

# Curvature-Displacement Resonance Transform Method on Clamped Orthotropic Plate under Concentrated Loading

T.C. Nwofor, Nkanunye C. Ozuru-Douglas

*Department of Civil Engineering, Faculty of Engineering, University of Port Harcourt PMB 5323, Port Harcourt, Nigeria*

## **Abstract**

*This paper presents a unique solution to a clamped orthotropic rectangular plate's exact bending solution under concentrated loading. This new approach, which is the curvature-displacement resonance method, has become an effective tool in solving the plate problem, has gained so much attention lately. The problem of point load on the clamped orthotropic plate was successfully solved by this method. The deflection equation and results were separated into two categories under the sub-heading, the deflection at a point directly under the load application and the deflection at any point on the plate when the point load is stationed at the center of the plate. This method's accuracy was tested by comparing the results to values obtained via a previous researcher's finite integral transform method.*

**Keywords:** *Curvature-displacement method, exact bending solution, clamped orthotropic plate, concentrated loading.*

## **I. INTRODUCTION**

Plate analysis dominance in recent civil and mechanical engineering works has gained so much attention by various researchers in specialized construction works such as offshore marine structures, air-crafts, bridges, buildings, etc. Different researchers have done extensive works on this subject, but there is still a need for further research to achieve an accurate and simplified design- analysis. Timoshenko and Woinowsky-Krieger<sup>1</sup> expanded the study of plates in 1959. They were able to study the strength of plates and shells and made interesting publications which now serve as a reference for many researchers today. The study of orthotropic rectangular plates is crucial since its material analysis is used in composite structural analysis such as steel bridge deck, reinforced concrete slab, flat panels stiffened by orthogonal ribs in airplanes, etc. Various researches have done extensive work on plates and came up with many methods of obtaining the bending solutions of thin orthotropic

Rectangular plates. The formulated methods are mainly based on numerical methods, while some are based on analytical methods.

A wide range of analytical solutions for thin plates with two opposite sides is available, like Navier's and Levy's solution<sup>2</sup>. However, the analytical solutions considering plates with combinations of boundary conditions are not easily ascertained. This gives rise to several methods that complex cases such as clamped plate are solved using superposition<sup>1,3,4</sup>. The numerical methods have now been widely used compared to the few analytical methods due to their flexibility and ease of use for different boundary conditions in solving plate problems such as the finite element method (FEM), the finite difference method, differential quadrature method, Rayleigh-Ritz method, the finite strip method, method of meshless, boundary element method, discrete singular convolution method, and wavelet collocation method.

According to Taylor and Govindjee<sup>5</sup>: in their research on a solution for problems on a clamped rectangular plate, they presented an efficient analytical method to solving very accurate solutions to a clamped rectangular problem plate. Their method was based upon the classical double cosine series expansion and exploitation of the Sherman-Morrison-Woodbury formula. They further concluded that if the cosine expansion involves M terms and N terms in the two plate axes directions, then the classical method for this problem involves solving a system of (MN) x (MN) equations.

Yang and Qian<sup>6</sup>: in their research on the analysis bending solutions of clamped rectangular thick plates, used the decoupling and the modified Navier's solution to study a simple analysis of all-round clamped rectangular thick plates. They proposed that their study does not require the derivations of complicated matrix solutions to calculate the coefficients but address the solution to the problem directly. They further ascertained that their method had become a simpler procedure in analyzing and solving the bending of clamped rectangular thick plates. They further showed



that plate supports problems like the points support spring support can be solved easily analytically using the proposed procedure on Mindlin's higher-order shear deformation plate theory to develop inspiring extensions in the field expectantly.

According to Rui and Others<sup>7</sup> in their study on the analytical bending solutions of free orthotropic rectangular thin plates under arbitrary loading, used the double sine finite integral transform known as the effective tool for plate problem solutions to solve plate problems of all boundary conditions which now serves as an elegant approach to analytical solutions of plate bending problems.

