SSRG International Journal of Civil Engineering
ISSN: 2348 – 8352 /doi:10.14445/23488352/IJCE-V6I1P105
Volume 6 Issue 1, 25-36, January 2019

Stiffness Modifications for Vibration Solutions of Multistory Frames (An Approach from Buckling to Vibration)

Nwofor T.C. and Obianime, Tamunoene Sunday

Department of Civil Engineering, University of Port Harcourt, P.M.B 5323 Port Harcourt, Rivers State, Nigeria.

Abstract

Vibration analysis has been approached through a common knowledge of buckling in this study. The introduction of the effective vibration length in vibration analysis makes vibration solutions very easy and design-oriented - effortlessly computing the frames' frequencies. Modification of Stiffness introduced to the concept of effective vibration length has enabled its application to every type of frame (rigid or flexible). The study shows the convergence of results by the present method to results by other reliable sources (error ranging from 0 – 2.5%). It has been observed that in multi-story steel frames, the average percentage difference between the present manual method and computer-based analysis increases as the height of the buildings increases. Hence this new method needs to be investigated further to improve results where the error should be less than one percent. Areas of improvement suggested are developing a mathematical model of the column alignment chart and taking the average of all the joint stiffness coefficients in a parent multi-story frame as the top stiffness coefficient determining the effective length of the reduced frame. However, taking the reduced frame stiffness coefficient as that of any joint with the highest coefficient has yielded results that are very close to results by computer-based dynamic analysis and other very reliable sources. Effective vibration length offers a new and easier way of providing vibration solutions.

I. INTRODUCTION

The relationship between buckling and vibration in building collapse requires more detailed investigation for safer designs of buildings. In developing economies, failure of buildings is frequent, and losses in lives and materials are enormous.

In recent times, there have been the collapses of the Rana Plaza building in Bangladesh (2013), Synagogue Church building in Lagos, Nigeria (2014), Reigners Bible Ministry in Uyo, Nigeria (2017), etc.

Investigations have revealed that these collapses are due to structural and vibrational failures. The Rana Plaza collapse has been associated with structural failure arising from continuous vibration of generators placed at the top of the Building. These vibrations went on for years and thus weakened the structural integrity of the building.

Preliminary vibration analysis requires the fundamental frequencies and guidelines are now found in the international code council and other standards. Following the American Institute of Steel Construction (AISC) effective length principles, it is easy to show that columns carry their unique, effective length factors (Shanmugam and Chen, 2001). Wood (1974) was convinced that a frame assumes a single controlling effective length at incipient collapse, as is the case with the vibration frequency. Horne's (1975) method is based on an elastic analysis for a small sway load solved for the frames controlling effective length.

A strong relationship between the effective buckling length and its corresponding effective vibration length has already been established by Johnary (2016). He used this principle to solved the vibration problems of rigid frames. This principle of vibration solution is adopted with some modifications to the structures' Stiffness before using the column alignment chart to obtain the effective lengths of the reduced frames. Thus flexible frames can also be analyzed by this method.

The modified model is then used to solve some flexible frames already analyzed by exact methods to validate the new concept.

II. METHODOLOGY

This research aims to establish the similarity between buckling and vibration in their fundamental modes of deformation and then solve for vibration in both flexible and rigid frames, from the known knowledge of buckling in common design usage, such as the effective Euler length.

The basic procedure is as follows:

1) Modify the adopted method of analysis to cover both rigid and flexible frames by introducing stiffness modifications.
2) Validate this modification by reanalyzing frames already analyzed by exact methods with the new concepts and comparing results.
3) To answer frequently asked questions of the fundamental frequencies in multi-story frame design (period, drift, etc.) by analyzing several frames with the new method

4) To determine the dynamic responses of frames under known masses.
5) To compare results with other theoretical methods such as Rayleigh's method and results by computer analysis using STAAD PRO software.

III. DERIVATION OF EFFECTIVE VIBRATION LENGTH (H)

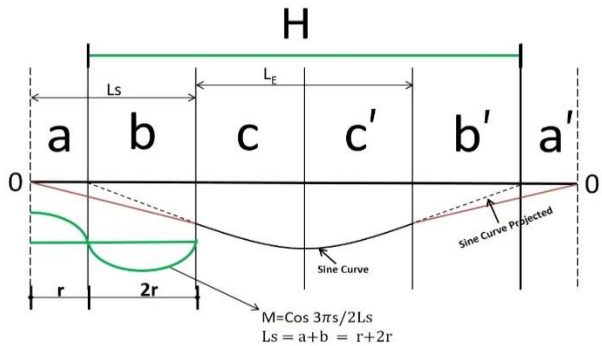


Fig 1. Fixed –Fixed-beam: effective vibration length H, from effective Euler-length, $L_E = c + c'$ (Johnarry 2016)

Concerning Fig. 1,

Effective-Euler-Length, $L_E = c + c' = \frac{L}{2}$

Hence

$$L - L_E = 2(r + 2r) = \frac{L}{2}$$

$$But\ \ L = a + b + c + a' + b' + c'$$

Now the effective vibration length,

$$H = b + c + c' + b'$$

Now,

$L = L_E + 6r$, and $L = H + 4r$

And also, $r = L - L_E / 6$

The effective vibration length is thus given by the equation

$$H = L_E + \frac{L - L_E}{3} \qquad (1)$$

Where L_E= Euler Effective length.

This equation can be applied algebraically to all cases between fixity and inflection, and the Euler effective length may be gotten from column alignment charts in preliminary Band analysis.

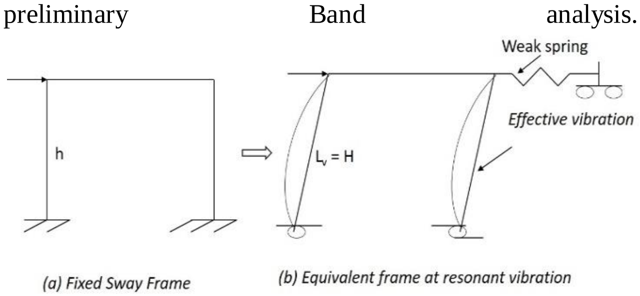


Fig.2. Effective vibration length L_v = H

A. Vibration through buckling

Equation 1 is key to vibration solution as the effective Euler length in buckling can be easily replaced by the effective vibration length as given by this expression.

