Analysis of Bridge Deck for Abnormal Load using the Acceleration-Displacement Ratio for a Balcony Function

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Abstract

The paper reviews a different approach to reduce risk from abnormal loads and to limit the occurrence of progressive collapse in large panel bridge structures. A bridge deck model is developed and based on a function $[W_m = A_m(Sin\ m\pi x/a)(y/b)]$ *was analysed using the acceleration-displacement method of plate analysis. A philosophy for establishing general structural integrity is developed to assure bridging of local damage while maintaining overall stability, thus eliminating the need to design for any particular abnormal load. In this approach, values for moment and deflections were obtained at different points of the bridge deck, considering an abnormal load of 150tons.This approach gave a central defection value of about 0.006073pa4/D and a moment maximum under the wheel load closest to the center line of the bridge deck of 3353.689KNm of dimension 20m by 11m. This has proven to be a more conservative approach as compared to similar research work carried out in this area.*

Keywords: *bridge deck,balcony,curvature, displacement, point load, acceleration.*

I. INTRODUCTION

Bridge deck design is a vital aspect of bridge design, since it is the main contact area between applied load and load transfer. Most bridges are designed to carry axle loads on daily basis but there is still need to consider some abnormal loads which may occur over time as a result of haulage of very heavy equipment which are used in construction sites near or within the axis of the location of such a bridge. With the boundary conditions which are limited by the balcony function in this research, it is expected that the location of such a load on the fixed end of the bridge may cause an overturning on the opposite (free) end of it using the acceleration-displacement ratio method (Johnarry, 2011).

Plates are common structural elements employed in many engineering applications and are subject to a wide variety of loads ranging from distributed loads, point loads, sinusoidal load, hydrostatic loads etc. (Onyia, 2008).

A lot of work have been done in the past on plate among which are; Lagrange's biharmonic equation, Johansen (1962, 1972) Yied-line analysis, Hillerborg (1956, 1959, 1975, 1982) Strip Method, Kantorovitch and Krylov (1954) approximations and recently Johnarry (1972, 2011, 2013), Ephraim and Orumu (2002, 2013), Timoshenko and Woinowsky (1959), Otoyo*et al.*(2015) etc. Bridge deck as a plate structure can be analyzed using various method of exactanalysis, but adopting the classical approach of Navier, which considered plate thickness in the general plate equation as a function of rigidity,D and transformingthe differential equation into algebraic expression by use of fourier trigonometric series, the acceleration-displacement ratio method can be used to modify the procedure to obtain a solution for the balcony function.

II. THEORETICAL FORMULATION

The basic concept of bridge analysis is based on plate theory wholes solution is seen below:

Plate Solution

The flexure of the plate is described by the bi-harmonic equation,

$$D\left[\partial^4 w/\partial x^4 + \partial^4 w/\partial y^4 + 2\partial^4 w/\partial x^2\ \partial y^2\right] = q \qquad 1$$

Alternate statement of equation 1

$$= D\left(w_{xxxx} + 2w_{xx\,yy}\right) = q \qquad 2$$

The equation can be solved as,

D J. w = q

Where, (D=flexural rigidity of thin plate.)

Therefore, $$w = \frac{(q/D)}{J} \quad (3)$$

Transform into acceleration-displacement ratio,

$$\partial^4 w/\partial x^4 = J_{xx} \cdot (\partial^2 w/\partial x^2)/w$$

$$\iint (\partial^4 w/\partial x^4)/w \ \partial x \partial y = J_{xx} \iint (\partial^2 w/\partial x^2) \partial x \partial y \quad (4)$$

For the all-round - simply supported case of a plate, take

$$w = \Sigma \Sigma A_{mn} \ (Sin \ m \ \Pi \ x/a) \ Sin \ n \ \Pi \ y/b \quad (5)$$

$$w = \Sigma \Sigma A_{mn} \ \Phi_x \Phi_y \quad (6)$$

Introducing equation 5 for equation 4, we have

$$J_{xx} = A_{mn} \ m^3 \ n \ \pi^4/16 \ a^2$$

$$\partial^4 w/\partial y^4 = J_{yy} \left(\partial^2 w/\partial y^2\right)/w \quad (7)$$

$$J_{yy} = A_{mn} \ m \ n^3 \ \pi^4/16 \ b^2 \quad (8)$$

The function in consideration is $W = \Sigma A_m (Sin \ m \pi x/a)(y/b)$ which will be analyzed and values for its moments, amplitude, and deflections are obtained.

