SSRG International Journal of Civil Engineering
ISSN: 2348 – 8352 /doi:10.14445/23488352/IJCE-V7I10P103

Volume 7 Issue 10, 16-, October 2020
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Mathematical Model For Optimization of Modulus of Elasticity of Polystyrene Lightweight Concrete Using Scheffe's Model

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ABSTRACT
The modulus of elasticity (MOR) of lightweight polystyrene concrete is a function of the constituent materials' proportions, namely, cement, water, polystyrene, fine and coarse aggregates. The conventional methods used to determine the mix proportions that will yield a desired modulus of elasticity are laborious, time-consuming, and expensive. The model can prescribe all the mixes that will produce the desired modulus of elasticity of concrete. It can also predict the modulus of elasticity of polystyrene lightweight concrete if the mix proportions are specified. The adequacy of the mathematical model was also tested.

Keywords*: Optimization, Modulus, Elasticity, Polystyrene, Lightweight, Concrete*

INTRODUCTION

Polystyrene lightweight Concrete is a construction material in which strength is very important. The strength is of such utmost importance that it is used as a yardstick for judging other polystyrene lightweight concrete properties such as permeability, durability, fire, and abrasion resistance. The strength is usually given in the form of compressive strength and flexural strength. The flexural strength is the solid property that indicates its ability to resist failure in bending [1]. And the modulus of elasticity (MOR) of concrete, as defined by International Concrete Repair Institute, is a measure of the ultimate load-bearing capacity of a concrete beam tested in flexure http://www.google.com/search. Various methods have been used to study and determine the modulus of elasticity of concrete [2]. These methods are based on the conventional approach of selecting arbitrary mix proportions, subjecting concrete samples to the laboratory, and then adjusting the mix proportions in subsequent tests. These methods are time-consuming and expensive. In this paper, a mathematical model based on Scheffe's model for concrete optimization theory is formulated to optimize the modulus of elasticity of polystyrene lightweight concrete. Every activity that must be successful in human endeavor requires planning. The target of planning is the maximization of the desired outcome of the venture. To maximize gains or outputs, it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is optimization [3]. In optimization science, the desired property or quantity to be optimized is the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables. The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. The constraint at any time is the amount of money available to the entrepreneur at the time of investment. Hence or otherwise, an optimization process seeks the maximum or minimum value and, at the same time, satisfying several other imposed requirements [4]. The function is called the objective function, and the specified requirements are known as the constraints of the problem. Concrete is a mixture of several components: cement, fine aggregate, coarse aggregate, and water. Concrete is a composite inert material comprising a binder course (cement) and mineral filler (body) or aggregate and water. Admixture could be added, but for a given set of materials, the proportion of the components influences the concrete mixture's properties, hence, the need to optimize concrete properties such as strength. Mathematical modeling is creating a mathematical representation of some phenomenon to understand that phenomenon [5] better. [6] described a model as an abstract that uses mathematical language to control the behavior of a given system. [7] modeling is a mathematical equation of the dependent variable (Response) and independent variable (Predictor). [8] stated that the area of application of mathematical modeling includes engineering and natural sciences. [8] studies on high-performance concrete, which contains many constituents and often subjected to several performance constraints, can be difficult and time-consuming. Different works by [9] and [7] demonstrated mathematical modeling in civil engineering. In the past, ardent researchers have done works in the behavior of flexural strength of polystyrene lightweight concrete under its components' influence. With given proportions of aggregates, polystyrene lightweight concrete's compressive strength depends primarily upon age, cement content, and the cement-water ratio [10]. Of all the desirable properties of hardened concrete, such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the overall criterion quality of the hardened concrete [4]. Every activity that must be successful in human endeavor requires planning whose target maximizes the desired outcome of the venture [3]. The optimization process seeks the maximum or minimum value and satisfies a number of other imposed requirements [4]. Modern research in polystyrene lightweight concrete seeks to understand better its constituent materials and possibilities of improving its qualities [11]. The concrete mix optimization task implies selecting the most suitable polystyrene lightweight concrete constituents from the database [12]. The optimization of mixed designs requires detailed knowledge of polystyrene lightweight concrete properties [13]. The task of polystyrene lightweight concrete mix optimization implies selecting the most suitable concrete aggregates from a database [12]. Mathematical models have been used to optimize some mechanical properties of lightweight polystyrene concrete [14].

Scheffe's Equation Method

[14] showed that a polynomial could approximate the response function (property) in a multi-component system. A polynomial of degree n in q variable has $C_q^n + n - 1$ coefficients and in the form:

$$Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \cdots + \sum b_{i1i2\dots in} X_{i1} X_{i2} X_{in}$$

$$1 \le i \le q \quad 1 \le i \le j \le q \quad 1 \le i \le j \le k \le q$$

Scheffe's simplex lattice designs provide a uniform scatter of points over the $(q - 1)$ simplex.

$$\sum X_i = 1 \text{ or } X_1 + X_2 + X_3 + X_4 = 1 \qquad (1)$$

Where;

X_1= Water/Cement Ratio
X_2 = Binder (Cement)
X_3 = Fine Aggregates (Sand)
X_4 = Coarse Aggregates (88% Granite + 12% EPS)

Multiply equation. 1 by b_0, we have

$$b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 = b_0$$

Multiplying eqn. 1 again by $X_1, X_2, X_3,$ and X_4 In turn, we have

$$X^2{}_1 = X_1, -X_1X_2, -X_1X_3 - X_1X_4$$
$$X^2{}_2 = X_2, -X_1X_2, -X_2X_3 - X_2X_4$$
$$X^2{}_3 = X_3, -x_1X_3, -X_2X_3 - X_3X_4$$
$$X^2{}_4 = X_4 - X_1X_4 - X_2X_4 - X_3X_4$$

Substitute the functions of b_0 (Equation. 3.23 and $X^2{}_i$ (i=1, 2, 3 and 4) in Equation we have

$$Y = b_oX_1 + b_oX_2 + b_oX_3 + b_oX_4 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3 \quad (2)$$

$$+b_{11}(X_1 - X_1X_2 - X_1X_3 - X_1X_4)$$
$$+b_{22}(X_2 - X_1X_2 - X_2X_3 - X_2X_4)$$
$$+b_{33}(X_3 - X_1X_3 - X_2X_3 - X_3X_4)$$
$$+b_{44}(X_4 - X_1X_4 - X_2X_4 - X_3X_4)$$

Re-arranging the equation, we have

$$Y = (b_o + b_1 + b_{11})X_1 + (b_o + b_2 + b_{22})X_2 + (b_o + b_3 + b_{33})X_3 + (b_o + b_4 + b_{44})X_4 + (b_{12} - ** b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{14} - b_{33} - b_{44})X_3X_4$$

Let $\propto_i = b_o + b_i + b_{ii}$ and $\propto_{ij} = b_{ij} + b_{ii} + b_{jj}$

Then, this becomes

$$Y = \propto_i X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_{12} X_1X_2 + \propto_{13} X_1X_3 + \propto_{14} X_1X_4 + \propto_{23} X_2X_3 + \propto_{24} X_2X_4 + \propto_{34} X_3X_4 \qquad (3)$$

In compact form, the equation can be stated as:

$$Y = \sum \propto_i X_1 + \sum \propto_{ij} X_iX_j$$

Where, $1 \le I \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le q$, respectively.