Now the equation of free vibration under uniform mass is given by

$$M^* . \ddot{y} + K\, y = 0 \qquad (2)$$

Where M^*= uniform mass

$$Let\ y = A\ Sin\omega t$$

$$\frac{dy}{dt} = w.A\ Cos\omega t \qquad and$$

$$\ddot{y} = -w^2 A\ sin\omega t$$

$$\therefore\ -M^*.w^2 A\ sin\omega t + K.Asin\omega t = 0$$

From which

$$Ky - w^2 M^* y = 0 \qquad (3)$$

Now for a beam (supply supported), the flexure related beam vibration is expressed as

$$EI\frac{d^4y}{dx^4} + M^*\ddot{y} = 0$$

But $M^*\ddot{y} = -\omega^2$ M*y

Therefore

$$EI\frac{d^4y}{dx^4} - \omega^2 M^* y = 0 \qquad (4)$$

Therefore by satisfying (2) and (3), ω^2 can be found.

The general solution to equation (4) is;

$$Y = A\sin - ut + B cos - ut + C\sinh - ut + D\ cosh - ut$$

Through boundary condition, this general equation ends up as

$$Y = A\ sin - ux :\ with\ u = \frac{\pi}{L}$$

$$\dot{y} = -u\ A\ cos - ux = velocity$$

$$\ddot{y} = u^2 A\ sin - ux = acceleration$$

$$\dddot{y} = -u^3 A\ cos - ux$$

$$\dddot{y}' = u^4 A\ sin - ux$$

Hence equation (4) becomes

$$EI.u^4 A\ sin - ux - \omega^2 M^*.A\ sin - ux$$

$$\therefore\ u^4 (EI) = \omega^2 M^*$$

$$u^4 = \frac{\omega^2 M^*}{EI} \qquad (5)$$

$$And\ \ \omega^2 = \frac{u^4 EI}{M^*} \qquad (6)$$

But $u^4 = \frac{\pi}{L}$

Hence

$\omega^2 = (\frac{\pi}{L})^4 \cdot \frac{EI}{M^*} \quad = \frac{EI\pi^4}{ML^4}$ (7)

For this case, the actual length L (Euler length) = H (pin-pin beam)

Where H = effective vibration length.

Hence,

$\omega^2 = \frac{EI\pi^4}{M^* H^4}$ (8)

The uniform mass M^* must be converted to the effective uniform mass e_m.

Therefore the vibration of any column, which is always determined by the angular velocity, ω^2 is given by the new expression;

$\omega^2 = \frac{EI\pi^4}{H^4 . e_m}$ (9)

Where;

H= effective vibration length

e_m = effective unit mass

Two new factors are introduced by this equation, the effective vibration length H and the effective unit mass, e_m

The effective vibration length, H, is found from the column alignment chart

The universally accepted expression gives the period of vibration T;

T = 2**π**/**ω**

B. Sway column and beam stiffness Modifications

Before entering the column alignment chart, the columns' Stiffness relative to the fixed-fixed beam connecting them has to be modified depending on the column's support conditions.

$K_c = sK_{c,0} \;\; or \;\; (I_{c.e}) = s(I_{c,0})$ (10)

Where "s"is a modification factor.

s=1 for fixed un-braced condition

s =3/4 for pinned un-braced condition

s=3/16 for roller, un-braced condition

Also the stiffness of the fixed-fixed beams connecting the columns may be modified as follows,

$K_b = s.\ K_b$ (11)

s = 1.5 (far-end rotation = near-end rotation indicating double curvature)

s = 0.5 (far-end rotation equal but opposite near-end rotation indicating single curvature)

s = 0.75 for far-end pinned.

In frames, the connecting beams are usually in double curvature, and their Stiffness is increased by a modification s = 1.5.

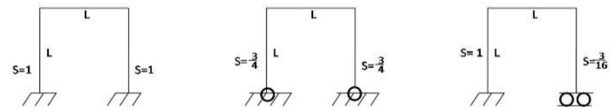


Fig 3 Sway Stiffness modification factors

C. Implementation of the procedure on simple frames

To show how the concept of effective-euler length is implemented by using the stiffness modification factors to estimate the effective vibration length, some simple single-story frames shall be analyzed to illustrate how to apply this method of analysis.

Figure 4 shows a pinned portal frame with members of equal length. The evaluation of the natural angular frequency (ω^2) is desired.

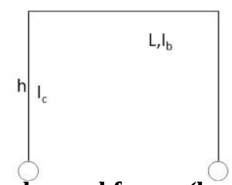


Fig. 4 Pinned portal frame (beam length = column length).

Step1: determination of the effective length

The Stiffness at the top and bottom joints is evaluated

At the joints G, the stiffness coefficient is given by;

$G = K_c/K_b$

And K = I / L

$G_{top} = \frac{[I_c/_h]}{I_b/_L} . s = 1 \; x \; 0.75$ (far-end pinned)

$G_{bottom} = \frac{[I_c/_h]}{I_b/_L} = \infty, \; (I_b/_L = 0)$

If the far end of a bar is pinned, s = 0.75, and $K_c = sK_{c,0}$= 0.75 x 1 = 0.75

Using these values in the column alignment chart gives the effective length of the frame, L_E = 2.25h.

Step 2: determination of effective vibration length

H = Le + (h – Le)/3 = 1.83h (equation 1)

Step 3: determine the overturning moment per column and evaluation of the equivalent uniform mass.

Note here that the mass carried by the beam (dead mass) will be shared equally between the two columns. So the lumped mass per column is M/2.

L_m = Lumped mass = m/2 per column.

Consider single Col

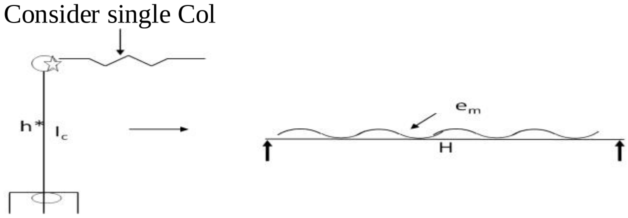


Fig. 5. Overturning moment due to lumped mass and effective unit mass simply supported over the vibration length H.

The overturning moment per column is $(L_m)(H) = MH/2$. Here the effective vibration length is used to determine the overturning moment. The original concept used the actual lengths of the bars.

