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Optimization of Compressive Strength of Polystyrene Lightweight Concrete Using Scheffe's Pseudo and Component Proportion Models

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Abstract
Expanded polystyrene beads are industrial waste that can be used for the construction of lightweight concrete. Although the major setback in the use of this material has been the challenge of obtaining a reliable compressive strength of the associated concrete suitable for residential and commercial purposes. This often comes with multiple trail mixes that are time-consuming and cost-intensive, hence the need to develop a mathematical model that will optimize the compressive strength of polystyrene lightweight concrete. The materials used for this study were (i) Ordinary Portland cement (ii) Water (iii) Sand (iv) coarse aggregate and (vi) Expanded Polystrene beads. The materials were batched according to their weights, except for coarse aggregates and polystyrene beads which were mixed and batched together as a single material in the volume. Thus, giving a total of four components instead of five. The study adopted Scheffe's simplex lattice design for both pseudo and component proportion models to generate their respective mixes. The first 10 mixes from each model served as the actual mixes, while the last 10 served as the control mixes. The constituents were manually mixed in the laboratory and the results used for model optimization were based on the 28th-day test. All specimens were cured based on NIS 87 (2004). The laboratory compressive results for the 28th-day test were obtained. The study showed that using Scheffe's Pseudo component model, an optimized compressive strength value of27.920 N/mm^2 can be obtained from water, cement, sand, and coarse aggregate (at 12% partial replacement with polystyrene aggregates) mix ratio of 0.455, 1, 1.820, and 2.980 respectively. On the other hand, Scheffe's component proportion model showed that compressive strength of 27.550 N/mm^2 can be attained from water, cement, sand, and coarse aggregate (at 12% replacement) mix ratio of 0.482, 1, 1.850, and 3.360 respectively. The results from the two models show that polystyrene lightweight concrete can attain a concrete strength that is suitable for residential purposes and can also be used as partitions in high rising buildings due to their lightweight.

Keywords: *Lightweight, Concrete, Mathematical Optimization, Polystyrene, Scheffe's Model, and Compressive Strength.*

I. INTRODUCTION

Continuous natural resource depletion in conjunction with the rising cost of conventional raw materials in construction has instigated the exploration of waste materials as alternatives within the construction industry[1]. If properly processed, waste materials have demonstrated a high degree of effectiveness as construction materials that can meet the required design conditions without difficulty [2],[3]. Adverse environmental problems have ensued as a result of the persistent and increasing extraction of natural aggregate materials for construction purposes. Most commonly, the effects of these actions impact more on rural areas where the quarrying activities take place and one of the most common effects is erosion [4],[5]. Expanded Polystyrene (EPS) beads can produce lightweight concrete by aggregating with various contents within a concrete mixture, at varying properties of densities. It has also identified general low strength as the reason for the drought in the literature on the use of EPS for modern structural designs [3]. Hence, it becomes imperative to mathematically optimize the strength of polystyrene lightweight concrete. The compressive strength is by far the most important strength property used to judge the overall quality of concrete. It may often be the only strength property of the concrete that may be determined since with a few exceptions almost all the properties of concrete can be related to its compressive strength. Compressive strength is usually determined by subjecting the hardened concrete, after appropriate curing, usually 28 days, to increasing compressive load until it fails by crushing and determining the crushing force. Mathematically, it is given as:

$$f_c = \frac{F}{A_c} \qquad (i)$$

Where: f_c is the compressive strength in MPa (N/mm^2)

F is the maximum load at failure, in N

A_c is the cross-sectional area of the specimen, in mm

II. MIXTURE EXPERIMENT AND MODEL FORMS

A mixture experiment is one in which the response is assumed to be dependent on the relative proportions of the constituent materials and not on their total amount [6]. The constituents of the mixture can be measured by volume or mass. For such experiments, two basic requirements must be satisfied namely; the sum of the proportions of the constituents must add up to 1 and none of the constituents will have a negative value. The above statements can respectively be stated mathematically as:

$$\boldsymbol{X_1 + X_2 + \cdots + X_q = \sum_{i=1}^{q} X_i = 1} \qquad (\boldsymbol{ii})$$

$$\boldsymbol{0 \leq X_i \leq 1} \qquad (\boldsymbol{iii})$$

Where

q is the number of mixture components.

X_i *(i = 1 to q)* is the volume or mass proportion of component *i* in the mixture.

It should be noted that since the total proportions of the constituents are constrained to 1, only *q-1* of the variables or constituents can be independently chosen. From Equation (iv),

$$\boldsymbol{X_q = 1 - \sum_{i=1}^{q-1} X_i} \qquad (\boldsymbol{iv})$$

If the response – which in this case is the 28-day compressive strength – is denoted by *y*, and X_1, X_2,...X_q are the constituents of the mixture – in this case are cement, water, sand, and coarse aggregates (polystyrene beads and granite chippings at 12% and 88% respectively), then we can write that:

$$\boldsymbol{y = F(X_1, X_2, X_3 \ldots\ldots\ldots.., X_q)} \qquad (v)$$

Mixture models have not only had their application in concrete mix designs but also in other real-life applications to include agriculture, food industry, pharmacy, etc. Mixture experiments were used to evaluate cement clinker oxidation by [7].