Therefore Equation 3.26 is the mathematical model based on Scheffe's second-degree polynomial.

$$\text{ч}_{4,2} = a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{14}X_1X_4 + a_{23}X_2X_3 + a_{24}X_2X_4 + a_{34}X_3 \qquad (5)$$

$$\text{ч}_{4,3} = \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 + \alpha_4X_4 + \alpha_{12}X_1X_2 + \alpha_{13}X_1X_3 + \alpha_{14}X_1X_4 + \alpha_{23}X_2X_3 + \alpha_{24}X_2X_4 + \alpha_{34}X_3X_4 + \text{ч}_{12}X_1X_2(X_1 - X_2) + \text{ч}_{13}X_1X_3(X_1 - X_3) + \text{ч}_{14}X_1X_4(X_1 - X_4) + \text{ч}_{23}X_2X_3(X_2 - X_3) + \text{ч}_{24}X_2X_4(X_2 - X_4) + \text{ч}_{34}X_3X_4(X_3 - X_4) + \alpha_{123}X_1X_2X_3 + \alpha_{124}X_1X_2X_4 + \alpha_{134}X_1X_2X_4 + \alpha_{234}X_2X_3X_4 \qquad (6)$$

Also,

$$a_i = \text{ч}_i \qquad (7)$$

And for a (4,2) polynomial

$$a_{ij} = 4_{ij} - 2\text{ч}_i - 2\text{ч}_j \qquad (8)$$

Equation 3.31 is the general form of Scheffe's second degree polynomial

Lightweight aggregate

[26] It is possible to produce coarse aggregates from fly ash by pelletisation techniques for use in structural grade concrete. They also studied properties like

bulk density, specific gravity, water absorption, and aggregate crushing value. The concrete made with the bonded partial replacement of polystyrene with coarse aggregate has a high slump, low density, and minimum structural grade concrete, as recommended in IS 456-2000.The permeability indicating tests such as sorptivity, water absorption rate, rapid chloride permeability test, etc. indicates satisfactory durability characteristics. [16] discussed lightweight concrete and lightweight aggregate concrete and its classification. It is also reported on properties of various lightweight aggregate concrete. It is also discussed on the proportioning of lightweight aggregate concrete by weight method. [17] produced lightweight tetrapod aggregates from high calcium fly ash with properties the light weight, strong, highly penetrating, and interlocking. They also obtained the results of the physical and mechanical properties of the produced regular fly ash aggregate. They also optimized the percentage of lime content for the best performance. [18] reported on the interactions between sintered fly ash lightweight aggregates and the Portland cement matrix-matrix to resolve factors other than aggregate strength, influencing the concrete strength. Aggregates of variable properties were produced, and concretes of equal effective water/cement ratio were prepared and tested for strength and microstructure. It was found that differences in concrete strength could not always be accounted for by differences in the aggregate strength. The physical process is identified as densification of the interfacial transition zone due to the absorption of the aggregates; this process has considerable influence at an early age. The chemical processes were associated with the pozzolanic activity of CH's aggregate and deposition in the pores in the aggregates' shell; these processes became effective only at a later age, beyond 28 days. The enhancement in strength due to these influences ranged between 20 and 40%. Such influences should be taken into account in the design of a lightweight aggregate of optimal properties. [19] described the details of the investigation on the use of fly ash based lightweight aggregate as a coarse aggregate in polymer concrete with sand fly ash and polyester resin as other components. They observed that the addition of lightweight aggregate reduces the density of polymer concrete and decreases its compressive strength. The tensile strength / compressive strength ratio for such polymer concrete was much higher than that of conventional concrete. [20] studied the segregation phenomenon in polymer concrete with granite and sintered fly ash aggregate. They observed no segregation at the coarse aggregate contents when sintered fly ash aggregate is used and crushed granite stone aggregates particles settle towards the base resulting in noticeable segregation in the mix. A study was made by [19] on geopolymer concrete containing sintered fly ash aggregates and granite aggregates. They observed higher compressive strength in geopolymer concrete containing crushed granite aggregates than sintered fly ash aggregates based on geopolymer concrete. The ultrasonic pulse velocities of values of more than 4 km/sec in geopolymer concrete with the above two aggregates indicate their dense microstructure. The effect of polymer on performances of lightweight aggregate concrete was studied by [21]. They observed higher compressive strength and flexural strength in lightweight aggregate concrete when ethylene-vinyl acetate latex ranges from 5% to 15%. The ratio of flexural strength to compressive strength was highly improved, the brittleness was decreased, and the toughness was improved in the lightweight aggregate concrete due to polymers.

Scheffe's Simplex lattice design

A simplex is a geometric figure with the number of vertices being one more than the variable factor space, q. It is a projection of n-dimensional space onto an n-1 dimensional coordinate system. Thus, if q is 1, the number of vertices is two and the simplex is a straight line; when it is 2, the simplex is a triangle and a tetrahedron when 3. A lattice is an ordered arrangement of points in a regular pattern. [22] first introduced simplex lattice design in his study of joint action on related hormones. [14], however, expanded and generalized the simplex lattice design. His work is often seen as a pioneering work in simplex lattice mixture design. Lattice designs are presently often referred to as Scheffe's simplex lattice designs. It was assumed that each component of the mixture resides on a vertex of a regular simplex-lattice with q-1 factor space. If the degree of the polynomial to be fitted to the design is n, and the number of components is q, then the simplex lattice, also called a *{q,n}* simplex will consist of uniformly spaced points whose coordinates are defined by the following combinations of the components: the proportions assumed by each component take the n+1 equally spaced values from 0 to 1, that is;

$$X_i = \ 0, \frac{1}{n},\ \frac{2}{n}\ , \dots\dots\dots..1 \tag{9}$$

And the Simplex lattice consists of all possible combinations of the components where the proportions of equation (2.13) for each component are used [23].