An alternative variational approach developed by Osadebe and Aginam<sup>8</sup> on the study of bending analysis of an anisotropic rectangular plate with all edges clamped showed that their method by-passes the tedious and rigorous solutions of convectional classical plate differential equation. Their method, which was based on total potential energy as a modification of the Ritz variational approach, indicates that the formulation of the deformed surface of the clamped plate under uniform distributed load is approximated to be the sum of products of constructed polynomials in the x and y axes. They further used the constructed polynomials as substituted in the plate equation and solved it through the minimization principle. This method's solution was done for the first, second, third, and fourth terms representing the four approximations, respectively. They compared their results with that of Timoshenko and Woinowsky Krieger<sup>1</sup>.

Most recent studies have been carried out on plate structures using the finite integral transform method and the acceleration-displacement methods<sup>9,10,11</sup> for plates' bending solution. An exact solution for the deflection of a clamped rectangular plate under uniform load was also investigated<sup>12</sup>. They came up with an exact solution where each term of their series is a hyperbolic and trigonometry one and satisfies a fully fixed plate's edge conditions. They also presented their solution in three terms, in which case and the first term refers to a strip's case. In comparison, the remaining two terms represent the effect on the boundary that satisfies an all-around clamped plate's boundary conditions. To show how their method works, they used numerical values of the deflections obtained in their work and compared them with other known solutions.

## II. GOVERNING EQUATION FOR ORTHOTROPIC RECTANGULAR PLATE

According to Schade<sup>13</sup>, orthotropic plate theory refers to materials with different elastic properties and two orthogonal directions.

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P(x, y) \quad (1)$$

Where,

$D_x$  is the unit of flexural rigidity around the y-axis

$D_y$  is the unit of flexural rigidity around the x-axis

$H = D_1 + 2D_{xy}$ , which is the effective torsional rigidity in which  $D_1 = \nu_2 D_x = \nu_1 D_y$  and  $\nu_1$  and  $\nu_2$  are Poisson's ratios.

$P$  is the pressure load over the surface.

### A. Moment Calculation

According to the theory of plates,

$$M_x = - (D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}) \quad (2)$$

$$M_y = - (D_y \frac{\partial^2 w}{\partial y^2} + D_1 \frac{\partial^2 w}{\partial x^2}) \quad (3)$$

$$M_{xy} = - (2D_{xy} \frac{\partial^2 w}{\partial x \partial y} + D_1 \frac{\partial^2 w}{\partial y^2}) \quad (4)$$

Where  $M_x$  and  $M_y$  are the bending moments,  $M_{xy}$  is the torsional moment.

The boundary condition that satisfies the function is

$$W|_{x=0,a} = 0, \quad W|_{y=0,b} = 0 \quad (5)$$

$$\frac{\partial w}{\partial x}|_{x=0,a} = 0, \quad \frac{\partial w}{\partial y}|_{y=0,b} = 0 \quad (6)$$

### B. Curvature - Displacement Transform Method Solution

For a fully clamped rectangles plate, the displacement function expressed as trigonometry function will be employed.

$$W = A_{mn} (\cos \frac{n\pi x}{b} - 1) (\cos \frac{m\pi y}{b} - 1) \quad (6)$$

The bi-harmonic equation for an orthotropic condition for uniform loading is given by

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x \partial y} + D_y \frac{\partial^4 w}{\partial y^4} = q \quad (7)$$

Re-writing the above expression;

$$D_x W_{xxxx} + 2HW_{xxyy} + D_y W_{yyyy} = q \quad (8)$$

The expression above is a direct opposite of Fourier's series transformation of the load method. Each of the differentials in equation (2) is made to have a point-wise constant value over the domain.