A. SCHEFFE'S SIMPLEX LATTICE DESIGN

According to [8], "a simplex is a geometric figure with the number of vertices being one more than the number of variable factor space, *q*. It is a projection of n-dimensional space onto an n-1 dimensional coordinate system".Consequently, if *q* is 1 it, therefore, implies that the simplex is a straight line and the number of vertices is 2. When q is 2, then it implies that the simplex is a triangle, and the number of vertices is 3.When *q* is 3 a tetrahedron with 4 vertices. Hence, it is an ordered arrangement of points in a regular pattern. The work of [9], presents a vivid explanation of lattice design and is often regarded as the pioneering work in simplex lattice mixture design. Presently, they are often referred to as "***Scheffe's simplex lattice designs***". His assumptions hold that "each component of the mixture resides on a vertex of a regular simplex-lattice with *q*-1 factor space. If the degree of the polynomial to be fitted to the design is *n* and the number of components is *q* then the simplex lattice also called a *{q,n}* simplex will consist of uniformly spaced points whose coordinates are defined by the following combinations of the components: the proportions assumed by each component take the *n*+1 equally spaced values from 0 to 1, that is;

$$\boldsymbol{X_i = 0, \quad \frac{1}{n}, \quad \frac{2}{n}, \ldots\ldots\ldots..1} \qquad (\boldsymbol{vi})$$

and the simplex lattice consists of all possible combinations of the components where the proportions of Equation (*iv*) for each component are used [6]. The second-degree Scheffe's polynomial for *q* components is given as:

$$\boldsymbol{y = \sum_{1 \leq i \leq q} \beta_i X_i \sum_{1 \leq i \leq j \leq q} \beta_{ij} X_i X_j} \qquad (\boldsymbol{vii})$$

The number of terms in the Scheffe's polynomial, *N* is the minimum number of experimental runs necessary to determine the polynomial coefficients and is given as:

$$\boldsymbol{N = C_n^{(q+n-1)} = \frac{(q+n-1)!}{(q-1)!(n)!}} \qquad \textbf{(viii)}$$

Consider a four component mixture. The factor space is a tetrahedron. If a second degree polynomial is to be used to define the response over the factor space then each component (X_1, X_2....X_4) must assume the proportions X_i = 0, $^1/_2$, and 1. The {4, 2} simplex-lattice consists of the ten points at the boundaries and the vertices of the tetrahedron: **(X_1, X_2, X_3, X_4)** = (1,0,0,0) , (0,1,0,0) , (0,0,1,0), (0,0,0,1), (1/2,1/2,0,0), (1/2,0,1/2,0), (1/2,0,0,1/2), (0,1/2,1/2,0), (0,1/2,0,1/2) and (0,0,1/2,1/2). The four points defined by (1,0,0,0) , (0,1.0,0) , (0,0,1,0) and (0,0,0,1), represent single component mixtures at the vertices of the tetrahedron.(1,0,0, 0).Since the q = 4 and n = 2,thenthe governing equation of Scheffefor this study is as follows:

$$y = \beta_1 X_2 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \quad \text{(ix)}$$

III. MATERIALS AND METHODS

The materials used for this study were (i) Ordinary Portland cement (ii) Water (iii) Sand (iv)coarse aggregate and (vi) Expanded Polystrene beads. Lafarge brand of Ordinary Portland cement was obtained from a major cement dealer in Calabar. Potable water conforming to the specification of [10] was used for all specimen preparations and curing. River sand was obtained from Calabar River beach in Calabar, Nigeria. Coarse Aggregate was obtained from the quarry site of Crush Rock Industries at Akamkpa, in the Cross River State of Nigeria. Lastly, the polystyrene beads were obtained from a local distributor in Owerri, Nigeria. The materials were batched according to their weights, except for coarse aggregates and polystyrene beads which were mixed and batched together as a single material in the volume. Hence, the total number of components was 4 and a second-degree polynomial was used in designing the experiments. That is, q = 4 and n = 2.Minitab 16 software by Minitab incorporated was used to generate the initial pseudo mixes. To obtain real ratios for real-life application, the pseudo components as shown in Appendix 2a and 2b were first transformed into real ratios using the following equation…R = AP…..(x). Where; R is the real component ratio vector; A is the transformation matrix obtained from trial mixes; P is the vector containing the pseudo ratios. Hence, the workable mix ratios at the vertices of the simplex are the elements of A. For instance, referring to the data in Appendix 2a and Appendix 3, the transformation matrix A for the first four values is given thus;

$$A = \begin{pmatrix} 0.45 & 0.50 & 0.46 & 0.44 \\ 1.00 & 1.00 & 1.00 & 1.00 \\ 1.50 & 2.00 & 2.50 & 3.00 \\ 2.00 & 4.00 & 5.00 & 6.00 \end{pmatrix}$$

Hence, to obtain the actual mix for mix number 13 in Appendix 3, the vector for the pseudo component P is given thus;

$$P = \begin{pmatrix} 0.000 \\ 0.250 \\ 0.000 \\ 0.750 \end{pmatrix}$$

Where P_i (i = 1, 2, 3, 4) for the four components of water, cement, sand, and coarse aggregates at 12% partial replacement respectively at the design points.