The maximum moment considering e_m as a simply supported distributed load gives the maximum moment = $\frac{e_m(H^2)}{8}$

Equating overturning moment and maximum moment due to the equivalent unit load

$$(L_m)(H) = \frac{e_m\,(H^2)}{8},, \qquad \backslash evI\backslash = H$$

$$e_m = 8(\frac{L_m.H}{H^2})$$

Step 4: determine ω^2, the angular frequency.

Recall equation (9),

$(\omega^2)_{col} = (\frac{97.4E(I_c)}{[(H)^4.e_m]}$

$(\omega^2)_{col=}\frac{8.6E(I_c)}{(h^3.h.e_m)} = \frac{8.6E(I_c)}{[(h^3\left(\frac{hH}{H^2}\right)8L_m]}\quad ;;\ L_m = \frac{M^*}{2}$

$(\omega^2)_{col} = \frac{8.6E(I_c)}{\left[(4M^*).h^3\left(\frac{h}{H}\right)\right]}\quad ;\quad H = 1.83h$

$\omega^2 = \frac{3.97EI}{[M^*h^3]}\ ;$

Check for exact $\frac{4EI}{(M^*h^3)}$; the error may arise from manual effective length graph interpretation.

The portal frame of Fig. 3.5 is symmetrical, and the result gotten using the present method of study and an exact method of analysis gave very close results with little error. Fig. 3.8 is a portal frame of unequal lengths (asymmetric frame) with a very rigid beam. R. C. Coates and others have analyzed this frame in their book, "STUCTURAL ANALYSIS," second edition and are now analyzed using the present method.

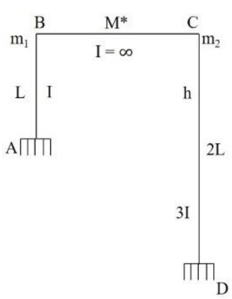


Fig 6. Portal frame of unequal length.

Each column shall be analyzed for vibration separately. As expected, the mass on the girder shall be shared between both columns. Let the mass on column AB be M_1 and that carried by column CD be M_2. So that $M = M_1 + M_2$.

Considering column AB, because the beam joining the two columns is very stiff (rigid), the stiffness coefficients at the top are zero, and that at the bottom is also zero (fixed end).

$G_{top} = \infty = 0$, and $G_{bottom} = \infty = 0$

The effective length as interpreted from the alignment chart is equal to 1. Therefore,

$L_E = L$

And the vibration length, H, as expressed by equation 1, is also 1, because,

$H = l + \frac{l-l}{3} = L$

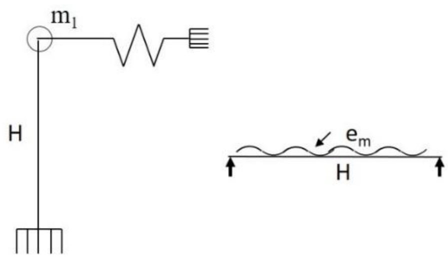


Fig. 7. Overturning moment due to lumped mass on column AB and effective unit mass supported H's vibration length.

The overturning moment about point B considering M_1 is given as

$$M_{overturn} = M_1 H$$

And the maximum moment along the span of column AB with the equivalent mass considered as simply supported is given as;

$\frac{e_m H^2}{8}$

Comparing both moments,

$M_1 H = \frac{e_m H^2}{8}$

Thus the equivalent unit mass $e_m = 8\frac{M_1}{H}$

And the angular velocity of column AB is,

$\omega_{AB}^2 = \frac{97.41\,EI}{e_m.H^4} = \frac{97.41.EI_c}{8\frac{M_1}{H}.H^4} = \frac{12.175EI_c}{M_1 H^3}$

$\frac{12.175EI_c}{M_1 L^3}$

Carry out the same analysis on column CD give

$\omega_{CD}^2 = 4.5656\frac{EI}{M_2 L^3}$

Because frame vibrates with a single frequency,

$$\omega_{AB}^2 = \omega_{CD}^2$$

$\frac{12.175}{M_1} = \frac{4.5656}{M_2}$

$M_1 = \frac{12.175M_2}{4.5656}$

Recall that,

$M_1 + M_2 = M^*$

$M_2 = 0.2727M^*$

$M_1 = 0.7273M^*$

$\omega_{AB}^2 = \omega_{CD}^2 = \dfrac{12.175}{0.7273} = \dfrac{EI}{ML^3}$

$= 16.74\dfrac{EI}{ML^3}$

The exact value is $16.5\frac{EI}{ML^3}$. The 1.45% difference in the result is an error arising from the Alignment chart reading.

D. Analysis of multi-story frames

For the simple application of equation (9) in computing the natural angular frequency of any multi-story frame, the said frame must be reduced to a single frame.

1. Reduction of Multi-Story Frames

For the ease of columns:

The floor modeling ration must be defined as;

$$r_i = \frac{I_i}{(h_i)^2} \quad \ldots\ldots(12)$$

I_i = second moment of inertia of the ith column
h_i = height of the ith column
r_i= modeling ratio of the ith floor column

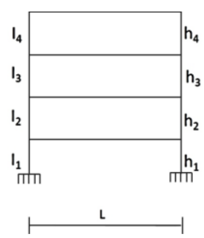


Fig. 8. Four-story frame with flexible girders

The overall modeling ratio of the ratio (re) is taken as the geometric average of all the individual modeling ratios in the frame

Thus;

$$r_e = (r_1.r_2.r_3.\ldots\ldots.r_n)^{\frac{1}{n}} = \frac{I_{e-col}}{(\sum h_i)^2} \ldots\ldots\ldots\ldots\ldots(13)$$

$$I_{e-col} = effective\ second\ moment\ of\ inertia\ of\ columms\ in\ reduced\ frame$$

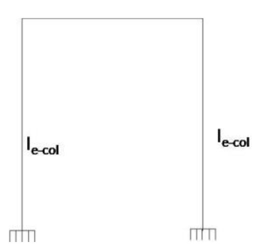


Fig 9.Reduced frame of Fig 8.