Transforming the differential into a normal relative curvature-displacement ratio ( $R_{ad}$ )

$R_{ad} = R_{xx}, R_{yy}, R_{xy}$  thus  $(x = \frac{\partial^2 w}{\partial x^2})$  or relative acceleration

$$R_{xx} = \left[ \frac{(\frac{\partial^2 w}{\partial x^2}) - (\frac{\partial^2 w}{\partial x^2})_0}{w} \right] \quad (9)$$

Taking the first term of the fourth differential of equation (1) for the x-term, we have;

$$\frac{\partial^4 w}{\partial x^4} = C_{xx} \left[ \frac{(\frac{\partial^2 w}{\partial x^2}) - (\frac{\partial^2 w}{\partial x^2})_0}{w} \right] = C_{xx} \cdot R_{xx} \quad (10)$$

Similarly;

$$\frac{\partial^4 w}{\partial y^4} = C_{yy} \left[ \frac{\left( \frac{\partial^2 w}{\partial y^2} \right) - \left( \frac{\partial^2 w}{\partial y^2} \right)_o}{w} \right] = C_{yy} \cdot R_{yy} \quad (11)$$

$$\frac{\partial^4 w}{\partial x^4} = C_{xx} \left[ \frac{\left( \frac{\partial^2 w}{\partial x^2} \right)_{rel}}{w} \right] \quad (12)$$

$$\frac{\partial^4 w}{\partial y^4} = C_{yy} \left[ \frac{\left( \frac{\partial^2 w}{\partial y^2} \right)_{rel}}{w} \right] \quad (13)$$

where C is a constant.

The constant C is derived from the X, Y, and XY strips, which is then substituted into equation (1) to give the value of deflection.

### C. Analysing the X-strip;

The X-strip parameters are taken from equation (1) and analyzed.

$$D_x \frac{\partial^4 w}{\partial x^4} = D_x C_{xx} \left[ \frac{\left( \frac{\partial^2 w}{\partial x^2} \right) - \left( \frac{\partial^2 w}{\partial x^2} \right)_o}{w} \right] = D_x C_{xx} \cdot R_{xx} \quad (14)$$

Multiplying equation (9) by the deflection 'w' and integrating (comparing potentials) yields:

$$D_x \int_o^b \int_o^a \left( \frac{\partial^4 w}{\partial x^4} \right) w \partial x \partial y = D_x C_{xx} \left[ \int_o^b \int_o^a \left( \frac{\partial^2 w}{\partial x^2} \right) \partial x \partial y - \int_o^b \int_o^a \left( \frac{\partial^2 w}{\partial x^2} \right)_o \partial x \partial y \right] \quad (15)$$

Recall that

$$W = \sum \sum A_{mn} \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right] \text{From equation} \quad (16)$$

Differentiating 'w' up to the fourth power yields:

$$W_x = -A_{mn} \left( \frac{m\pi}{a} \right) \left( \sin \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \quad (17)$$

$$W_{xx} = -A_{mn} \left( \frac{m^2 \pi^2}{a^2} \right) \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \quad (18)$$

$$W_{xxx} = A_{mn} \left( \frac{m^3 \pi^3}{a^3} \right) \left( \sin \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \quad (19)$$

$$W_{xxxx} = A_{mn} \left( \frac{m^4 \pi^4}{a^4} \right) \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \quad (20)$$

Imputing all the variables in equation (15) yields:

$$D_x \int_o^b \int_o^a \left( \frac{m^4 \pi^4}{a^4} \right) \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \cdot A_{mn} \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right] \partial x \partial y = C_{xx} \cdot D_x \int_o^b \int_o^a \left( \frac{\partial^2 w}{\partial x^2} \right) \partial x \partial y - \int_o^b \int_o^a \left( \frac{\partial^2 w}{\partial x^2} \right)_o \partial x \partial y \quad (21)$$

Simplifying equation (21), we have;

$$D_x \frac{m^4 \pi^4}{a^4} A_{mn}^2 \int_o^b \int_o^a \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right] \partial x \partial y =$$

$$D_x \int_o^b \int_o^a -A_{mn} C_{xx} \left( \frac{m^2 \pi^2}{a^2} \right) \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \quad (22)$$