Therefore the real mix "R" for mix 13 is given thus;

$$R = \begin{pmatrix} 0.45 & 0.50 & 0.46 & 0.44 \\ 1.00 & 1.00 & 1.00 & 1.00 \\ 1.50 & 2.00 & 2.50 & 3.00 \\ 2.00 & 4.00 & 5.00 & 6.00 \end{pmatrix} \begin{pmatrix} 0.000 \\ 0.250 \\ 0.000 \\ 0.750 \end{pmatrix} = \begin{pmatrix} 0.46 \\ 1.00 \\ 2.63 \\ 5.50 \end{pmatrix}$$

This implies that, the actual trial mix ratios for mix 13 are as follows: Water = 0.46%, Cement = 1%, Sand = 2.63% and Coarse aggregate = 5.50%. Similar calculations were made for the other points. Afterward, trail mixes were carried out based on the transformed components to mold the blocks, cylinders, and beams required for the laboratory test and optimization. On the other hand, the proportions of the components Z_i where gotten from the formula: $Z_i = \frac{R_i}{R_1+R_2+R_3+R_4}$ ………(***xi***), Where i = 1, 2, 3, 4 of the real components.The constituents were manually mixed in the laboratory and the results used for model optimization were based on the 28th-day test. All specimens were cured based on [11]. The experiment was conducted in Strength of Material Lab, Workshop five (5) Cross River University of Technology Calabar, Nigeria. Twenty 150mm X 150mm different cubes were molded to determine the compressive strength. This was determined by subjecting the hardened concrete to increasing compressive load until the point of failure, and determining the crushing force following [12].

IV. RESULTS AND DISCUSSION OF FINDINGS

A. *SCHEFFE'S PSEUDO COMPONENT MODEL.*

Table 1 and Table 2 show the estimated regression coefficients with the associated statistics and the Anova table respectively. Table 3 shows the observed strengths and the fitted values (predicted) along with the residuals.

a) Model equation

It is seen in Table 1 that both the linear and quadratic regression sources are significant at a 95% confidence limit since each has a p-value less than 0.05. The quadratic model is chosen since it is of a higher degree than the linear model. The estimated model coefficients are then as given in Table 1. Thus the coefficients of Scheffe's second-degree polynomial are given as:

$\beta_1 = 31.343,\quad \beta_2 = 27.044,\quad \beta_3 = 18.625,$
$\beta_4 = 14.431,\quad \beta_{12} = -10.548,$
$\beta_{13} = 3.384,\quad \beta_{14} = 7.225,$
$\beta_{23} = 14.252,\quad \beta_{24} = 12.501,$
$\beta_{34} = 2.224$

If we let the components cement, water, sand, and coarse aggregates (12% replacement of polystyrene beads with88% granite chippings)be represented respectively by X1, X2, X3, and X4, then the model equation in terms of pseudo units is:

$$\begin{aligned} Y &= 31.343X_1 + 27.044X_2 + 18.625X_3 + 14.431X_4 \\ &- 10.548X_1X_2 + 3.384X_1X_3 + 7.255X_1X_4 \\ &+ 14.352X_2X_3 + 12.501X_2X_4 \\ &+ 2.224X_3X_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (\boldsymbol{xii}) \end{aligned}$$

TABLE 1:
Estimated Regression Coefficients for Compressive strength(Scheffe's pseudo components model)

Model		**Unstandardized Coefficients**		**Standardized Coefficients**	T	Sig.
		B	**Std. Error**	**Beta**		
1	X1	31.343	3.286	.482	9.538	.000
	X2	27.044	3.269	.377	8.273	.000
	X3	18.625	3.131	.296	5.949	.000
	X4	14.431	3.141	.237	4.595	.001
	X1 * X2	-10.548	13.923	-.032	-.758	.466
	X1 * X3	3.384	13.290	.011	.255	.804
	X1 * X4	7.255	13.189	.023	.550	.594
	X2 * X3	14.352	14.183	.045	1.012	.335
	X2 * X4	12.501	13.846	.042	.903	.388
	X3 * X4	2.224	13.919	.006	.160	.876

TABLE 2:
Analysis of Variance for Compressive strength (Scheffe's pseudo component model)

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	11296.897	10	1129.690	101.265	.000c
	Residual	111.558	10	11.156		
	Total	11408.454d	20			

b) TEST FOR LACK-OF-FIT

Table 2 shows that there is an insignificant lack-of-fit, the p-value for lack-of-fit being 0.00 which is less than 0.05. The conclusion, therefore, is that Equation . (x) is adequate for predicting the 28th-day strength of expanded polystyrene concrete. The other statistics in Table 1, lend credence to the adequacy of the model

TABLE 3:
Residuals for compressive strength (Scheffe's pseudo component model)

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	14.430715	31.343069	23.410754	4.2029372	20
Residual	-4.0920582	8.2292471	.1028656	2.4208079	20
Std. Predicted Value	-2.137	1.887	.000	1.000	20
Std. Residual	-1.225	2.464	.031	.725	20

c) MODEL COMPRESSIVE STRENGTH FOR PSEUDO COMPONENT MODEL.

Data in Table 4and figure 1 shows the mathematically generated compressive strength of the polystyrene lightweight concrete using the beta values obtained from the Scheffe'spseudo component model. Amodel compressive strength of 31.343 N/mm^2 from mix ratio number 1 was obtained, this is 0.253 higher than the laboratory result. Other notable model values are 27.097, 27.044, 26.557, 26.423, 26.416, 25.830, 25.405, 24.701 and 24.698MPa obtained from mix number 11, 2, 5, 8, 15, 6, 14, 7 and 12 respectively.