From equation (13)

$I_{e-col} = r_e(\sum h_i)^2$

And by putting $\sum h_i = h_e$

$I_{e-col} = r_e(h_e)^2 \ldots\ldots(14)$

And $I_{e-Frame} = 2I_{e-col}$.............. (15)

$I_{e-col} = effective\ second\ moment\ of\ inertia\ of\ columms\ in\ reduced\ frame.$

For the case of beams:

The equivalent top beam in the reduced frame should be modeled as given in equation 16

$$I_{b,e} = I_{b-roof} + \sum(I_b i_{CG} + A_i h_j^2) \qquad (16)$$

h_j = distance of floor r_i to top –floor.
Ai = area of ith beam.
I_{b-roof}= second moment of inertia of the roof beam
$I_{b,e}$= second moment of inertia of the equivalent top beam in the reduced frame
I_{biCG}= second moment of inertia about the centroid of the ith beam.

The expression given by equation (16) for the second moment of inertia of the equivalent top beam quickly tends to infinity when the number of floors is more than two in a frame, thereby converting flexible girders to a rigid reduced frame. To extend the concept of the present method to cover flexible frames, bearing in mind that a reduced frame must remain flexible if the parent frame is composed of flexible members, the following has been proposed ;

1. The equivalent top beam may be expressed as the sum of all the beams in the frame, divided by the number of bays of the frame.

$$I_{b.e} = \frac{\sum I_{bi}}{n_b} = second\ moment\ area\ of\ any\ beam$$

$$n_b = no\ of\ bays$$

2.The relative Stiffness at the top of the reduced frame may be taken as the Stiffness of any joint, within the frame, with the highest (critical) value.

The approximation that gives the highest value of the effective vibration length should be adopted for the analysis.

The second proposition has been found to give the closest approximations to exact results.

Concerning figure 8
With I_1=I_2 =I_3=I_4
h_1=h_2 =h_3=h_4

$$K_{critical} = \frac{2k_c}{\text{Kb}}$$

$$for\ any\ of\ the\ joints\ that\ has\ 2\ columns\ connected\ to\ it$$

K_c = Column Stiffness = $\frac{I_c}{L_c}$
K_b = Beam Stiffness = $\frac{I_b}{L_b}$
I_c = moment of inertia of column
I_b = moment of inertia of the beam
L_c = Length of column= h
L_b = Length of beam = L

These modifications to the original concept are used to solve problems that have already been analyzed by exact methods.

Consider the two-story frame in Fig. 10. This frame is taken from Anil Chopra's Dynamics of structure (2nd edition – problem 10.11). The frame carries lumped masses at the floor levels, and the natural frequency (fundamental) of vibration is desired. All members of the frame are of the same material and cross-section, indicating that it is flexible. The solution applying the proposed stiffness modification is as follows;

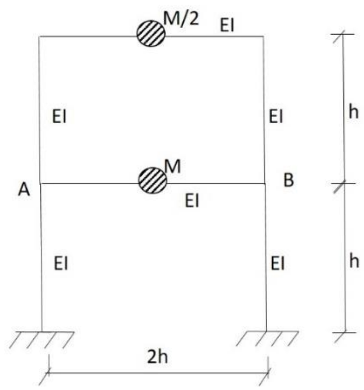


Fig. 10. Two-story frame with lumped masses

Step 1: Reduce the frame to a single-story frame

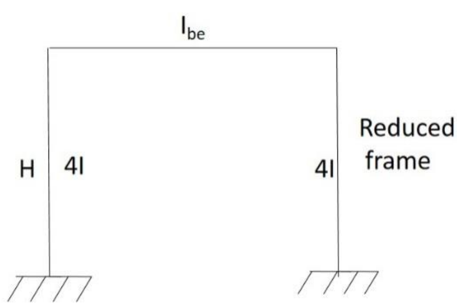


Fig. 11. The reduced frame of the two-story flexible frame.

The floor modeling ratio

$$r_i = r_1 = r_2 = \frac{I}{h^2},$$

And the overall floor modeling ratio is;

$$r_e = (r_1 \times r_2)^{\frac{1}{2}}$$

$$r_e = \frac{I}{h^2}$$

Thus the second moment of inertia of the columns in the reduced frame becomes;

$$I_{e-col} = r_e.(\sum h)^2 = \frac{I}{h^2} \times 2^2 \times h^2 = 4I$$

The Stiffness of the reduced frame is achieved by using the second proposition, considering joints A or B. Thus,

$K_{reduced\ frame} = 2\frac{K_c}{K_b} = \frac{2\frac{I}{h}}{\frac{I}{2h}} = 4$

From chart the effective length factor = 1.45 and L_E = $1.45\sum h$

Hence H = $1.3\sum h$ = 2.6 h (equation 1)

Step 2.Determining overturning moment:

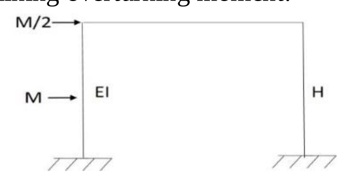


Fig 12:Overturning moment at the base of reduced frame

The overturning at the base of Fig. 3.9 is given as

$M_{ov} = \frac{m}{2}(H) + m\left(\frac{H}{2}\right) = mH$

Determine M_{max} considering e_m

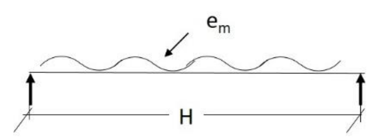


Fig 13: Simply supported equivalent uniform mass

$M_{max} = \frac{e_m H^2}{8}$

Putting $M_{max} = M_{overturn}$

$\frac{e_m H^2}{8} = M.H$

$$\therefore\ e_m = \frac{8M}{H}$$

Determine ω^2 ,

$$\omega^2 = \frac{97.41 \times 2 \times 4EI}{8\frac{M}{H}.H^4} = \frac{779.28}{8M \times 2.6^3 h^3} = 5.5222\frac{EI}{Mh^3}$$

$$\omega = 2.35\left(\frac{EI}{Mh^3}\right)^{\frac{1}{2}}$$

Exact solution by Anil Chopra's Dynamics of Structures (problem 10.11)

Gives $\omega = 2.41\left(\frac{EI}{Mh^3}\right)^{\frac{1}{2}}$

Error = 2.5%

This result is very close to the exact value. The little error is from the interpretation of the alignment chart

2. Idealized Steel Frames

In other to evaluate the dynamic responses of multi-story frames, using the present method, some frames have been idealized due to the limited material on already analyzed multi-story frames.