By integrating equation (22) and factorizing, we have;

$$C_{xx} = \frac{(3/4)m^2(\pi^2)}{a^2} \cdot A_{mn}$$

Similarly going the same process for the **y-strip**, we have;

$$C_{yy} = \frac{(3/4)n^2(\pi^2)}{b^2} \cdot A_{mn}$$

Also, for the twist transform (xy), we have;

$$C_{xy} = (0.375)(m^2 n^2) \frac{\pi^4}{(a^2 \cdot b^2)} \cdot A_{mn}$$

$$D_x C_{xx} \left[ \frac{\left( \frac{\partial^2 w}{\partial x^2} \right)}{w} + 2HC_{xy} + D_y C_{yy} \left( \frac{\partial^2 w}{\partial y^2} \right) \right] = q \quad (23)$$

Substituting all the deduced variables into equation (3.17) yields:

$$D_x A_{mn} \frac{3 m^2 \pi^2}{4 a^2} \left[ - \frac{m^2 \pi^2 \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi x}{b} - 1 \right)}{\left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi x}{b} - 1 \right)} \right] + 2HA_{mn} \cdot \frac{3 m^2 n^2 \pi^4}{8 a^2 b^2} + A_{mn} \cdot D_y \frac{3 n^2 \pi^2}{4 b^2} \left[ - \frac{n^2 \pi^2 \left( \cos \frac{n\pi y}{b} \right) \left( \cos \frac{m\pi x}{a} - 1 \right)}{b^2 \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi x}{b} - 1 \right)} \right] = q \quad (24)$$

$$A_{mn} = \frac{q}{0.75\pi^4 \left[ D_x \frac{m^4}{a^4} + H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right]} \quad (25)$$

Therefore, the deflection 'w' for an orthotropic rectangular plate is hereby given as;

Substituting  $A_{mn}$  (equation 25) back into equation (6) yields:

$$W = \frac{q \left( \cos \frac{m\pi y}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right)}{0.75\pi^4 \left[ D_x \frac{m^4}{a^4} + H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right]} \quad (26)$$

### III. A CLAMPED RECTANGULAR ORTHOTROPIC PLATE UNDER CONCENTRATED LOAD

In the case of a point load 'P,'  $q^*$ , which is a uniform loading, only exists over an isolated center area where the total load is 'P.'

By comparison of potentials

$$q^* = \frac{q}{1} = \frac{\int_o^b \int_o^a q^* \cdot w dx dy}{\int_o^b \int_o^a w dx dy} \quad (27)$$

$$q^* \partial x \partial y = P \quad (28)$$

N/B:  $q^*$  absorbs  $\partial x \partial y$  and becomes P and frees 'w' from integration. We are now able to solve the point load case without reference to the load width.

Implying that:

$$q = (P/a, b) \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right]_p \quad (29)$$

The isolated distributed load  $q^*$  has now transformed into the plate-wide load, and a location function,  $\left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right]_p$  and this now represents the shape function in the point load zone in the mid-plane.

Therefore, the deflection will now be looked at in different conditions, namely; type1 and type 2 deflection

**A. Type 1 Deflection ( under the load point)**

Replacing 'q' with equation (29) and substituting into equation (26) yields:

$$W = \frac{P \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right]_p}{0.75\pi^4 ab \left[ D_x \frac{m^4}{a^4} + H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right]} \quad (30)$$

**B. Type 2 Deflection ( outside the load point)**

$$W = \frac{\Sigma \Sigma (P/a, b)}{0.75\pi^4 \left[ D_x \frac{m^4}{a^4} + H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right]} \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right] \left[ \left( \cos \frac{m\pi x}{a} - 1 \right) \left( \cos \frac{n\pi y}{b} - 1 \right) \right]_p \quad (31)$$

**IV. IMPLEMENTATION OF THE PROCEDURE**

The proposed formulation for a fully clamped orthotropic rectangular plate will be validated under the following conditions;

- 1.) Deflection 'W' will bear the answers as the  $(Pa^2/D_x)$  coefficient for a fully clamped orthotropic plate rectangular under concentrated loading at the center. The result will be compared to Bidgoli and other<sup>14</sup>.
- 2.) A case study of a bridge deck under concentrated loading 'P' will be examined to determine the deflections at various specified points on the deck regarding various aspect ratios of the deck. The comparison will be made for the value of deflection when the point load is acting and the value of deflection outside the location of the point load.