TABLE 4:
Model compressive strength for the pseudo component model

S/N	X_1	X_2	X_3	X_4	X1 * X2	X1 * X3	X1 * X4	X2 * X3	X2 * X4	X3 * X4	Laboratory Result	Model Result
1	1	0	0	0	0	0	0	0	0	0	31.09	31.343
2	0	1	0	0	0	0	0	0	0	0	26.31	27.044
3	0	0	1	0	0	0	0	0	0	0	18.79	18.625
4	0	0	0	1	0	0	0	0	0	0	16	14.431
5	0.5	0.5	0	0	0.25	0	0	0	0	0	26.04	26.557
6	0.5	0	0.5	0	0	0.25	0	0	0	0	24.9	25.830
7	0.5	0	0	0.5	0	0	0.25	0	0	0	25.46	24.701
8	0	0.5	0.5	0	0	0	0	0.25	0	0	25.14	26.423
9	0	0.5	0	0.5	0	0	0	0	0.25	0	25.53	23.863
10	0	0	0.5	0.5	0	0	0	0	0	0.25	16.54	17.084
11	0.5	0.25	0.25	0	0.125	0.125	0	0.063	0	0	28.01	27.097
12	0.25	0.25	0.25	0.25	0.063	0.063	0.063	0.063	0.063	0.063	25.2	24.698
13	0	0.25	0	0.75	0	0	0	0	0.188	0	16.49	19.934
14	0.5	0	0.25	0.25	0	0.125	0.125	0	0	0.063	25.72	25.405
15	0.5	0.25	0	0.25	0.125	0	0.125	0	0.063	0	26.86	26.416
16	0	0.25	0.75	0	0	0	0	0.188	0	0	22.16	23.428
17	0	0.25	0.25	0.25	0	0	0	0.063	0.063	0.063	25.06	16.857
18	0.25	0.125	0.5	0.125	0.031	0.125	0.031	0.063	0.016	0.063	25.26	23.898
19	0.25	0.25	0	0.5	0.063	0	0.125	0	0.125	0	22.76	23.617
20	0.125	0.125	0.25	0.5	0.016	0.031	0.063	0.031	0.063	0.125	16.96	21.074

Source: Author's computation, 2020.

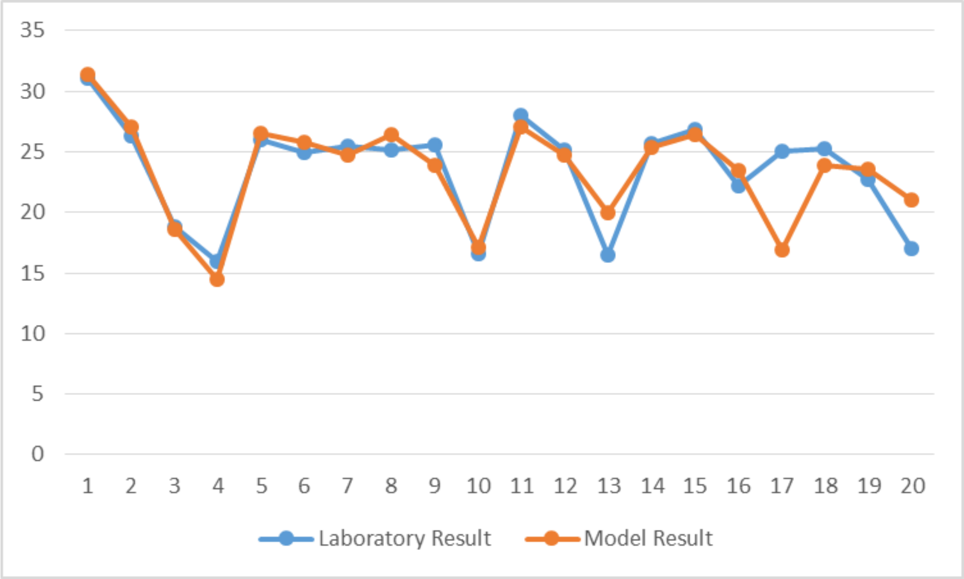

Figure 1: Graph showing the laboratory values against the mathematically optimized values of compressive strength of polystyrene lightweight concrete using the pseudo component model.

d) TEST OF HYPOTHESIS FOR THE PSEUDO COMPONENT MODEL

H_0: There is no significant difference between the laboratory compressive strength and the modelcompressive strength.

H_1: There is a significant difference between the laboratory compressive strength and the modelcompressive strength.

Calculated t value = 0.053

Table t value = 2.04

Decision: Since the table value (2.04) is greater than the calculated value (0.053), the alternate hypothesis (H_1) was rejected and (H_0) accepted at a 95% confidence level. Hence there is no significant difference between the laboratory results and the model generated values for the 28th-day tensile strength using Scheffe's pseudo component model.

e) OPTIMIZATION RESULT OF COMPRESSIVE STRENGTH USING SCHEFFE'S PSEUDO COMPONENT MODEL.