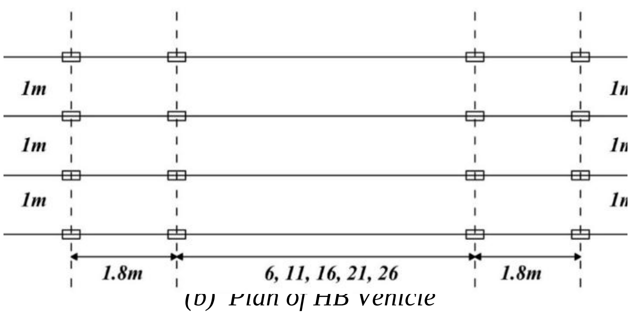

Figure 1:Dimension of HB Vehicle (a) Top View of a HB Loading(b)Plan of HB Vehicle

Normal differentials are reduced to critical resonant norms for summation.

Mass x acceleration = $[K]$ x W_i 9

Acceleration/W_i = $[K]/mass$ = constant

For a steady state of motion, the above equation must be obeyed.

Given the plate equation;

$$W_{xxxx} + W_{yyyy} + 2W_{xxyy} = q/D \quad (10)$$

Transforming the differential into normal relative acceleration-displacement ratios R_{xx}, R_{yy}

$$\partial^4 w/\partial x^4 = G_{w4x} \left[\left(\partial^2 w/\partial x^2\right) - \left(\partial^2 w/\partial x^2\right)_0\right]/w = G_{w4x} R_{xx} \quad (11)$$

$$\partial^4 w/\partial x^4 = G_{w4x} \left[(\partial^2 w/\partial x^2)_{rel}\right]/w$$

So, multiply by *w* and integrating we have;

$$\iint \left(\partial^4 w/\partial x^4\right) wdxdy = G_{w4x} \iint \left(\partial^2 w/\partial x^2\right)_{rel} dxdy \quad (12)$$

Implementationof the procedure for a typical structure.

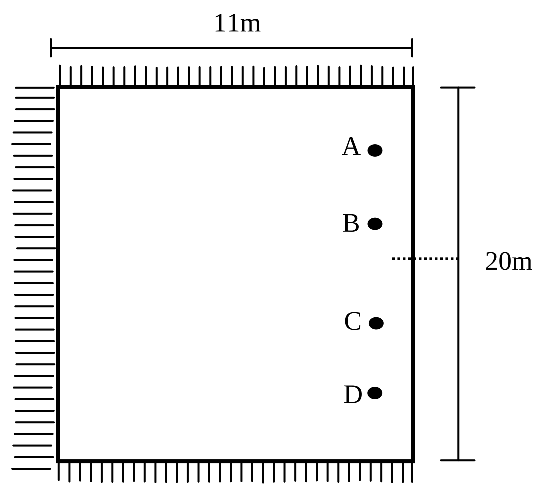

Figure 2:Bridge Deck

The bridge model above is adopted for the purpose of this research. The deck is dimensioned 20mx 11m with the two longitudinal ends simply supported and the transverse end having one end approximate simply supported and the other end free. The term approximate is used to depict the actual configuration of the boundary condition, where that part of the deck is being supported/ reinforced to carry pedestrian footings and other light weight load that need to use the bridge as well.

Calculationfor deflection:

Axle D is positioned at 3.7m from the simply supported end of the longitudinal axis of the bridge deck, while Axle C is place at 5.5m from same end as axle D, Axle B is place at 11.5m from same end which extends finally to axle A which is also positioned at 13.3m away from the simply supported end as well. Each carrying an axle load of 375KN and the effect of these loading is analysed using the acceleration-displacement method where a balcony function as seen below is adopted.

$$W_m = A_m (Sin\ m\pi x/a)(y/b) \quad 14$$

From Equation 3.2

$$\delta^4 w/\delta x^4 + 2\delta^4 w/\delta x^2 \delta y^2 + \delta^4 w/\delta y^4 = q/D \quad 15$$

Where$w$ is the deflection.