The frames considered herein are steel sway frames modeled as plane frames. Properties of the frames are given in detail. These frames are analyzed by the method being studied, and results are compared to solutions by STAAD PRO computer software.

3. Computer Analysis (staad pro software program)

taad Pro V8i computer software is employed in the dynamic analysis of all idealized multi-story frames.

The frequencies and periods of multi-story frames are analyzed by Modal Analysis and compared with Rayleigh's Method. The computer software uses Rayleigh's method to determine the maximum dynamic deflection.

Two percent (2%) of the total beam gravity load is taken as wind load to stimulate vibration in the program, acting on the external columns of all idealized steel frames. These frames are analyzed with the present manual method and results compared with Staad Pro Software's computer analysis, using both modal analysis and Rayleigh's method. The 2% gravity load can also be taken as a zero wind load combination.

4. Manual Vibration Solution to Idealized Steel Sway Frames

The properties of each frame are completely outlined. All idealized frames are made up of flexible members. Proposition no. 2 is used to determine the relative Stiffness of the reduced frames. The frames are firstly modeled inside the computer program, and the section properties have gotten from the computer used for the manual solution. Seven frames are worked on with this new method.

General Frame Properties

Inter-Story height – 4m

Beamwidth – 8m

Uniform undistributed load on beam – KN/m

Total mass on beams – 3.81NSec^2/mm

Elastic modulus of Steel -205,000N/mm^2

The density of Steel – 76.8195KN/m^3

Column Sections – UC 305 x 305 x 118

Moment of Inertia of Columns (I_c) – 277 x 10^{6}mm^4

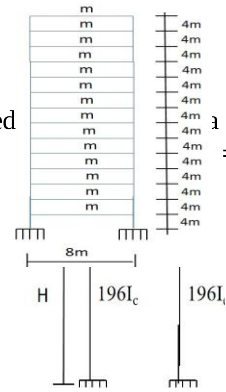
Beam Sections –UB 686 x 254 x 170

Moment of Inertia of beams (I_b) – 1700 x 10^6 mm^4

Area of beams sections – 0.0217m^2

14 Story Single Bay Frame

Red



a single-story frame

$$= (14)^2 I_c = 196 I_c$$

$$I_{e-col} = (14^2) I_c$$
$$= 14^2 \times 277 \times 10^6$$
$$= 5.4292 \times 10^{10} mm^4$$

The effective length factor = 1.1

And the effective length L_e= 1.0667∑h

1. Vibration Length H = 1.0667∑h = 1.0667 x 14 x 4000 = 59,735.2mm
2. Mass on beams

$$Beam\ load = (0.0217m^2 \times 8m \times 76.8195 \,{}^{KN}/_{m^3}) + \frac{3KN}{m} \times 8m$$
$$57.336\text{KN}$$
$$\text{Mass} = 3.81 \,{}^{N.s^2}/_{mm}$$

3. Overturning Moment

$$M_{ov} = MH\left(1 + \frac{91}{14}\right) = 7.5MH$$

Maximum Moment (M_{max}) due to equivalent unit mass.

$$M_{max} = \frac{e_m H^2}{8}$$
$$M_{max} = M_{ov} = \frac{e_m H^2}{8} = 7.5MH$$
$$\therefore e_m = 60\frac{M}{H}$$

5. Angular frequency (ω)

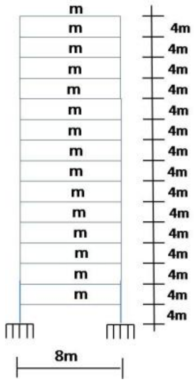
$$\omega^2 = \frac{97.41\, E(\sum I_{e-col})}{e_m . H^4}$$
$$\omega^2 = \frac{97.41 \times 205000 \times 2 \times 5.4292 \times 10^{10}}{60 \times 3.81 \times (59{,}735.2)^3}$$
$$\omega^2 = 44.4996$$
$$\omega = 6.67\, rad/sec$$
$$Period\ T = \frac{2\pi}{\omega} = \frac{2\pi}{6.67} = 0.94 secs$$

Computer Analysis (Staad Pro) Results

	Period	**frequency**
Modal Analysis	0.92 sec	1.09 circles/sec
Rayleigh's	0.8985 sec	1.113 circles/sec

Maximum Deflection = 275mm

15 Story Single Bay Frame



e frame to a single-story frame

$$I_{e-\,col} = (15)^2 I_c = 225 I_c$$

$$K_{critical} = K_{reduced\ frame} = \frac{2K_c}{1.5kg}$$

$$K_c = \frac{I_c}{L_c} = \frac{27700cm^4}{400cm} = 69.25cm^3$$

$$K_c = \frac{I_b}{L_b} = \frac{240{,}000cm^4}{720cm} = 212.5cm^3$$

$$K_{reduced\ frame.} = \frac{2 \times 69.25}{212.5} = 0.65 = K_{critical}$$

The effective length factor from chart = 1.1
Effective length $L_e = 1.1\sum h$
Vibration Length H = 1.0667∑h = 1.0667 x 60000 = 64,002mm (equation 1)

2. Mass on beams

$$Beam\ load = (0.0217m^2 \times 8m \times 76.8195\, {}^{KN}/_{m^3}) + \frac{3KN}{m} \times 8m$$

$$57.336\text{KN}$$

$$\text{Mass} = 3.81\, {}^{N.s^2}/_{mm}$$

3. Overturning Moment

$$M_{ov} = MH\,(1+7) = 8MH$$

Maximum Momentum (M_{max})

$$M_{max} = \frac{e_m H^2}{8}$$

$$M_{max} = M_{ov}$$

$$= \frac{e_m H^2}{8} = 6MH$$

$$\therefore e_m = 64\frac{M}{H}$$

4. Angular frequency (ω)

$$\omega^2 = \frac{97.41\, E(\sum I)}{e_m . H^4}$$

$$\omega^2 = \frac{97.41 \times 205000 \times 2 \times 225 \times 277 \times 10^6}{64 \times 3.81 \times (64002)^3}$$

$$= \frac{2.489142083 \times 10^{18}}{6.392718576 \times 10^{16}} = 38.83$$

$$\omega = 6.23$$

Period, T = $\frac{2\pi}{\omega} = \frac{2\pi}{6.23} = 1.01secs$

Computer Analysis (Staad Pro) Results for Period and frequency

Modal Analysis 1.02 sec 0.9804 circles/sec
Rayleigh's 1.001 sec 0.9985 circles/sec
Maximum Deflection = 341.06mm
Period by Euro-code 8, T = 0.085 x $60^{0.75}$ = 1.83secs