Thus, for the quadratic lattice {q,n} approximating the response surface with second-degree polynomials, (n = 2) the following levels of every factor must be used; $0, \frac{1}{2}$, and 1; for a cubic polynomial $(n = 3): 0, \frac{1}{3}$, and 1, and for a fourth − degree polynomial $(n = 4): 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ and 1.

(10)

Consider a four-component mixture. The factor space is a tetrahedron. If a second-degree polynomial is to be used to define the factor space's response, then each component (X_1, $X_2 \dots X_4$) must assume the proportions X_i = 0, ½, and 1. The (4,2) simplex lattice consists of the ten points at the boundaries and the vertices of the tetrahedron:

(X_1, X_2, X_3, X_4) = (1,0,0,0), (0,1,0,0), (0,0.1.0), (0,0,0,1), (1/2, 1/2, 0,0), (1/2, 0, 1/2, 0), (1/2, 0,0,1/2), (0,1/2,1/2,0), (0,1/2,0,1/2) and (0,0,1/2,1/2). The four points defined by (1,0,0,0), (0,1,0,0), (0,0,1,0) and (0,0,0,1) represent single component mixtures at the vertices of the tetrahedron. (1,0,0,0) for instance, is a mixture at a vertex with $X_1 = 1$ and $X_2 = X_3 = X_4 = 0$. The other mixtures are binary blends of two component mixtures at the middle of the edges of the tetrahedron. Thus the mixture (1/2,0, 1/2,0) is a binary blend of equal amounts of X_1 and X_3 (X_2 and X_4 being zero) at the midpoint of the edge connecting vertex 1 and vertex 3. Figure 1 shows the ten points of a (4,2) sim

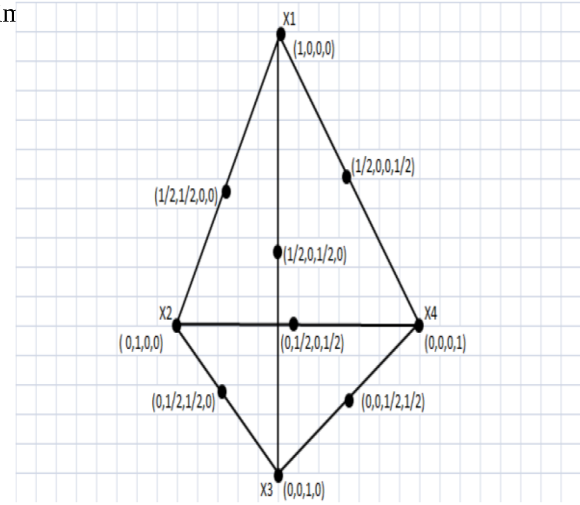

Figure 1: A (4,2) Simplex lattice showing the pseudo ratios at the design points.

Canonical polynomial for Scheffe's mixture model.

The general form of a polynomial of degree *n* in *q* variables is given (Akhnazarova and Kafarov, 1988) as:

$$\hat{y} = b_o + \sum_{1\le i\le q} b_i X_1 + \sum_{1\le i\le j\le q} b_{ij} X_i X_j + \sum_{1\le i\le j\le k\le q} b_{ijk} X_i X_j X_k + + \sum b_{i_1 i_2 \ldots\ldots i_n} X_{i_1} X_{i_2} X_{i_n} \quad (11)$$

The number of terms in equation (11) is C_n^{q+n}; that is $(q+n)$Combination n.

[14], by substituting the identity $X_1+X_2+ \ldots + X_q = 1$ in equation (11) reduced the number of terms in the polynomial to $C_n^{q+n}$and this number of terms is equal to the number of points associated with the simplex lattice design. This can be illustrated by considering the derivation of a second degree polynomial for a ternary system [24]. For such a system, the general form of the polynomial reduces to:

$$\hat{y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 \quad (12)$$

Since

$$X_1 + X_2 + X_3 = 1 \quad (13)$$

Multiplying Equation (13) by b_0 gives:

$$b_0 = b_0X_1 + b_0X_2 + b_0X_3 \quad (14)$$

Multiplying Equation (13) successively by X_1, X_2, and X_3, and rearranging gives

$$X_1^2 = X_1 - X_1X_2 - X_1X_3$$

$$X_2^2 = X_2 - X_1X_2 - X_2X_3$$

$$X_3^2 = X_1 - X_1X_3 - X_2X_3 \quad (15)$$

Substituting Equation (14) and (15) into Equation (12) and simplifying gives

$$\hat{y} = (b_0 + b_1 + b_{11}) X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{23} - b_{22} - b_{33})X_2X_3 \quad (16)$$

If we let:

$$\beta_i = b_0 + b_i + b_{ii}, \text{ and } \beta_{ij} = b_{ij} + b_{ij} \quad (17)$$

Then:

$$\hat{y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_3 \quad (18)$$

A similar analysis when the number of components is four and *n* is 2 gives

$$\hat{y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \quad (19)$$

Again, the number of terms is ten as against 15 in the original form of the polynomial.

In summary, the reduced second degree polynomial for *q* components is given as:

$$\hat{y} = \sum_{1\le i\le q} \beta_i X_1 + \sum_{1\le i\le i\le q} \beta_{ij} X_i X_j \quad (20)$$

The reduced form is called the canonical polynomial or simply the {q,n} polynomial. The number of terms in the reduced polynomial is the minimum number of experimental runs necessary to determine the polynomial coefficients and is given as:

$$N = C_n^{(q+n-1)} = \frac{(q+n-1)!}{(q-1)!(n)!} \quad (21)$$

Considering Equation (20), the term β_i represents the expected response to pure component X_i. The non-linear part of $\beta_i X_i X_j$ is called the synergism if it is greater than the linear portion and antagonism if it is less. The term β_{ij} is known as the quadratic coefficient of binary synergism of the components *i* and *j*.

Determination of the parameters of the {q, 2} polynomial

There is a one-to-one relationship between the number of points on the simplex lattice and the number of terms in the canonical polynomial as a result of which the parameters in the reduced polynomial can be expressed as simple functions of the expected responses at the points of the {q,n} simplex lattice [23]. The determination of the estimates of the coefficients of the {q,n} simplex lattice again can be illustrated using the {4, 2} simplex lattice. The design matrix for this simplex lattice is shown in Table 2 below.