Table 5 shows the optimized mix ratios generated by the optimizer based on the pseudo value matrix. The optimized data shows that mix 125 will produce the highest strength of 27.920 N/mm^2 with corresponding water, cement, sand, and coarse aggregate (at 12% replacement) of 0.455, 1, 1.820, and 2.980 respectively. This optimized value of 27.920 N/mm^{2}conforms to the BS 206:2013 and ASTM C 39 standards. This implies that the optimized mix can produce a compressive strength that is suitable for residential structures at 12% partial replacement of the coarse aggregates. Other notable optimized compressive strength results were obtained from mix 124, 107, 123, 122, 106, 105, 121, 90 and 104 with a corresponding optimized compressive strength value of 27.900, 27.870, 27.870, 27.850, 27.840, 27.810, 27.790, 27.780 and 27.770 N/mm^2 respectively. However, all optimized mixes and compressive strength results as presented in Table 5 are all suitable for residential purposes as per [13] and [14].

TABLE 5:
Optimized polystyrene concrete mixtures and corresponding compressive strength using Scheffe's pseudo component model.

SN	Water (%)	Cement (%)	Sand (%)	C.A. (%)	C.S. (N/mm^2)	SN	Water (%)	Cement (%)	Sand (%)	C.A. (%)	C.S. (N/mm^2)
1	0.482	1.000	1.820	3.280	26.160	64	0.463	1.000	1.810	3.040	27.490
2	0.480	1.000	1.830	3.280	26.230	65	0.454	1.000	1.960	3.320	27.040
3	0.478	1.000	1.840	3.280	26.290	66	0.456	1.000	1.920	3.240	27.210
4	0.479	1.000	1.810	3.220	26.340	67	0.457	1.000	1.910	3.220	27.260
5	0.475	1.000	1.850	3.280	26.360	68	0.457	1.000	1.900	3.200	27.310
6	0.477	1.000	1.820	3.220	26.420	69	0.458	1.000	1.890	3.180	27.360
7	0.473	1.000	1.860	3.280	26.430	70	0.458	1.000	1.880	3.160	27.400
8	0.474	1.000	1.850	3.260	26.500	71	0.460	1.000	1.850	3.100	27.500
9	0.475	1.000	1.830	3.220	26.490	72	0.460	1.000	1.840	3.080	27.540
10	0.475	1.000	1.820	3.200	26.560	73	0.460	1.000	1.830	3.060	27.570
11	0.476	1.000	1.800	3.160	26.540	74	0.462	1.000	1.810	3.020	27.620
12	0.471	1.000	1.870	3.280	26.510	75	0.462	1.000	1.800	3.000	27.650
13	0.471	1.000	1.860	3.260	26.580	76	0.452	1.000	1.970	3.320	27.130
14	0.473	1.000	1.840	3.220	26.580	77	0.453	1.000	1.960	3.300	27.180
15	0.473	1.000	1.830	3.200	26.640	78	0.453	1.000	1.950	3.280	27.230
16	0.474	1.000	1.810	3.160	26.630	79	0.453	1.000	1.940	3.260	27.280
17	0.475	1.000	1.800	3.140	26.690	80	0.454	1.000	1.930	3.240	27.330
18	0.469	1.000	1.880	3.280	26.590	81	0.455	1.000	1.910	3.200	27.400
19	0.469	1.000	1.870	3.260	26.650	82	0.455	1.000	1.900	3.180	27.440
20	0.471	1.000	1.840	3.200	26.720	83	0.456	1.000	1.890	3.160	27.480
21	0.472	1.000	1.810	3.140	26.780	84	0.456	1.000	1.880	3.140	27.520
22	0.467	1.000	1.890	3.280	26.670	85	0.457	1.000	1.870	3.120	27.560
23	0.467	1.000	1.880	3.260	26.730	86	0.457	1.000	1.860	3.100	27.600
24	0.469	1.000	1.850	3.200	26.810	87	0.458	1.000	1.840	3.060	27.660
25	0.469	1.000	1.840	3.180	26.870	88	0.459	1.000	1.830	3.040	27.690
26	0.470	1.000	1.820	3.140	26.870	89	0.459	1.000	1.820	3.020	27.730
27	0.471	1.000	1.810	3.120	26.930	90^{9}	0.460	1.000	1.800	2.980	27.780
28	0.472	1.000	1.790	3.080	26.920	91	0.450	1.000	1.980	3.320	27.220
29	0.464	1.000	1.900	3.280	26.750	92	0.450	1.000	1.970	3.300	27.270
30	0.465	1.000	1.890	3.260	26.810	93	0.451	1.000	1.960	3.280	27.320