IV. DISCUSSION

Table 1 is a summary of all the results from the vibration analysis of all idealized steel frames. The closeness of the periods by the computer-based analysis (modal analysis and Rayleigh's method) can be seen clearly. This can thus be used to validate the results by the present manual method. Fig.13 shows the period-height charts. In all the charts in the figure, the period of vibration increases as the height of the frames increases. The two different methods used in the computer-based analysis yielded almost similar results. The results of all methods of analysis used to analyze the 15 story frame of 60m overall height seem to agree. In Figure 14, the period of vibration for each frame is plotted against the associated maximum deflection. The three charts in the figure indicate that taller buildings are associated with higher periods and higher deflections. The height-deflection plot of Figure 15 vividly shows this attribute. The highest deflection is observed in the 20 story frame. The 14 to 20 – story frames are of the same Stiffness and indicate linear increase with respect to both period and deflection.

Table 1: Summary of results for all idealized frames Analysis

No. of Floors	Inter-Floor Height (m)	Total Height of Building (m)	Periods(sec)			Maximum Deflection (mm) @ 2% gravity load
			Present Manual method	Computer Analysis		
				Modal Analysis	Rayleigh's Method	
14	4	56	0.942	0.920	0.899	275
15	4	60	1.010	1.020	1.001	341.060
16	4	64	1.072	1.106	1.083	403.800
17	4	68	1.140	1.195	1.171	474.720
18	4	72	1.201	1.288	1.262	554.700
19	4	76	1.270	1.380	1.360	644.500
20	4	80	1.332	1.455	1.426	716.430

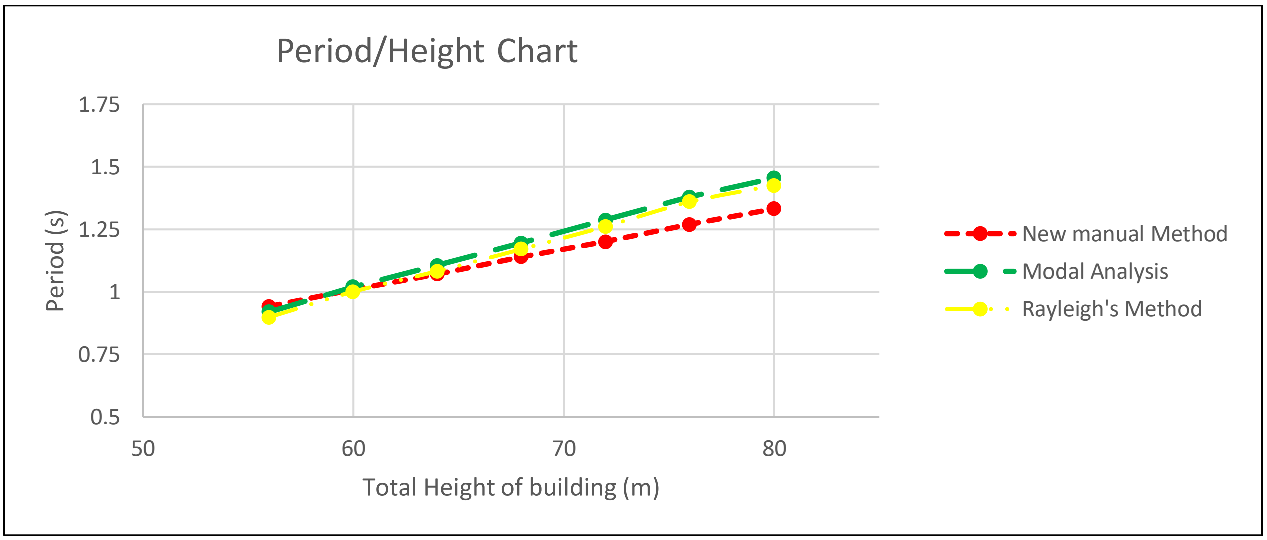


Fig 13. Plot comparing Periods and Heights of buildings.