Table 1: Design matrix for (4,2) simplex lattice

S/No.	X_1	X_2	X_3	X_4	Response
1	1	0	0	0	y_1
2	0	1	0	0	y_2
3	0	0	1	1	y_3
4	0.5	0.5	0	0	y_4
5	0.5	0	0	0.5	y_{12}
6	0.5	0.5	0.5	0	y_{23}
7	0	0	0.5	0.5	y_{34}

At point 1,

$X_1 = 1, X_2 = 0, X_3 = 0.$

Substituting these values into Equation (20) gives

$\beta_1 = y_1$ (22)

Substituting these values into the equation. (2.23) gives

Similar substitutions at points 2 and 3 give

$\beta_2 = y_2$ and $\beta_3 = y_3$ (23)

At the fourth point,

$y_{12} = 0.5\beta_1 + 0.5\beta_2 + 0.5 * 0.5\ \beta_{12} = 0.5\ \beta_1 + 0.5\ \beta_2 + 0.25\ \beta_{12}$ (24)

Substituting β1 = y1 and β2 = y2 into Equation (16) and rearranging gives

$\beta_{12} = 4y_{12} - 2y_1 - 2y_2$ (25)

Similarly,

$\beta_{13} = 4y_{13} - 2y_1 - 2y_3$
$\beta_{23} = 4y_{23} - 2y_2 - 2y_3$ (26)

Where

$a_i = X_i(2X_i - 1)$ and $a_{ij} = 4X_iX_j$, i, j = 1, 2………..q, i < j (32)

Then the variance of estimate of **yi** is given as

$$Var\ (y_i) = Var\ (y)\left(\sum_{1\le i\le q} \frac{a_i^2}{r_i} + \sum_{1\le i\le j\le q} \frac{a_{ij}^2}{r_{ij}}\right) \quad (33)$$

When the number of replicate observations is equal to **r** at all observation points,

$$Var\ (y_i) = \frac{Var\ (y)}{r}\left(\sum_{1\le i\le q} a_i^2 + \sum_{1\le i\le j\le q} a_{ij}^2\right) \quad (34)$$

In summary, for the {q, 2} canonical polynomial,

$\beta = y_i$ and $\beta_{ij} = 4y_{ij} - 2y_i - 2y_j = 4y_{ij} - 2\ (y_i + y_i)$ (27)

Variance

It is assumed that the errors are uncorrelated and identically distributed with zero means. The variance of the predicted response is var(y), then the variance of the response can be written as a function of the variances of the averages of responses at the lattice points. If **r**i and **r**ij are the numbers of replicate observations at points **i** and **ij** and **yi** and **yij** are respectively the average responses at those points, then the variances of **yi** and **yij** are given as

$$var\ (y_i) = \frac{var\ (y)}{r_i} \quad (28)$$

$$var\ (y_{ij}) = \frac{var\ (y)}{r_{ij}} \quad (29)$$

If their expressions replace the coefficients in the canonical equation in terms of average responses
$\beta_i = y_i$ and $\beta_{ij} = 4y_{ij} - 2y_i - 2y_i$

We obtain

$$ŷ = \sum_{1\le i\le q} y_i X_i + \sum_{i\le j\le q} (4y_{ij} - 2y_i - 2y_j)X_i \quad (30)$$

$$ŷ = \sum_{1\le i\le q} a_i\, y_1 + \sum_{1\le i\le j\le q} a_{ij}\, y_{ij} \quad (31)$$

Augmented Simplex Lattice design (ASL)

The simplex lattice design, in its original form, is saturated. It contains just only the design points at the vertices and edges necessary to formulate the model equation. It, however, does not give any information about the inside of the simplex. As a way of improving the model, additional points within the simplex are included in the design. These points are incorporated to improve the model and are also used in testing the adequacy of the fitted model. Hence they are also known as checkpoints.

The usual practice is to augment the simplex with the following points:

(i) The centroid of the simplex

(ii) The points lying midway between the centroid and each of the vertices.

Figure 2.2 is an augmented simplex lattice made up of 15 points, the original 10 points as in Figure 2.1, and additional 5 checkpoints. The checkpoints were at the centroid of the simplex (point number 11) and point's midway between the centroid and each of the vertices (point numbers 12, 13, 14, and 15). The inclusion of the test or checkpoints does not affect the form of the model equation

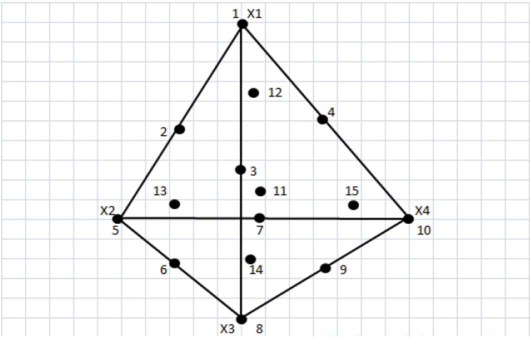

Figure 2: An augmented {4, 2} Simplex lattice showing the design points.

For the augmented simplex lattice design, parameter estimates will differ slightly from those obtained using only the simplex lattice points. If the total number of observations, including the replicates, is *M*, then the least square estimates of the regression coefficients are given in matrix form [23] as:

$$\beta = (X^1X)^{-1} X^1 y \qquad (35)$$

Where:

β is a vector whose elements are the least square estimates of the regression coefficients.

X is an *M xq* matrix whose elements are the mixture component proportions and the component proportions' functions.

y is the vector (length, *M*) of the observations or responses at the various observation points.

Axial designs

Axial designs consist mainly of complete mixtures of *q*-component blends where most of the points are positioned inside the simplex. They are recommended for use when component effects are to be measured and screening experiments, particularly when first degree models are to be fitted [23]. The axis of component ***i*** is the imaginary line extending from the base $X_i = 0$, $X_j = 1$ (q-1) for all $j \neq i$. The length of the axis is the shortest distance from the opposite (*q*-2) dimensional boundary. The design points are positioned only on the component axes. A more comprehensive description of the axial designs can be obtained from [23].

Simplex-centroid design.