31	0.465	1.000	1.880	3.240	26.870	94	0.451	1.000	1.950	3.260	27.370
32	0.466	1.000	1.860	3.200	26.890	95	0.452	1.000	1.940	3.240	27.410
33	0.467	1.000	1.850	3.180	26.950	96	0.452	1.000	1.930	3.220	27.450
34	0.467	1.000	1.840	3.160	27.010	97	0.452	1.000	1.920	3.200	27.490
35	0.468	1.000	1.820	3.120	27.020	98	0.453	1.000	1.910	3.180	27.530
36	0.469	1.000	1.810	3.100	27.070	99	0.453	1.000	1.900	3.160	27.570
37	0.470	1.000	1.790	3.060	27.070	100	0.454	1.000	1.880	3.120	27.640
38	0.462	1.000	1.910	3.280	26.840	101	0.455	1.000	1.870	3.100	27.680
39	0.463	1.000	1.900	3.260	26.900	102	0.455	1.000	1.860	3.080	27.710
40	0.463	1.000	1.890	3.240	26.950	103	0.456	1.000	1.850	3.060	27.740
41	0.465	1.000	1.860	3.180	27.040	104^{10}	0.456	1.000	1.840	3.040	27.770
42	0.465	1.000	1.850	3.160	27.090	105^{7}	0.457	1.000	1.830	3.020	27.810
43	0.467	1.000	1.820	3.100	27.160	106^{6}	0.457	1.000	1.820	3.000	27.840
44	0.467	1.000	1.810	3.080	27.210	107^{3}	0.458	1.000	1.810	2.980	27.870
45	0.460	1.000	1.920	3.280	26.920	108	0.448	1.000	1.990	3.320	27.320
46	0.460	1.000	1.910	3.260	26.980	109	0.448	1.000	1.980	3.300	27.370
47	0.461	1.000	1.900	3.240	27.040	110	0.449	1.000	1.970	3.280	27.410
48	0.461	1.000	1.890	3.220	27.090	111	0.449	1.000	1.960	3.260	27.450
49	0.463	1.000	1.860	3.160	27.180	112	0.449	1.000	1.950	3.240	27.500
50	0.463	1.000	1.850	3.140	27.230	113	0.450	1.000	1.940	3.220	27.530
51	0.464	1.000	1.840	3.120	27.280	114	0.450	1.000	1.930	3.200	27.570
52	0.465	1.000	1.820	3.080	27.310	115	0.451	1.000	1.920	3.180	27.610
53	0.465	1.000	1.810	3.060	27.350	116	0.451	1.000	1.910	3.160	27.640
54	0.466	1.000	1.790	3.020	27.370	117	0.451	1.000	1.900	3.140	27.680
55	0.458	1.000	1.920	3.260	27.070	118	0.452	1.000	1.890	3.120	27.710
56	0.459	1.000	1.910	3.240	27.120	119	0.452	1.000	1.880	3.100	27.740
57	0.459	1.000	1.900	3.220	27.180	120	0.453	1.000	1.870	3.080	27.760
58	0.459	1.000	1.890	3.200	27.230	121^{8}	0.453	1.000	1.860	3.060	27.790
59	0.460	1.000	1.880	3.180	27.270	122^{5}	0.454	1.000	1.850	3.040	27.850
60	0.461	1.000	1.860	3.140	27.320	123^{4}	0.454	1.000	1.840	3.020	27.870
61	0.461	1.000	1.850	3.120	27.360	124^{2}	0.455	1.000	1.830	3.000	27.900
62	0.462	1.000	1.840	3.100	27.410	125^{1}	0.455	1.000	1.820	2.980	27.920
63	0.463	1.000	1.820	3.060	27.440						

B. SCHEFFE'S COMPONENT PROPORTION MODEL

The estimated regression coefficients for the components proportion model are given in Table 6 while the Anova table is presented in Table 7.

TABLE 6:
Estimated Regression Coefficients for Compressive strength (Scheffe's Component proportion model)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
	Z1 * Z2	-2974.527	960.365	-1.043	-3.097	.007
	Z1 * Z3	4304.017	1245.223	3.084	3.456	.004
1	Z1 * Z4	230.796	287.997	.300	.801	.435
	Z2 * Z3	-566.429	454.686	-.879	-1.246	.232
	Z3 * Z4	-79.677	43.924	-.501	-1.814	.090

TABLE 7:
Analysis of Variance for Compressive strength (Scheffe's component proportion model)

Model		Sum of Squares	df	Mean Square	F	Sig.
	Regression	11360.639	5	2272.128	712.785	.000c
1	Residual	47.815	15	3.188		
	Total	11408.454d	20			

a) MODEL EQUATION

From Table 6 the estimated coefficients for the Scheffe's second degree polynomial are given as:

$$\beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 0\,, \quad \beta_4 = 0,$$
$$\beta_{12} = -2974.527,$$
$$\beta_{13} = 4304.017\,,$$
$$\beta_{14} = 230.796,$$
$$\beta_{23} = -566.429\,, \quad \beta_{24} = 0,$$
$$\beta_{34} = \ -79.677$$

If we let the components' proportions of cement, water, sand, and Coarse aggregates (12% replacement of polystyrene beads and 88% granite chippings) be represented respectively by Z1, Z2, Z3, and Z4, then the model equation in terms of components' proportions is:

$$\begin{aligned} Y &\\ = {} & -2974.527Z_1Z_2 + 4304.017Z_1Z_3 \\ & + 230.796Z_1Z_4 - 566.429Z_2Z_3 \\ & - 79.677Z_3Z_4 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\,(\boldsymbol{xiii}) \end{aligned}$$

This model suggests that components $Z_1, Z_3,\ and Z_4$ themselves contribute nothing to the response of the mixture. Similarly, components $Z_2Z_3,\ and Z_2Z_4$ do not also contribute to the response.

TABLE 8:
Residuals for compressive strength (Scheffe's component proportion model)

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	16.282448	31.237665	23.513605	3.9924089	20
Residual	-2.9450693	2.7992806	.0000149	1.5863758	20
Std. Predicted Value	-1.811	1.935	.000	1.000	20
Std. Residual	-1.650	1.568	.000	.889	20

b) MODEL COMPRESSIVE STRENGTH FOR COMPONENT PROPORTIONAL MODEL.