Where:

P = Point load

$D_y = 4D_x, D_{xy} = 0.85D_x, \nu_1 = 0.075, \nu_2 = 0.3$

And  $D_1 = \nu_2 D_x = \nu_1 D_y, H = D_1 + 2D_{xy}$

The deflection results were obtained in terms of  $Pa^2/D_x$

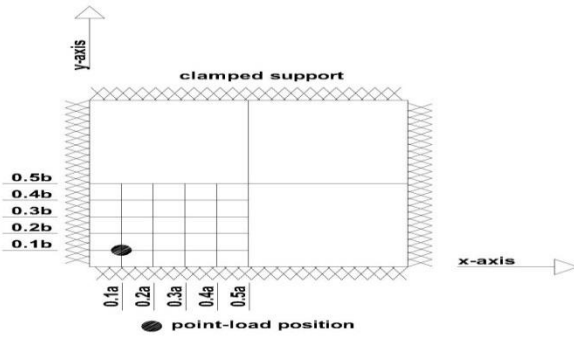
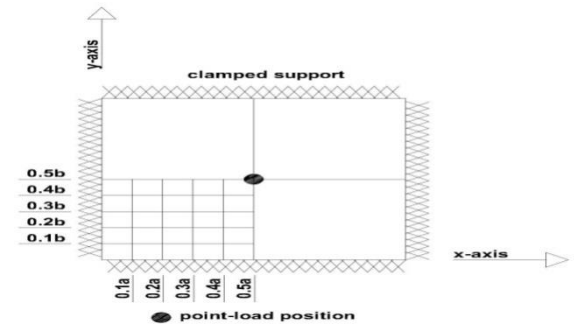


Figure 1: Structural diagram showing the position of point load on the clamped plate



**V. RESULTS**

**A. Validation of Result**

To validate the result of this study, the deflection results using this method of plate analysis for a point load at the center of the plate ( $w_{x/2}$  and  $w_{y/2}$ ) concerning all the aspect ratio (b/a) specified were obtained and compared to the ones gotten by Bidgoli et al.<sup>14</sup>, using the same mechanical parameters.

Comparing further with the above the researcher<sup>14</sup>, it can be seen from Table 1 below that our results are in close agreement with the values obtained with an average percentage difference of 9.68%.

Also, a graph of the deflection (w) concerning the aspect ratio (b/a) was plotted for the present method against that obtained by Bidgoli et al.

Tables 1: Deflection values compared to Bidgoli et al (2015)

b/a	$W_{x=a/2, y=b/2}$ ( $Pa^2/D_x$ ) Present	$W_{x=a/2, y=b/2}$ ( $Pa^2/D_x$ ) Bidgoli, et al	Difference (%)
1.0	0.002170	0.002402	9.658618
1.1	0.0025377	0.002808	9.626068
1.2	0.0028828	0.003192	9.686717
1.3	0.0031953	0.003540	9.737288
1.4	0.0034692	0.003844	9.75026
1.5	0.0037024	0.004102	9.741589

1.6	0.0038958	0.004314	9.694019
1.7	0.0040522	0.004485	9.649944
1.8	0.0041753	0.004620	9.625541
1.9	0.0042697	0.004724	9.61685
2.0	0.0043396	0.004803	9.648137

**B. Deflection Values at Various Points**

Table2 shows results for deflections when the point load is placed at 0.1b in the y-axis of the plate and moving it along the x-axis direction of the plate from 0 to 1.0a with an interval of 0.1a, which indicates that the values obtained from Type 1 maintain a gradual rise from zero (0) at the clamped support to a maximum of 0.000255 at the 0.5a. In contrast, type 2 deflection results show an increase of deflection value from zero at the support to a maximum of 0.0000583.