Scheffe [25] introduced the simplex-centroid design in which the number of Distinct Points is 2*q* – 1, *q* being the number of components. The points correspond to *q* permutations of single components or (1, 0, 0,….,0), *q*C2 permutations of all binary mixtures or (0.5, 0.5, 0, …,0), *q*C3 permutations of (1/3, 1/3, 1/3, 0, 0, 0….) and so on, with finally the overall centroid point (1/*q*, 1/*q*, …., 1/*q*) or *q*-nary mixture. Simplex-centroid designs contain as many coefficients as there are points in the design and take the form:

$$\hat{y} = \sum_{1\le i\le q} b_i X_i + \sum_{1\le i<j\le q} b_{ij} X_i X_j + \sum_{1\le i<j<k\le q} b_{ijk} X_i X_j X_k + \cdots + b_{12\ldots\ldots q} X_1 X_2 \ldots\ldots X_q \qquad (36)$$

Unlike the simplex-lattice design, for a given number of components, there exists only one simplex-centroid design.

Materials and methods

The materials used for the experiment were: Polystyrene, fine aggregate (river sand), coarse aggregate (granite). Ordinary limestone cement (Lafarge cement), water (fresh drinkable water). The experiment was carried out in Civil Engineering Laboratory at the Cross River University of Technology Calabar.

Sand: The Sand was obtained in accordance with British Standard Institution, BS 882: 1992. The researcher purchases the sand from the local qua rivers in Akpabuyo.

Cement: The Eagle cement brand of OPC with properties conforming to British standard was used.

Water: The potable drinking water conforming to the specification of British Standards Institution, BS EN 1008: 2002, was used.

Coarse aggregate: They are natural gravel and sand that are usually dug or dredged from a pit, river, lake, or seabed. For this study, it was obtained Akpabuyo River in Cross River State.

Polystyrene: This was obtained from the market through a local distributor in Owerri, Imo State.

After collected and stored in a dry area, all the materials were subjected to chemical analysis to determine Polystyrene lightweight concrete's elemental composition using Scheffe's model, as presented in Tables 2 and 3, respectively.

Experimental method

The Minitab statistical software 16(23) was used in designing the experiment based on an augmented (4,2)

Scheffee's simplex lattice design. The experimental design simplex is indicated in Figure 1, whereas the augmented (4,2) simplex's design matrix is shown in Table 2. The design contains twenty (20) mixes at the tetrahedron's vertices and edge, augmented with five more mixes within the simplex. These five points were used as checkpoints to validate the models developed. There were also replicate points at the tetrahedron's vertices and centroid, making it a total of twenty points. However, the design was based on Pseudo components, and randomization was applied. The actual and Scheffe's pseudo-components Mathematical optimization methods (requirement of the simplex);

$X_1+X_2+X_3+X_4=1$ (37)

Table 2: Actual (Zi) and Pseudo (xi)components for Scheffe's (4, 2) Simplex Lattice

S/N	X_1	X_2	X_3	X_4	Response	Z_1	Z_2	Z_3	Z_4
1.	1	0	0	0	Y_1	0.45	0.50	0.46	0.44
2.	0	1	0	0	Y_2	1	1	1	1
3.	0	0	1	0	Y_3	1.5	2.0	2.5	3.0
4.	0	0	0	1	Y_4	3	4.0	5.0	6.0
5.	½	½	0	0	Y_{12}	0.475	1	2.75	3.5
6.	½	0	½	0	Y_{13}	0.455	1	2.0	5.0
7.	½	0	0	½	Y_{14}	0.445	1	2.25	4.5
8.	0	½	½	0	Y_{23}	0.48	1	2.25	4.5
9.	0	½	0	½	Y_{24}	0.47	1	2.5	4.5
10.	0	0	½	½	Y_{34}	0.45	1	2.75	5.5
Control Points									
11.	½	¼	¼	0	C_1	0.465	1	1.88	3.75
12.	¼	¼	¼	$^1/_4$	C_2	0.463	1	2.25	4.5
13.	0	¼	0	$^3/_4$	C_3	0.46	1	2.63	5.5
14.	½	0	¼	$^1/_4$	C_4	0.48	1	2.13	4.25
15.	½	¼	0	$^1/_4$	C_5	0.46	1	2.0	4.0
16.	0	$^1/_4$	¾	0	C_6	0.47	1	2.38	4.75
17.	0	½	¼	$^1/_4$	C_7	0.475	1	2.13	4.75
18.	¼	$^1/_8$	½	$^1/_8$	C_8	0.46	1	2.25	4.50
19.	¼	$^1/_4$	0	½	C_9	0.458	1	2.38	4.75
20.	$^1/_8$	$^1/_8$	¼	½	C_{10}	0.454	1	2.56	5.13

Components Transformation of Polystyrene

The Pseudo ratio was transformed to real the component ratios used for the blending of the polystyrene. The relationship between the real component ratios and the Pseudo components is as shown below:

R = AP (38)

From Equ. 38, R is a vector containing the real ratios of the components, P is a vector containing the pseudo ratios, and *A* is a transformation matrix which can be obtained from trial mixes given as:

$$A = \begin{bmatrix} 0.45 & 0.50 & 0.46 & 0.44 \\ 1 & 1 & 1 & 1 \\ 1.5 & 2.0 & 2.5 & 3.0 \\ 3 & 4.0 & 5.0 & 6.0 \end{bmatrix}$$

The element of each column of [A] represents the proportions of the components at the vertex in the following order of water (X_1), cement (X_2), and (X_3), and Coarse aggregate (X_4).

The totality of all the polystyrenes was blended using a crushing machine. The aggregates were used in their dry condition, and batching was by weight. Manual mixing was employed. Here, the entire polystyrene was put into backs and sook in portable water inside a container. The polystyrenes were blended and cured in the open air for 28 days by sprinkling them with water twice daily.

Modulus of Elasticity

The statistical method developed by Scheffe was adopted for the study in accordance with the Royal Statistical Society Journal, Series B. 20, 1958. The theory was developed for experiments with mixtures of q components whose purpose was for the empirical prediction of the responses to any mixture of the components when the response depends only on the component's proportion and not on the total amount. The Scheffe model introduced the (q,m) simplex lattice designs. Simplex is simply the projection of a q-dimensional space onto a q-1 dimensional coordinate system; this can be done because the proportions of the mixture are constrained to sum to one. Thus, a feasible combination of four components: Sand, cement, water, and coarse aggregate, can be projected onto a two-dimensional triangular field. The lattice part of the simplex lattice design shows that points are spaced regularly on the simplex. The degree of the simplex lattice is defined by the degree of the polynomial that may be used to fit the response surface over the simplex. Scheffe indicated that the number of points in (q,m) lattice is given by:

$$^{q+m-1}C_m = q(q+1)\ldots\ldots\ldots(q+m-1)/m! \quad (39)$$

However, for a four-component mixture, i.e. (4,2) lattice, the number of points equals 4(4+1)/2! = 10.