Differentiating, we have;

$$\delta^4 w/\delta x^4 = A_m m^4 \pi^4/a^4 (Sin\ m\pi x/a)(y/b)$$

$\delta^4 w/\delta x^2 \delta y^2 = 0$

$\delta^4 w/\delta y^4 = 0$

also;

$$D\left[J_{XX}(\partial^2 w/\partial x^2)/w + J_{xy} + J_{yy}(\partial^2 w/\partial y^2)/w\right] = q \quad 16$$

Multiplying each term by w and integrating twice we have;

$$D\left[J_{XX}(\partial^2 w/\partial x^2) + J_{xy} w + J_{yy}(\partial^2 w/\partial y^2)\right] = q.w$$

$$D\int^a\int^b J_{XX} A_m \left(m^2\pi^2/a^2 (Sin\ m\pi x/a)(y/b) + 2J_{xy} A_m (Sin\ m\pi x/a)(y/b) + 0 = \int^a\int^b q\ X\ w\right)$$

$$D\left[J_{XX} A_m (m^2\pi^2/a^2)(Sin\ m\pi x/a)(y^2/2b) + 2J_{XX} A_m (a/amx)(Sin\ m\pi x/a)(y^2/2b) = P.(Sin\ m\pi x/a)(y/b)\right]$$

But;

$$\partial^4 w/\partial x^4 = J_{xx} R_{xx}$$

$$LHS = J_{xx}(x - x_0)/w = J_{xx} R_{xx} \quad 17$$

$$LHS\ X\ w = J_{xx}(x - x_0)$$

$$R_{xx} = (x - x_0)/w$$

Therefore,

$$(x - x_0) = R_{xx}\ X\ w$$

$$\partial^2 w/\partial x^2\ X\ w = m^2\pi^2/a^2 (Sin^2\ m\pi x/a)(y^2/b^2)$$

$$LHS\ X\ w = J_{xx} R_{xx} \quad 18$$

$$m^4\pi^4/a^4 (Sin^2\ m\pi x/a)(y^2/b^2) = J_{xx} m^2\pi^2/a^2 (Sin^2\ m\pi x/a)(y^2/b^2)$$

$$\int^a\int^b m^4\pi^4/a^4 (Sin^2\ m\pi x/a)(y^2/b^2) = J_{xx}\int^a\int^b m^2\pi^2/a^2 (Sin^2\ m\pi x/a)(y^2/b^2)$$

$$m^4\pi^4/a^4\ X\ (a/2)\ X\ (y^3/3b^2) = J_{xx} m^2\pi^2/a^2\ X\ (a/2)\ X\ (y^3/3b^2)$$

Therefore;

$$J_{xx} = m^2\pi^2/a^2 \quad 19$$

Also, $LHS = J_{yy}(y - y_0)/w = J_{xx} R_{xx}$

But; $\partial^4 w/\partial y^4 = 0$

so $J_{yy} = 0$

likewise; $\partial^4 w/\partial x^2 \partial y^2 = 0 = J_{xy}$

Collecting the terms together we have.

$$D\int^a\int^b J_{XX}\partial^2 w/\partial x^2 + 2J_{xy} w + J_{yy}\partial^2 w/\partial y^2 = \int^a\int^b q\ X\ w \quad 20$$

$$D A_m\left[m^2\pi^2/a^2\ X\ m^2\pi^2/a^2 (Sin\ m\pi x/a)(y/b)\right] = P(Sin\ m\pi x/a)(y/b)$$

Therefore,

$$A_m = \Sigma\Sigma P a^4/Dm^4\pi^4$$

Since the amplitude has been obtained, the deflection function now becomes;

$$w_m = \Sigma\Sigma P a^4/Dm^4\pi^4 (Sin\ m\pi x/a)(y/b)$$

Calculation for Moment

To obtain the moments values, we say;

$$m_x = \left[\delta^2 w/\delta x^2 + v\delta^2 w/\delta y^2\right]D \quad 21$$

(v = 0.3), for steel design

Therefore;

$$\delta^2 w/\delta x^2 = m^2\pi^2/a^2\,(Sin\;\; m\pi x/a)(y/b)$$

and $\delta^2 w/\delta x^2 = 0$

$$m_x = \sum\sum P\; a^4/Dm^4\pi^4 \text{ X } m^2\pi^2/a^2\,(Sin\; m\pi x/a)(y/b)$$