The deflection results trend conforms with theory, which indicates that the deflection of a clamped plate at the support is zero and tends to increase to a maximum value at the center of the plate.

**Tables 2: Deflection values for X= 0.1a to 1.0a and Y=0.1b for b/a=1.0**

A	TYPE 1 (Pa <sup>2</sup> /D <sub>x</sub> )	TYPE 2 (Pa <sup>2</sup> /D <sub>x</sub> )
0	0	0
0.1	0.0000515	0.0000145
0.2	0.0001034	0.0000173
0.3	0.000152	0.0000224
0.4	0.0002039	0.0000305
0.5	0.000255	0.0000583
0.6	0.0002039	0.0000305
0.7	0.000152	0.0000224
0.8	0.0001034	0.0000173
0.9	0.0000515	0.0000145
1	0	0

At a point 0.2b of the plate and moving the point load along the x-axis from 0.1a to 1.0a with an interval of 0.1, the results from Table 3 shows that the deflection for type 1 and type 2 which ranges from zeros at the supports to a maximum of 0.000788 and 0.00031 at mid-span respectively.

This indicates that, as the point load moves further away from the supports, deflection increases, which is in line with various study<sup>1</sup>.

**Tables3: Deflection values for X= 0.1a to 1.0a and Y=0.2b for b/a=1.0**

A	TYPE 1 (Pa <sup>2</sup> /D <sub>x</sub> )	TYPE 2 (Pa <sup>2</sup> /D <sub>x</sub> )
0	0	0
0.1	0.0001212	0.000026
0.2	0.000307	0.000061
0.3	0.000481	0.00012
0.4	0.000667	0.00022
0.5	0.000788	0.00031
0.6	0.000667	0.00022
0.7	0.000481	0.00012
0.8	0.000307	0.000061
0.9	0.0001212	0.000026
1	0	0

Moving further to a point 0.3b on the y-axis and considering various points on the x-axis from 0.1a to 1.0a, the results from Table 4 shows that the deflections for type 1 and type 2 maintains a steady rise from zero at the support to a maximum deflection value of 0.00138 and 0.0009 respectively.

It can be said that, as the point load moves further away from its clamped edges, the difference in values of the deflections for both type 1 and type 2 are reducing, which means that the effect of deflection value with respect to the position of the point load as it moves closer to the center of the plate is becoming less significant.

**Tables4: Deflection values for X= 0.1a to 1.0a and Y=0.3b for b/a=1.0**

a	TYPE 1 (Pa <sup>2</sup> /D <sub>x</sub> )	TYPE 2 (Pa <sup>2</sup> /D <sub>x</sub> )
0	0	0
0.1	0.000188	0.000037
0.2	0.000528	0.00015
0.3	0.000853	0.00036
0.4	0.001193	0.00069
0.5	0.001382	0.0009
0.6	0.001193	0.00069
0.7	0.000853	0.00036
0.8	0.000528	0.00015
0.9	0.000188	0.000037
1	0	0

Table 5 shows the variation of the deflection results of 0.4b and 0 ≤ a ≤ 1.0a in the x-direction which

indicates that the maximum deflection at the centre is 0.0019414 and 0.00172 for type 1 and type 2.

**Tables5: Deflection values for X= 0.1a to 1.0a and Y=0.4b for b/a=1.0**

A	TYPE 1 (Pa <sup>2</sup> /D <sub>x</sub> )	TYPE 2 (Pa <sup>2</sup> /D <sub>x</sub> )
0	0	0
0.1	0.000258	0.000064
0.2	0.000732	0.00028
0.3	0.001182	0.00069
0.4	0.001657	0.00133
0.5	0.001914	0.00172
0.6	0.001657	0.00133
0.7	0.001182	0.00069
0.8	0.000732	0.00028
0.9	0.000258	0.000064
1	0	0

Looking at Table6, it is seen that the value of deflection at the mid-span for both type1 and type 2 are the same, with a maximum value of 0.00217. A plot of the values will also show that at the center of the plate, the deflection curve for both cases converged.