The (q,m) simplex lattice designs are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation representing the response surface over the entire simplex region. The polynomial has exactly as many parameters as the number of points in the associated simplex lattice design. The response represents the property studied and is normally assumed to be a multi-varied function. In this study, the response is the modulus of elasticity.

Scheffe's modified polynomial equation using the restriction $\sum X_i = 1$ is represented as the equation (40).

$$Y = \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 + \alpha_{12}X_1X_2 + \alpha_{13}X_1X_3 + \alpha_{23}X_2X_3 \quad (40)$$

General form of the Equation (2) is

$$Y = \sum \alpha_i X_i + \sum \alpha_{ij} X_i X_j \quad (41)$$

where

$1\leq i\leq q$, $1\leq i\leq j\leq q$

q is the number of components of a mixture and ***i*** ranges from 1 to q.

X_i is the proportion of the *i* component in the mixture.

α_i and α_{ij} are the coefficients.

The values of the unknown coefficients are determined using the following equations:

$$\alpha_i = y_i \quad (42)$$

$$\alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (43)$$

The pseudo components which represent the proportion of the components of the *i* component in the mixture i.e. X_1, X_2, X_3 X_4, were transformed to actual mix proportions (components) Z_1, Z_2, Z_3, Z_4 using the following relationships and presented on Table 1.

$$X = BZ \quad (44)$$

$$Z = AX \quad (45)$$

Where *A* = matrix whose elements are from the arbitrary mix proportions chosen when Equation (38) is opened and solved mathematically.

B = the inverse of matrix *A*

Z = matrix of actual components

X = matrix of pseudo components obtained from the lattice.

The design matrix is shown in Table 2 for the X_i experimental points are called "Peudo-Components" and Z_i are the actual experimental components.

Table 3: Mix ratio of Modulus of Elasticity at 28 days

Mixes	Properties	Age (Days)														
		3			7			14			21			28		
1	Applied Load (kN)	64.589	65.343	63.157	104.96	106.18	102.63	145.33	147.02	142.10	153.40	155.19	150.00	159.86	161.72	156.31
	Modulus of Elasticity (N/mm^2)	0.91	0.92	0.89	11.66	11.80	11.40	16.15	16.34	15.79	17.04	17.24	16.67	17.76	17.97	17.37
	Average Modulus of Elasticity (N/mm^2)	0.91			11.62			16.09			16.98			17.70		
2	Applied Load (kN)	57.334	55.702	55.702	93.17	90.52	90.52	129.00	125.33	125.33	136.17	132.29	132.29	141.90	137.86	137.86
	Modulus of Elasticity (N/mm^2)	0.81	0.79	0.79	10.35	10.06	10.06	14.33	13.93	13.93	15.13	14.70	14.70	15.77	15.32	15.32
	Average Modulus of Elasticit	0.80			10.16			14.06			14.84			15.47		

	y (N/mm^2)															
3	Applied Load (kN)	49.631	49.705	49.656	80.65	80.77	80.69	111.67	111.84	111.73	117.87	118.05	117.93	122.84	123.02	122.90
	Modulus of Elasticity (N/mm^2)	0.70	0.70	0.70	8.96	8.97	8.97	12.41	12.43	12.41	13.10	13.12	13.10	13.65	13.67	13.66
	Average Modulus of Elasticity (N/mm^2)	0.70			8.97			12.42			13.11			13.66		
4	Applied Load (kN)	45.729	44.744	45.009	74.31	72.71	73.14	102.89	100.67	101.27	108.61	106.27	106.90	113.18	110.74	111.40
	Modulus of Elasticity (N/mm^2)	0.65	0.63	0.64	8.26	8.08	8.13	11.43	11.19	11.25	12.07	11.81	11.88	12.58	12.30	12.38
	Average Modulus of Elasticity (N/mm^2)	0.64			8.15			11.29			11.92			12.42		
5	Applied Load (kN)	55.478	55.313	54.758	90.15	89.88	88.98	124.83	124.45	123.21	131.76	131.37	130.05	137.31	136.90	135.53
	Modulus of Elasticity (N/mm^2)	0.78	0.78	0.77	10.02	9.99	9.89	13.87	13.83	13.69	14.64	14.60	14.45	15.26	15.21	15.06
	Average Modulus of Elasticity (N/mm^2)	0.78			9.96			13.80			14.56			15.18		
6	Applied Load (kN)	51.817	52.074	49.714	84.20	84.62	80.78	116.59	117.17	111.86	123.07	123.68	118.07	128.25	128.88	123.04
	Modulus of Elasticity (N/mm^2)	0.73	0.74	0.70	9.36	9.40	8.98	12.95	13.02	12.43	13.67	13.74	13.12	14.25	14.32	13.67
	Average Modulus of Elasticity (N/mm^2)	0.72			9.24			12.80			13.51			14.08		
7	Applied Load	54.037	51.113	56.091	87.81	83.06	91.15	121.58	115.01	126.21	128.34	121.39	133.22	133.74	126.51	138.83

	(kN)															
	Modulus of Elasticity (N/mm^2)	0.76	0.72	0.79	9.76	9.23	10.13	13.51	12.78	14.02	14.26	13.49	14.80	14.86	14.06	15.43
	Average Modulus of Elasticity (N/mm^2)	0.76			9.70			13.44			14.18			14.78		
8	Applied Load (kN)	54.749	50.707	53.838	88.97	82.40	87.49	123.19	114.09	121.14	130.03	120.43	127.87	135.51	125.50	133.25
	Modulus of Elasticity (N/mm^2)	0.77	0.72	0.76	9.89	9.16	9.72	13.69	12.68	13.46	14.45	13.38	14.21	15.06	13.94	14.81
	Average Modulus of Elasticity (N/mm^2)	0.75			9.59			13.27			14.01			14.60		
9	Applied Load (kN)	50.161	50.939	51.867	81.51	82.78	84.28	112.86	114.61	116.70	119.13	120.98	123.18	124.15	126.08	128.37
	Modulus of Elasticity (N/mm^2)	0.71	0.72	0.73	9.06	9.20	9.36	12.54	12.73	12.97	13.24	13.44	13.69	13.79	14.01	14.26
	Average Modulus of Elasticity (N/mm^2)	0.72			9.21			12.75			13.46			14.02		
10	Applied Load (kN)	46.301	46.409	49.001	75.24	75.41	79.63	104.18	104.42	110.25	109.96	110.22	116.38	114.60	114.86	121.28
	Modulus of Elasticity (N/mm^2)	0.66	0.66	0.69	8.36	8.38	8.85	11.58	11.60	12.25	12.22	12.25	12.93	12.73	12.76	13.48
	Average Modulus of Elasticity (N/mm^2)	0.67			8.53			11.81			12.47			12.99		
11	Applied Load (kN)	58.369	58.353	59.28	94.85	94.82	96.33	131.33	131.29	133.38	138.63	138.59	140.79	144.46	144.42	146.72
	Modulus of	0.83	0.83	0.84	10.54	10.54	10.70	14.59	14.59	14.82	15.40	15.40	15.64	16.05	16.05	16.30