Data in Table 9and Figure 2 shows the model compressive strength of the polystyrene lightweight concrete using Scheffe's component proportional model. The modeled results ranged between 16.96MPa and 27.99MPa, however, the actual mixes (first 10 mixes) showed better results than the control mixes (last 10 mixes) at mix 1, mix 5 and mix 2. Although the actual mixes gave better results, the results of the control mix also conform to the standards of [13] and [14] in mix 11, 15, 14, 12, 18, 19, 17, and 20 for residential structures.

TABLE 9:
Mathematically optimized compressive strength for component proportional model

S/N	Z_1	Z_2	Z_3	Z_4	Z1 * Z2	Z1 * Z3	Z1 * Z4	Z2 * Z3	Z2 * Z4	Z3 * Z4	Laboratory Results	Model Results
1	0.091	0.202	0.303	0.404	0.018	0.028	0.037	0.061	0.082	0.122	31.09	27.99
2	0.067	0.133	0.267	0.533	0.009	0.018	0.036	0.036	0.071	0.142	26.31	26.81
3	0.051	0.112	0.279	0.558	0.006	0.014	0.029	0.031	0.062	0.156	18.79	21.18
4	0.042	0.096	0.287	0.575	0.004	0.012	0.024	0.028	0.055	0.165	16	16.96
5	0.076	0.161	0.281	0.482	0.012	0.021	0.037	0.045	0.077	0.135	26.04	27.98
6	0.065	0.144	0.288	0.503	0.009	0.019	0.033	0.041	0.072	0.145	24.9	25.64
7	0.058	0.130	0.292	0.520	0.008	0.017	0.030	0.038	0.068	0.152	25.46	23.73
8	0.058	0.122	0.273	0.547	0.007	0.016	0.032	0.033	0.066	0.149	25.14	24.18
9	0.052	0.111	0.279	0.557	0.006	0.015	0.029	0.031	0.062	0.155	25.53	22.24
10	0.046	0.103	0.284	0.567	0.005	0.013	0.026	0.029	0.058	0.161	16.54	19.09
11	0.071	0.152	0.285	0.493	0.011	0.020	0.035	0.043	0.075	0.140	28.01	26.96
12	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.035	0.067	0.151	25.2	23.98
13	0.047	0.103	0.283	0.567	0.005	0.013	0.027	0.029	0.058	0.161	16.49	19.61

14	0.061	0.137	0.290	0.512	0.008	0.018	0.031	0.040	0.070	0.149	25.72	24.75
15	0.066	0.144	0.287	0.503	0.009	0.019	0.033	0.041	0.072	0.145	26.86	26.27
16	0.055	0.116	0.276	0.553	0.006	0.015	0.030	0.032	0.064	0.153	22.16	22.71
17	0.052	0.112	0.279	0.558	0.006	0.015	0.029	0.031	0.062	0.155	25.06	21.89
18	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.036	0.067	0.151	25.26	23.69
19	0.055	0.120	0.285	0.540	0.007	0.016	0.030	0.034	0.065	0.154	22.76	22.96
20	0.050	0.111	0.284	0.555	0.006	0.014	0.028	0.032	0.062	0.158	16.96	20.98

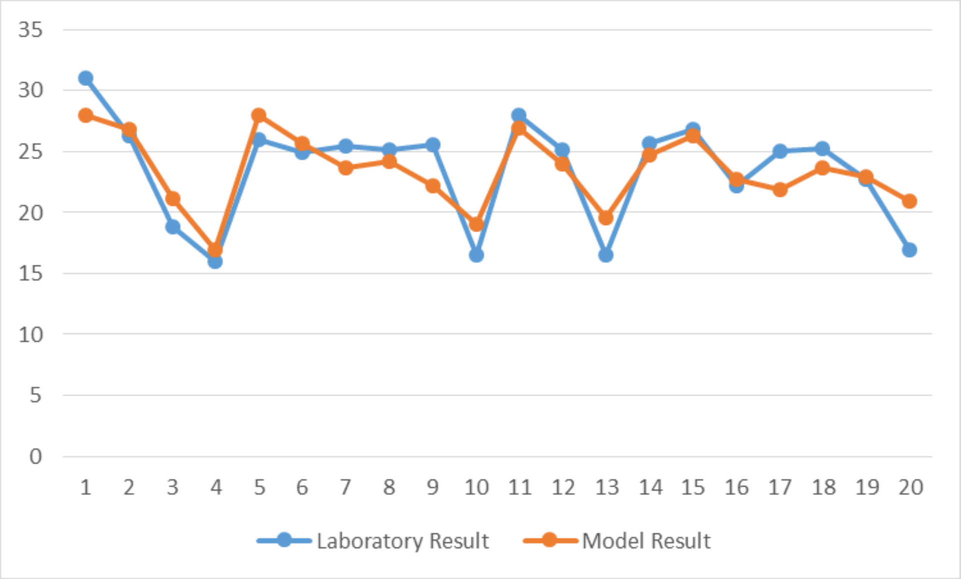

Figure 2: Graph showing the laboratory values against the mathematically optimized values of compressive strength of polystyrene lightweight concrete using the component proportion model.

c) TEST OF HYPOTHESIS FOR THE COMPONENT PROPORTION MODEL.