**Table 6: Deflection values for X= 0.1a to 1.0a and Y=0.5b for b/a=1.0**

A	TYPE 1 (Pa <sup>2</sup> /D <sub>x</sub> )	TYPE 2 (Pa <sup>2</sup> /D <sub>x</sub> )
0	0	0
0.1	0.000309	0.00010
0.2	0.000835	0.00037
0.3	0.001334	0.00087
0.4	0.001861	0.00165
0.5	0.002170	0.00217
0.6	0.001861	0.00165
0.7	0.001334	0.00087
0.8	0.000835	0.00037
0.9	0.000309	0.00010
1	0	0

**VI. CONCLUSION**

In this investigation, an orthotropic plate's exact bending solution with clamped edges under

concentrated loading by curvature-displacement resonance method is obtained. From the various results obtained, it can be concluded that the curvature-displacement method can be used to solve the plate problem under concentrated loading for the bridge deck, etc. Also, it can be said that the results of deflection at a point on the plate increase linearly with an increase in the aspect ratio.

**REFERENCES**

- [1] Timoshenko S., Woinowsky-Krieger S.Theory of plates and shells, Auckland, McGraw- Hill Book Company. (1959).
- [2] Navier, C. L. M. H. Science Bulletin Philomarthique Society, Paris. (1923).
- [3] Khalili, M.R., Malekzadeh, K, and Mittal, R.K. A new approach to static and dynamic analysis of compositeplates with different boundary conditions. Composite Structures, 69(8), (2005),149–155.
- [4] Fu, B.L. Applications of the reciprocal theorem to solving the equations of the deflection surface of the rectangular plates with various edge conditions. Applied Mathematical Mechanics, 3,(1982), 353–364.
- [5] Taylor, R. L. and Govindjee, S. Solution of clamped rectangular plate problems. Technical report: UCB/SEMM, University of California, Berkley, USA.,(2002).
- [6] Yang, Z. and Qian, X. Analysis Bending Solutions of Clamped Rectangular Thick Plate. Mathematics Problems in Engineering, School of Civil and Hydraulics Engineering, Dalian University of Technology, Dalian.,(2017).
- [7] Rui, L., Bin, T. and Yang, Z. Analytical bending solutions of free orthotropic rectangular thin plates under arbitrary loading. State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian.,(2013).
- [8] Osadebe, N. N. and Aginam, C. H. Bending and analysis of Isotropic rectangular plate with all edges clamped: variational symbolic solution. Journal of Emerging Trends in Engineering and Applied Sciences., 2,(2011), 846-852.
- [9] Nwofor, T.C. and Osere G. Bending solutions of fully clamped orthotropic rectangular thin plate using finite integral transform method. International Journal of Structural Mechanics and Finite Elements, 4(2),(2018),12-21.
- [10] Nwofor, T.C. and Etafo, I. Analysis of fixed offshore platform deck using acceleration-deflection resonance method. International Journal of Structural Mechanics and Finite Elements, 4(1),(2018),28-35.
- [11] Nwofor, T.C. and Nnoaham, B.C. Analysis of bridge deck for abnormal load using the acceleration-displacement ratio for a balcony function. SSRG International Journal of Civil Engineering, 5(3), (2018) ,35-41.
- [12] Imrak, C. E. and Gerdemili, I. An exact solution for the deflection of a clamped rectangular plate under uniform load. J. Applied Mathematical Science, 1(43), (2007), 2129-2137.
- [13] Schade, H. A.. Design Curves for cross stiffened plating under uniform bending load. Trans, SNAME, 49. (1942).
- [14] Bidgoli, A., M., Daneshmehr, A., R., and Kolahchi, R.. Analytical bending solution of fully clamped orthotropic rectangular plates resting on elastic foundation by finite integral transform method. Journal of Applied and Computational Mechanics, 1(2), (2015), 52-58.