	Elasticity (N/mm^2)															
	Average Modulus of Elasticity (N/mm^2)	0.83			10.59			14.67			15.48			16.13		
12	Applied Load (kN)	54.749	52.182	53.838	88.97	84.80	87.49	123.19	117.41	121.14	130.03	123.93	127.87	135.51	129.15	133.25
	Modulus of Elasticity (N/mm^2)	0.77	0.74	0.76	9.89	9.42	9.72	13.69	13.05	13.46	14.45	13.77	14.21	15.06	14.35	14.81
	Average Modulus of Elasticity (N/mm^2)	0.76			9.68			13.40			14.14			14.74		
13	Applied Load (kN)	47.088	46.111	48.761	76.52	74.93	79.24	105.95	103.75	109.71	111.83	109.51	115.81	116.54	114.12	120.68
	Modulus of Elasticity (N/mm^2)	0.67	0.65	0.69	8.50	8.33	8.80	11.77	11.53	12.19	12.43	12.17	12.87	12.95	12.68	13.41
	Average Modulus of Elasticity (N/mm^2)	0.67			8.54			11.83			12.49			13.01		
14	Applied Load (kN)	26.932	26.947	56.903	43.77	43.79	92.47	125.03	90.30	128.03	131.98	123.11	135.14	137.53	128.29	140.84
	Modulus of Elasticity (N/mm^2)	0.38	0.38	0.81	4.86	4.87	10.27	13.89	10.03	14.23	14.66	13.68	15.02	15.28	14.25	15.65
	Average Modulus of Elasticity (N/mm^2)	0.52			6.67			12.72			14.45			15.06		
15	Applied Load (kN)	26.932	26.947	56.497	43.77	43.79	91.81	129.00	93.17	127.12	136.17	132.29	134.18	141.90	137.86	139.83
	Modulus of Elasticity (N/mm^2)	0.38	0.38	0.80	4.86	4.87	10.20	14.33	10.35	14.12	15.13	14.70	14.91	15.77	15.32	15.54

	Average Modulus of Elasticity (N/mm²)	0.52			6.64			12.94			14.91			15.54		
16	Applied Load (kN)	26.932	26.947	50.641	43.77	43.79	82.29	111.22	80.33	113.94	117.40	118.44	120.27	122.34	123.43	125.34
	Modulus of Elasticity (N/mm²)	0.38	0.38	0.72	4.86	4.87	9.14	12.36	8.93	12.66	13.04	13.16	13.36	13.59	13.71	13.93
	Average Modulus of Elasticity (N/mm²)	0.49			6.29			11.31			13.19			13.74		
17	Applied Load (kN)	26.932	26.947	52.745	43.77	43.79	85.71	116.59	84.20	118.68	123.07	123.68	125.27	128.25	128.88	130.54
	Modulus of Elasticity (N/mm²)	0.38	0.38	0.75	4.86	4.87	9.52	12.95	9.36	13.19	13.67	13.74	13.92	14.25	14.32	14.50
	Average Modulus of Elasticity (N/mm²)	0.50			6.42			11.83			13.78			14.36		
18	Applied Load (kN)	26.932	26.947	55.495	43.77	43.79	90.18	121.58	87.81	124.86	128.34	121.96	131.80	133.74	127.10	137.35
	Modulus of Elasticity (N/mm²)	0.38	0.38	0.79	4.86	4.87	10.02	13.51	9.76	13.87	14.26	13.55	14.64	14.86	14.12	15.26
	Average Modulus of Elasticity (N/mm²)	0.52			6.58			12.38			14.15			14.75		
19	Applied Load (kN)	26.932	26.947	49.697	43.77	43.79	80.76	112.86	81.51	111.82	119.13	120.00	118.03	124.15	125.05	123.00
	Modulus of Elasticity (N/mm²)	0.38	0.38	0.70	4.86	4.87	8.97	12.54	9.06	12.42	13.24	13.33	13.11	13.79	13.89	13.67
	Average Modulus of Elasticit	0.49			6.23			11.34			13.23			13.79		

	y (N/mm²)															
20	Applied Load (kN)	26.932	26.947	49.217	43.77	43.79	79.98	111.84	80.77	110.74	118.05	113.47	116.89	123.02	118.24	121.81
	Modulus of Elasticity (N/mm²)	0.38	0.38	0.70	4.86	4.87	8.89	12.43	8.97	12.30	13.12	12.61	12.99	13.67	13.14	13.53
	Average Modulus of Elasticity (N/mm²)	0.49			6.20			11.24			12.90			13.45		

Modulus of elasticity result of lightweight polystyrene concrete of Scheffe's model is shown in Table 3. The mixes were 1-20 showing significant differences in average to predict the strength of lightweight polystyrene concrete. From the mix 1 of modulus of elasticity, it comprises an average strength (N/mm²) of 0.91 for 3days, 7days (11.62), 14days (16.09), 21days (16.98), and 28days (17.70), respectively. This revealed that the residuals fall appropriately close and are significant at the 0.05 level. However, it is adequate for the 28days British standard for the prediction of polystyrene lightweight concrete

RESULT AND DISCUSSION

The modulus responses using pseudo components, actual mix ratios, and responses are presented in Table 4.

Experimental Model for Modulus of Elasticity

The second-degree model (*Equ.* 47) was fitted to the data set of the 20 modulus of elasticity test responses at a 95% confidence limit ($\alpha = 0.05$ level of significance) using [23]. The One-Way Analysis of Variance (ANOVA) comparison is shown in table 4. The normal probability plot of the regression standardized residual is shown in Figure 1. In contrast, the cox response trace plot is presented in Figure 2, and the variation of modulus of elasticity in both linear and quadratic is adequately represented in Figure 3 4, respectively.

