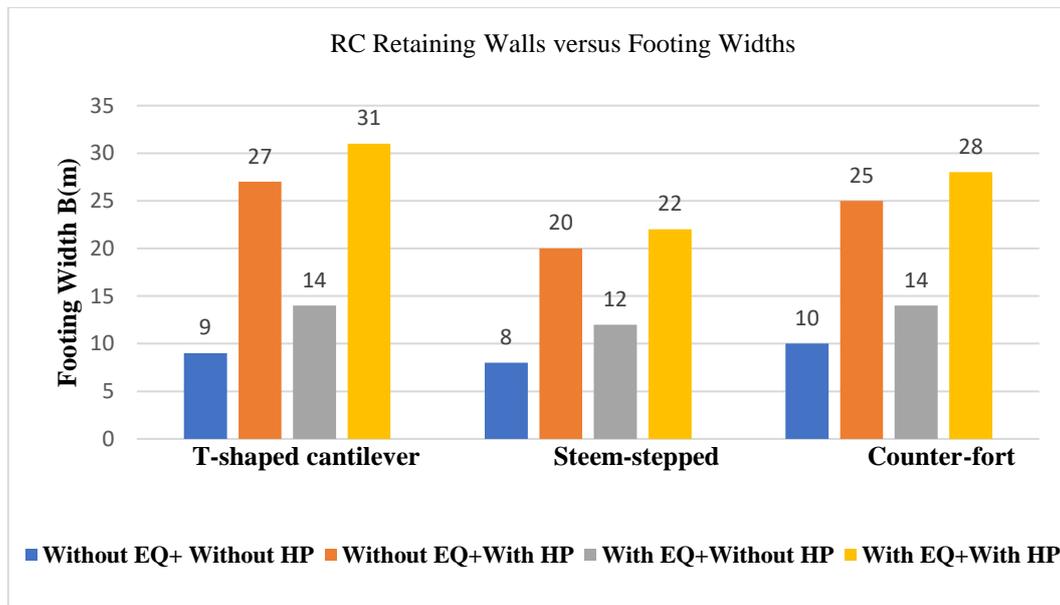


Fig.4. Reinforced concrete walls versus the relevant footing widths

As indicated in Fig.4, three different shaped cantilever walls were subjected to stability analyses and reinforcements. Each wall was analyzed under some loads; first loading: in case of no earthquake and no hydraulic pressure, second loading: in case of no earthquake motion and presence of hydraulic pressure, third loading: in case of presence of earthquake motion and no hydraulic pressure, and fourth loading: in case of presence of earthquake motion and hydraulic pressure.

It is determined that the extreme loading required the greatest footing widths and otherwise, the smallest widths revealed without earthquake and hydraulic pressure. When considered all loading cases, it can be said that steem-stepped cantilever walls provided the most suitable wall dimensions satisfying conditions regarding stability analysis and reinforcement. This process is followed by counter-type cantilever wall as second one and finally by T-shaped cantilever wall.

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