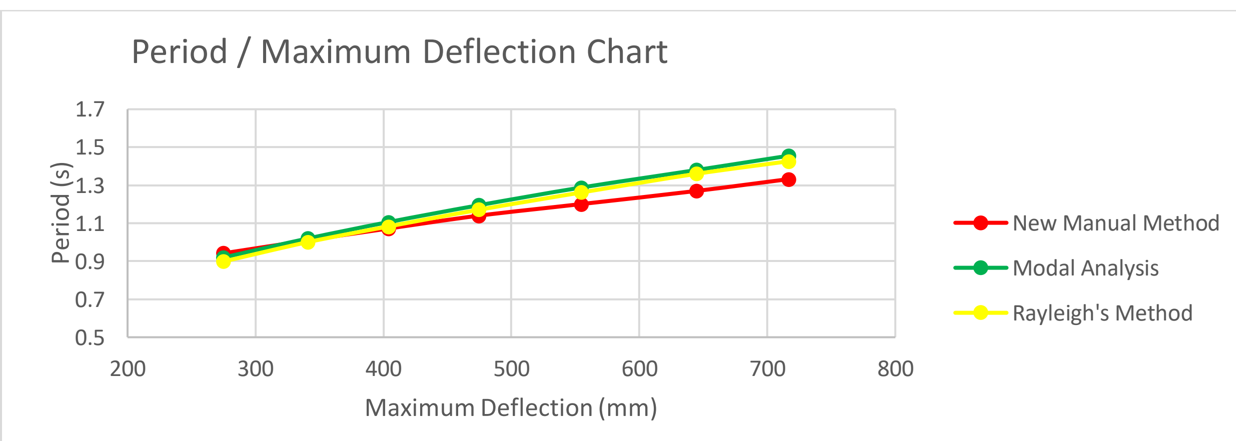


Fig14 Plot comparing Periods and Maximum Deflection of buildings

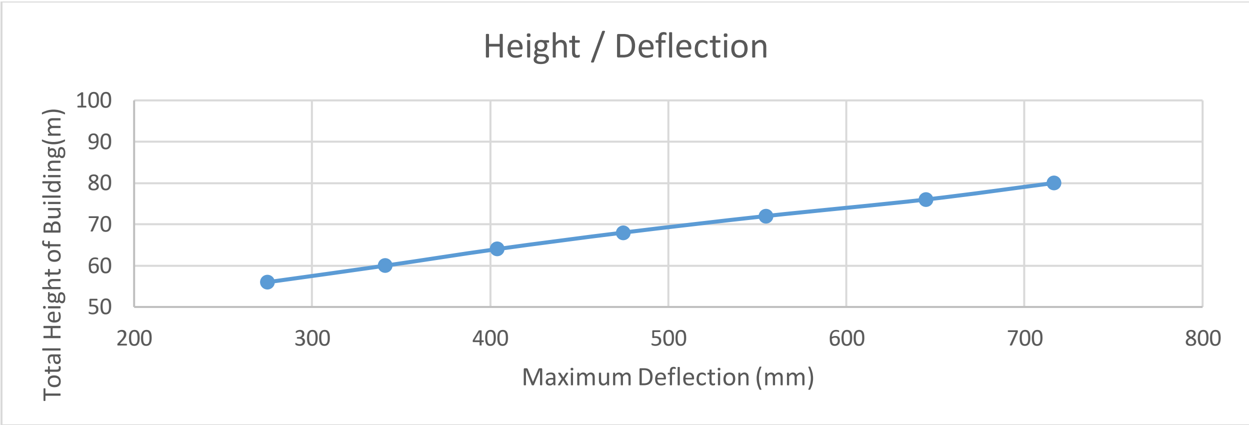


Fig.15 Height-Deflection Graph.

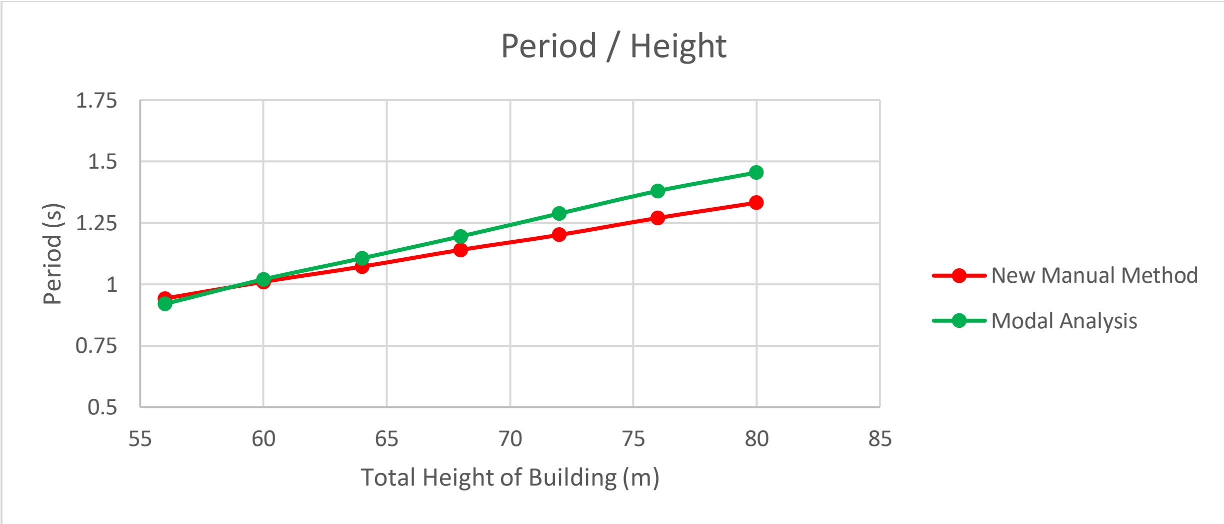


Fig.16 Period-Height Graphs for New Manual Method and Computer Analysis (Modal Analysis)

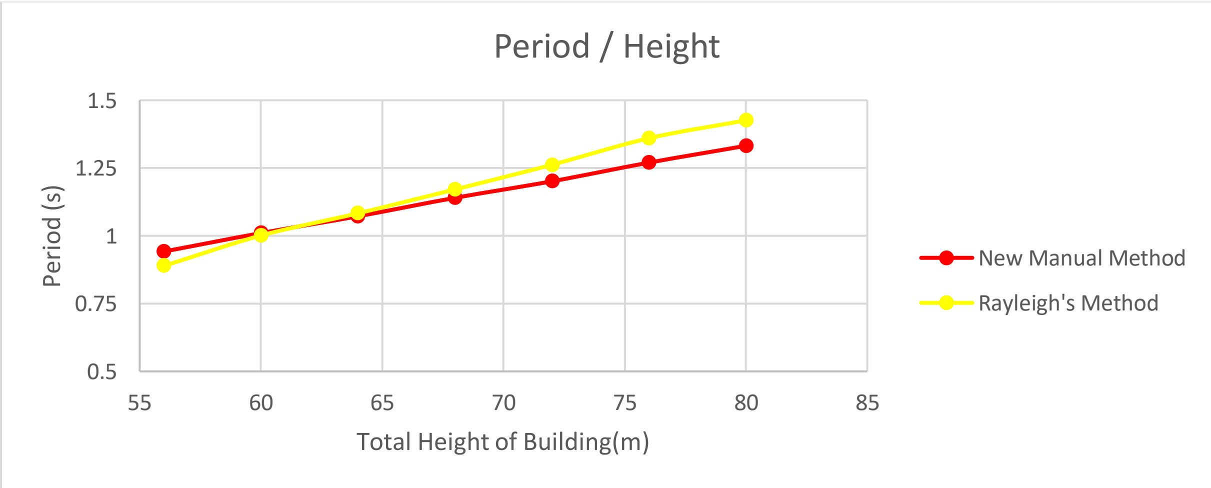


Fig.17. Period-Height Graphs for New Manual Method and Computer Analysis (Rayleigh's Method)