$$m_x = \sum\sum P\;\; m^2\pi^2/a^2\,(Sin\; m\pi x/a)(y/b) \quad 22$$

Also, since

$m_y = vm_x$

then,

$$m_y = v\sum\sum P\; a^2/m^2\pi^2(Sin\; m\pi x/a)(y/b) \quad 23$$

Also;

$$m_{xy} = D\left[\delta^2 w/\delta x\delta y\right](1 - v) \quad 24$$

$$\delta^2 w/\delta x\delta y = P\,a^4/Dm^4\pi^4 \text{ X } m\pi/a\,(Cos\;\; m\pi x/a)(1/b) \quad \{since \quad v = 0.3\}$$

$$m_{xy} = \sum\sum P\; a^3/m^3\pi^3 b\;\; (Cos\;\; m\pi x/a) \quad 25$$

Deflection at centre: Let a =20m; b =11m, x = a/2 ; y = b/2; p= 375KN

$$\Sigma^{\infty}\; m = 1,\,3,\,5,\,7,\,9 \qquad W_m = W_1 + W_3 + W_5 + W_7 + W_9 \;.......$$

$$W_1 = \frac{P \times a^4}{D \times 1^4 \times \pi^4}\left(Sin\;\frac{\pi}{2}\right)\left(\frac{1}{2}\right) = 0.00513$$

The calculation above is repeated for the series m = 3, 5, 7 and 9 to obtain the deflection.

Therefore,

$$\Sigma^{\infty}\; m = 1,3,5,7,9 \qquad W_m = 0.006073 \quad mm$$

Moment at the centre.

Let a =20m; b =11m, x = a/2; y = b/2; p= 375KN

$$M_1 = \frac{P \times a^2}{1^2 \times \pi^2}\left(Sin\;\frac{\pi}{2}\right)\left(\frac{1}{2}\right) = 0.05447$$

The calculation above is repeated for the series m = 3, 5, 7 and 9 to obtain the moment.

Therefore,

$$\Sigma^{\infty}\; m = 1,3,5,7,9 \qquad M_m = 0.05468 \quad kNm$$

III. RESULTS

Table 1 an 2 below shows the respective deflection and moment (m_x) values for various axle points as seen from the transverse section of the bridge deck. The axle load was loaded individually without considering the impact of other axle points on the bridge deck, and result obtained as seen below. It conforms to the principle that says; moment will occur maximally under the wheel load closest between the centre line of the load and the centre of the bridge deck. This is evident from the graph in Figure 3, thus Axle B is closest to the centreline and therefore has the highest moment value(1240.643KNm), also a steep gradient was observed for the moment.

Table 1:Table showing deflection values of the various axle load point.

y(m)	W_A(mm)	W_B(mm)	W_C(mm)	W_D(mm)
1	25.23	28.018	22.247	16.323
4	100.918	112.07	88.989	65.292
7	176.607	196.123	155.73	114.262
10	252.295	280.175	222.247	163.231

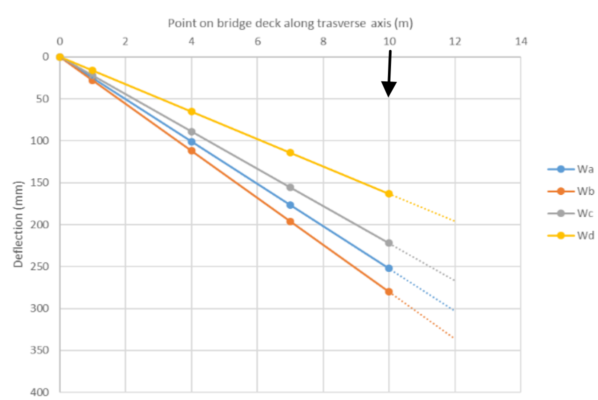

Fig. 3:A graph showing deflection of Axle load on bridge deck

Table 2:Showing moment (m_x) values along transverse axis of the bridge deck.