The bitter's values are, therefore;

$\beta_1 = 8.662,\quad \beta_2 = 7.590,\quad \beta_3 = 6.510,\quad \beta_4 = 5.950,\quad \beta_{12} = -2.035,\quad \beta_{13} = -1.719,\ \beta_{14} = -0.593,\ \beta_{23} = 1.408,\ \beta_{24} = .910,\ \beta_{34} = 1.848$ (46)

If we let the components water, cement, sand, and coarse aggregate be represented respectively by X_1, X_2, X_3, and X_4, then the model equation in terms of pseudo units is:

$$\hat{y} = 8.662X_1 + 7.590X_2 + 6.510X_3 + 5.950X_4 - 2.035X_1X_2 - 1.719X_1X_3 - 0.593X_1X_4 + 1.408X_2X_3 + .910X_2X_4 + 1.848X_3X_4 \quad (47)$$

From Table 4 and equation 47, the variance analysis was used to compare the lack-of-fit of modulus of elasticity. The comparison yielded an F-ratio of 256.992, which is greater than the .05 level of significance and shows an insignificant lack-of-fit of polystyrene for the p-value of lack-of-fit (0.00), which is less than 0.05. This means that equation 47 is adequate for predicting the 28th-day strength of expanded polystyrene concrete using Scheffe's model.

Table 4: One-Way Analysis of Variance (ANOVA) of Estimated Regression Coefficients of Modulus of Elasticity using Scheffe's pseudo components model

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
1	X_1	8.662	.614	.450	14.116	.000
	X_2	7.590	.610	.357	12.434	.000
	X_3	6.510	.585	.348	11.136	.000
	X_4	5.950	.586	.330	10.146	.000
	$X_1 * X_2$	-2.035	2.600	-.021	-.783	.452
	$X_1 * X_3$	-1.719	2.482	-.018	-.693	.504
	$X_1 * X_4$	-.593	2.463	-.006	-.241	.814
	$X_2 * X_3$	1.408	2.648	.015	.532	.607

$X_2 * X_4$	.910	2.586	.010	.352	.732
$X_3 * X_4$	1.848	2.599	.018	.711	.493

Model		Sum of Squares	Df	Mean Square	F	Sig.
	Regression	999.656	10	99.966	256.992	.000[c]
1	Residual	3.890	10	.389		
	Total	1003.546[d]	20			

Model	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	5.271002	8.661866	7.038411	.6833335	20
Residual	-.3523155	1.7330723	.0216634	.4519228	20
Std. Predicted Value	-2.586	2.376	.000	1.000	20
Std. Residual	-.565	2.779	.035	.725	20

The normal probability of regression standardized residual in Figure 2 shows that the residuals fall outside the reference line, indicating that the data does not follow a normal distribution. The cox response trace plot in Figure 3 show relative significant deviation from reference blend in the proportion of fitted modulus of elasticity strength of X_1 (0.0015), X_2 (0.1585), X_3 (0.5300) X_4 (0.3100), respectively.

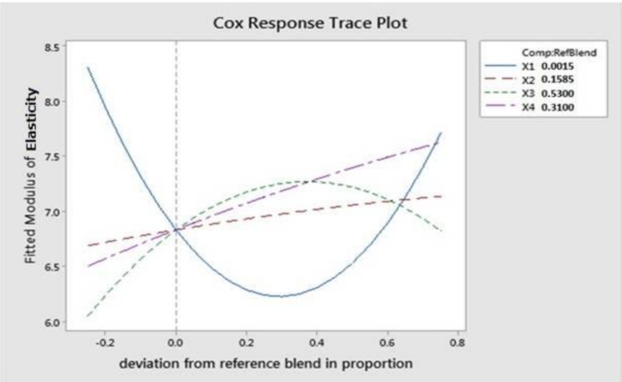

Figure 3: Cox response trace plot of modulus of elasticity

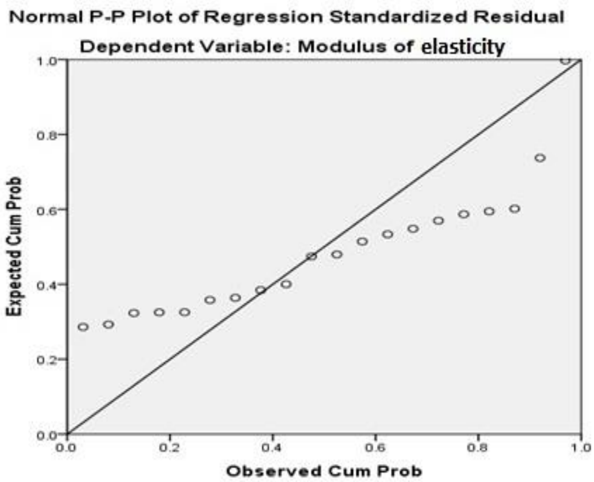

Figure 2: Normal probability plot of Modulus of Elasticity

Conclusion

There exist adequate durability characteristics of expanded polystyrene lightweight concrete with partially replaced with coarse aggregate. Hence, this research sought to investigate the mathematical model for optimizing the modulus of elasticity of lightweight polystyrene concrete using scheffe's model. Four components were generated to represent the study adequately. This includes sand, water, cement, and coarse aggregate. Scheffe's simplex lattice pseudo component model was adopted. The developed models were all tested for lack-of-fit and were found adequate for predicting the various responses within the bounds of the simplex at a 95% confidence limit. Each component plays a significant role in investigating the optimization of the modulus of elasticity of polystyrene lightweight concrete using scheffe's model. Through them, a lot of information and predictions were made.

Recommendations

1) The government should encourage and adopt the Scheffe's pseudo component model to adequately provide the various mixes of water, sand, cement, and coarse aggregate.
2) The maximum and minimum water absorption of the polystyrene lightweight concrete predictable by Scheffe's pseudo component model was found to be 1.08, and 0.50% should be encouraged.
3) The government should set standards and policies that can eliminate the reduction of the number of trial mixes and the use of arbitrary mixes. Doing so would help reduce the time, effort, and resources needed in meeting the requirements of a given response.
4) The mathematical models developed using Scheffe's pseudo component are adequate and should optimize lightweight polystyrene concrete.

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