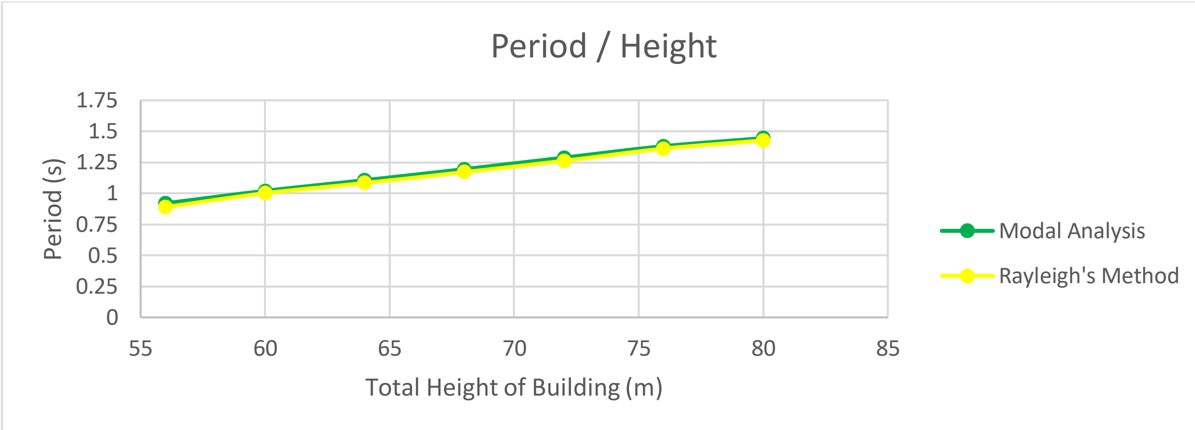


Fig.18Period-Height Graphs for Computer Analysis (Modal Analysis Vs. Rayleigh's Method)

Table 1 compares periods associated with the present manual method and computer (modal analysis) based analysis. It is shown by comparison that the average ratio of the periods obtained from the present manual method to those obtained from the computer-based modal analysis is 0.92. The average percentage difference between both methods is 7.3%. This deviation may arise from the use of 2% of the gravity beam load as wind load distributed along the external walls of the frames to stimulate vibration in the computer program. This difference may also propagate as the number of floors and frame height increases. Fig. 16 shows that as the frame's height increases, the percentage difference between both methods increases.

Also, from Table 1, the present manual method is tested against computer-based Rayleigh's method. The same trends observed are also visible here. Also, Figures 16 and 17 show similar trends. The average ratio of the periods obtained from the present manual method to those obtained from computer-based Rayleigh's method is 0.94, and the average percentage difference is 6.56%. The same reason cited above could be responsible for this deviation as well.

Looking at the Table and Figure 18, the convergence of both computer-based designs can be seen clearly. Rayleigh's solution closely approximates the solution by modal analysis.

V. CONCLUSION

This study has shown that the effective vibration length principle, as modified, gives accurate results to vibration problems of simple (single story) frames with results lying within 0 - 1.5% of cited reliable sources. Any frame can be reduced to a supported vibrating length carrying a uniformly distributed equivalent uniform mass.

The analysis method presented in this study approaches vibration from the buckling solution of any frame; the effective length of the frame leads to the effective vibration length, H.

Any multi-story frame can be modeled accurately by a single-story frame for most of the important vibration indices. For frames with rigid floors, the effective Euler length is equal to unity, and so is the effective vibration length by inspection. Frames with flexible floors have been successfully modeled so that the reduced frame still has the multi-story parent frame's characteristics. Results are within 2.5% of other reliable sources; -the error is due to the manual interpretation of the column alignment chart used to determine the studied frames' buckling lengths.

The conversion of lumped masses to constant equivalent uniform masses simplifies the solution. This conversion must always be possible since the differential equation of vibration requires uniform unit masses.

The two computer-based analytical methods (modal analysis and Rayleigh's method) show results in agreement and can thus validate the modifications introduced into the original concept.

This effective vibration length method offers a new and easier vibration solution for simple and multi-story frames.

RECOMMENDATION

1. The accuracy of this present method lies in the accurate interpretation of the column alignment chart. Mathematically modeling this chart would give accurate and consistent results. Applying the mathematical model would reduce the errors in interpreting the buckling length of frames.
2. The Stiffness of the single-story reduced frame should be studied further. The reduced frame's relative Stiffness may also be taken as the average of all the independent joint stiffnesses in the parent frame. This should be investigated.

REFERENCES

[1] AISC; Manual of steel construction: Load and Resistance Factor Design, 1st –Edn. AISC; Chicago, Illinois., (1986).

[2] Chopra, A.. Dynamics of Structures, theory, and applications to earthquake engineering, 3rd Edition 2007, Prentice-Hall, Englewood Cliffs Upper Saddle River., (2001).

[3] Coates R. C., Coutie M. G. Kong F. K., Structural Analysis, New York., Wiley 496,(1972).

[4] Goel, R.K. and Chopra, A.K. Period formulas for moment-resisting frame buildings, Journal of Structural Engineering, ASCE 123(11),(1997).

[5] Dr. Şeref Doğuşcan AKBAŞ. Free Vibration Characteristics Of Edge Cracked Functionally Graded Beams By Using Finite Element Method. International Journal of Engineering Trends and Technology (IJETT). 4(10) (2013) 4590-4597 .

[6] Iwicki, P. Buckling of Frame Braced by Linear Elastic Springs. Mechanics and Mechanical Engineering, 14(2), (2010), 201-213

[7] John M. Biggs; Introduction to Structural Dynamics. McGraw-Hill Inc., (1964).

[8] Johnarry T. N : Effective Vibration Length from Effective Buckling Length, International Review of Civil Engineering (I.RE.C.E), 7(2), (2010).

[9] Rakesh K. G and Anil K. C: Period Formulas for Moment-Resisting Frame Buildings, J. Structural Engineering, 123(11), (1997).

[10] Shanmugam, N.E and Chen W.F.; Structural engineering CE-STR-99-8. February 1993 School of Civil Engineering; Purdue University.

[11] Ward H. S. Dynamic Characteristics of a Multi-Story Concrete Building. Proc. Institute of Civil Engineers,. 43(1969), 553-555.

[12] Young-Soo, C., Ji-Soo, Y., Kug-Kwan, C. and Li-Hyung, L., Approximate estimations of natural periods for apartment buildings with shear-wall dominant systems, Journal of Earthquake and Structural Dynamics, 39(14),(2000).

[13] Yura J. A. The Effective Length of Columns in Unbraced Frames. American Institute of Steel Construction Engineering Journal (1971),37-42.