y(m	M_{xA} *(KNm)*	M_{xB} *(KNm)*	M_{xC} *(KNm)*	M_{xD} *(KNm)*
1	117.456	124.064	108.952	88.516
4	469.823	496.25	435.809	354.06
7	822.19	868.45	762.665	619.61
10	1174.55	1240.64	1089.52	885.15

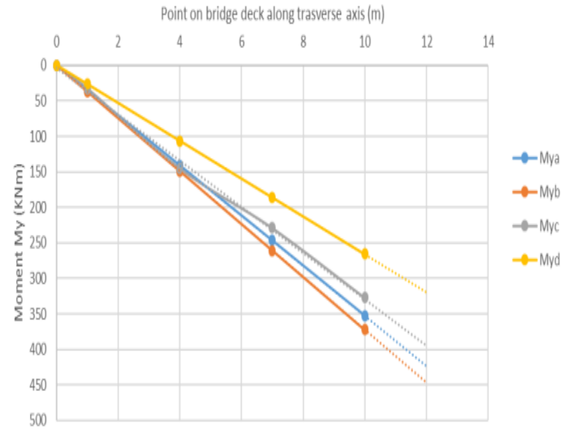

Fig. 5:A graph showing Moment, My of Axle load on bridge deck

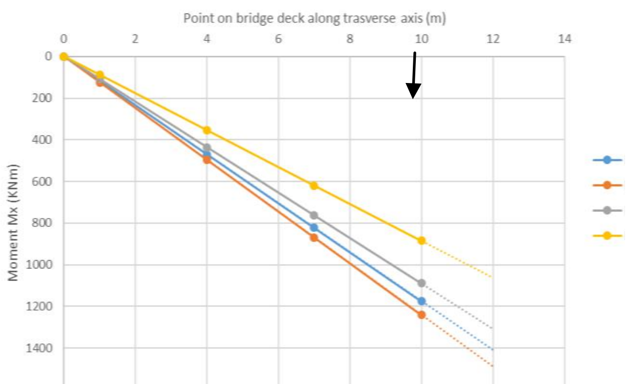

Fig. 4:A graph showing Moment, Mx of Axle load on bridge deck

Table 3:Showing moment (m_y) values along transverse axis of the bridge deck.

y(	M_{yA} (KNm	M_{yB} (KNm	M_{yC} *(KNm)*	M_{yD} *(KNm)*
1	35.23	37.21	32.68	26.55
4	140.9	148.8	145.1	106.2
7	246.6	260.5	228.7	185.8
10	352.3	372.1	326.8	265.5

Table 4:Results showing Twisting Moment

X*(m)*	M_{xy}(KNm)
13.3	531.794
11.5	582.726
5.5	475.727
3.7	357.465

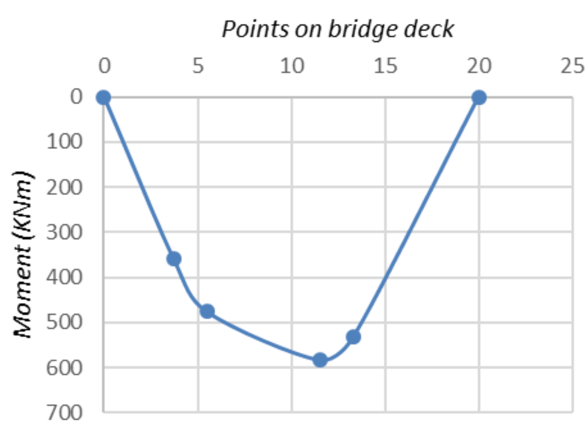

Figure 6: Graph showing Twisting Moment M_{xy}

When the bridge deck is under combine loading,(i.e. all axle load acting at the same time) each axle load will have an effect on the other depending on their respecting intensity. Axle A will have an effect on Axle B, also on Axle C and even on axle D and same happens to the other axle loads.

Table 5, shows the contribution of each axle load on another, these were listed individually, and to this effect, a summation of the load itself, and the collective effect/impact of other load on the axle under consideration are termed the combine load effect. This, thus gives the actual moment/deflection under each axle load, and that explains the graph below for the bridge deck model used in this research.

Table 5: Combine Moment Values for each Axle Load.

Load point	Mx, a (KNm)	Mx, b (KNm)	Mx, c (KNm)	Mx, d (KNm)
a = 13.3	1174.55	1017.77	491.87	333.41
b = 11.5	1017.77	1240.643	634.9	460.385
c = 5.5	491.87	634.9	1089.52	789.23
d = 3.7	333.41	460.389	789.23	885.158
Σ	**3017.6**	**3353.698**	**3005.52**	**2468.183**

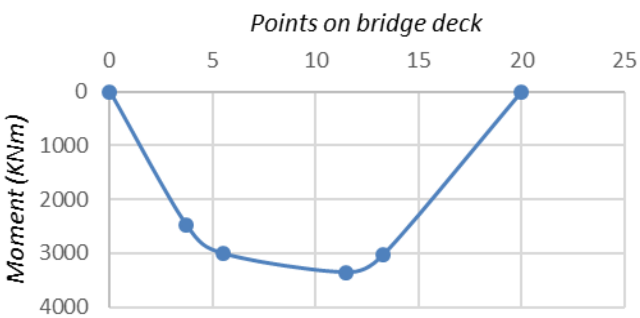

Figure 7: Moment along the Longitudinal Axis for Combine Load

Deflection was also obtained for combine loading. But a unique difference was spotted. Unlike the moment, the maximum deflection still occurred at the center of the bridge deck as seen in figure 7, along the longitudinal axis. Though with a very close value between the midpoint and axle B representing 765.941mm and 746.13mm respectively.

The gradient of the graph has a steep slope between the simply supported end and the axle D, which gradually curves on approaching axle C and with a curve whose apex is at a distance that describes the mid-point (10m) and finally returns back to the zero point in similar manner in which it descends.

Table 6: Total Deflection Values for each Axle Load.

Load point	Wmn, a (mm)	Wmn, b (mm)	Wmn, c (mm)	Wmn, d (mm)
a = 13.3	252.295	218.607	105.65	71.62
b = 11.5	218.607	280.175	143.38	103.97
c = 5.5	105.65	143.38	222.247	160.99
d = 3.7	71.62	103.97	160.99	163.231
Σ	**648.172**	**746.13**	**632.27**	**499.81**

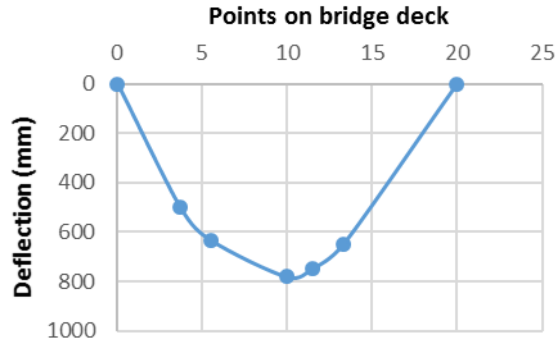

Figure 7: Deflection along the Longitudinal Axis for Combine Load

Based on the scope of this research, similar work done by Timoshenko and Woinowsky-Kreieger (1959)was compared based on assumptions/boundary condition adopted in this research. The deflection value at the centre, gave a value of0.006073pa^4/D as against 0.00710pa^4/D obtained by Timoshenko and woinowsky-kreieger. Also, the moment in similar manner gave 0.05468pa^2 as against 0.05668 pa^2.The shape function used on this research work is a combination of a trigonometric function and a polynomial function. But for Timoshenko and Woinowsky-Kreieger, the shape function was purely a trigonometric function, which tends to converge easily. Thus, this is the reason for the slight discrepancy.

IV. CONCLUSION

Though, the maximum bending moment occurred under the axle load closest between the centre line of the bridge and centre of loading, (axle B). The maximum deflection still occurred at the centre of the bridge deck.Also, the comparative analysis of similar work done by Timoshenko, using the Navier solution method, has really shown a great similarity in the result.

Hence, a conclusion can the drawn to the fact that the balcony function used in this research work, for bridge deck analysis using the acceleration displacement method gave valid results for moment, deflection and even shear forces, which is helpful in the design and detailing of bridge deck for construction benefits.

RECOMMENDATIONS

From this research, I strongly recommend that:

1. The acceleration-displacement method should be applied to every other boundary condition such as clamped plate.
2. Hydro-static loads should be analyzed using the acceleration-displacement method of plate analysis.

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