H_0: There is no significant difference between the laboratory compressive strength and the model compressive strength.

H_1: There is a significant difference between the laboratory compressive strength and the model compressive strength.

Calculated t value = 0.052

Table t value = 2.04

Decision: Since the table value (2.04) is greater than the calculated value (0.052), the alternate hypothesis (H_1) was rejected and (H_0) accepted at a 95% confidence level. Hence there is no significant difference between the laboratory results and the mathematically optimized results for the 28th-day tensile strength using Scheffe's component proportion model.

d) OPTIMIZATION RESULT FOR SCHEFFE'S COMPONENT PROPORTION MODEL.

Data in Table 10 shows the optimized mix ratios generated by the optimizer based on the component proportion value matrix. The optimized data shows that mix 1 will produce the highest strength of 27.550 N/mm^2 with corresponding water, cement, sand, and coarse aggregate (at 12% replacement) of 0.482, 1, 1.850, and 3.360 respectively. This optimized value of 27.550 N/mm^2 conforms to the standards of [13] and [14]. This implies that the optimized mix can produce a compressive strength that is suitable for residential structures at 12% partial replacement of the coarse aggregates. Other notable optimized compressive strength results were obtained from mix 4, 6, 3, 14, 9, 5, 2, 13 and 8 with a corresponding optimized compressive strength value of 27.550, 27.520, 27.510, 27.500, 27.490, 27.480, 27.460, 27.450 and 27.440 N/mm^2 respectively. Just as in the pseudo model, all optimized mixes and corresponding compressive strength results as presented in Table 10 are all suitable for residential purposes as per [13] and [14]. Also, these mixes are very useful in creating blocks for partitions in high rising buildings.

TABLE 10:
Optimized polystyrene concrete mixtures and corresponding compressive strength using Scheffe's component proportion model.

SN	Water (%)	Cement (%)	Sand (%)	C. A. (%)	C.S. (N/mm^2)	SN	Water (%)	Cement (%)	Sand (%)	C. A. (%)	C.S. (N/mm^2)
1^{1}	0.482	1	1.85	3.360	27.55	73	0.459	1	1.87	3.160	26.98
2^{8}	0.48	1	1.85	3.340	27.46	74	0.46	1	1.86	3.140	27.02
3^{4}	0.48	1	1.85	3.340	27.51	75	0.46	1	1.87	3.160	27.03
4^{2}	0.48	1	1.84	3.320	27.55	76	0.46	1	1.87	3.160	27.08
5^{7}	0.478	1	1.85	3.320	27.48	77	0.461	1	1.87	3.160	27.13
6^{3}	0.479	1	1.85	3.320	27.52	78	0.461	1	1.88	3.180	27.13
7	0.476	1	1.85	3.300	27.39	79	0.461	1	1.87	3.160	27.18
8^{10}	0.476	1	1.85	3.300	27.44	80	0.461	1	1.87	3.160	27.23
9^{6}	0.477	1	1.85	3.300	27.49	81	0.457	1	1.86	3.120	26.87
10	0.474	1	1.85	3.280	27.35	82	0.457	1	1.87	3.140	26.88
11	0.475	1	1.85	3.280	27.40	83	0.458	1	1.86	3.120	26.92
12	0.475	1	1.86	3.300	27.41	84	0.458	1	1.87	3.140	26.93
13^{9}	0.475	1	1.85	3.280	27.45	85	0.458	1	1.86	3.120	26.97
14^{5}	0.475	1	1.85	3.280	27.50	86	0.458	1	1.87	3.140	26.98
15	0.472	1	1.85	3.260	27.31	87	0.458	1	1.87	3.140	27.03
16	0.472	1	1.86	3.280	27.32	88	0.458	1	1.88	3.160	27.04
17	0.473	1	1.85	3.260	27.36	89	0.459	1	1.87	3.140	27.08
18	0.473	1	1.86	3.280	27.37	90	0.459	1	1.88	3.160	27.09
19	0.473	1	1.85	3.260	27.41	91	0.459	1	1.87	3.140	27.14
20	0.473	1	1.86	3.280	27.42	92	0.459	1	1.88	3.160	27.14
21	0.47	1	1.86	3.260	27.23	93	0.46	1	1.87	3.140	27.19
22	0.471	1	1.85	3.240	27.27	94	0.455	1	1.87	3.120	26.78
23	0.471	1	1.86	3.260	27.28	95	0.455	1	1.86	3.100	26.82
24	0.471	1	1.85	3.240	27.32	96	0.455	1	1.87	3.120	26.83
25	0.471	1	1.86	3.260	27.33	97	0.456	1	1.86	3.100	26.87
26	0.471	1	1.86	3.260	27.38	98	0.456	1	1.87	3.120	26.88
27	0.468	1	1.86	3.240	27.14	99	0.456	1	1.87	3.120	26.93
28	0.468	1	1.86	3.240	27.19	100	0.457	1	1.87	3.120	26.99
29	0.469	1	1.85	3.220	27.23	101	0.457	1	1.88	3.140	26.99
30	0.469	1	1.86	3.240	27.24	102	0.457	1	1.87	3.120	27.04
31	0.469	1	1.85	3.220	27.28	103	0.457	1	1.88	3.140	27.04
32	0.469	1	1.86	3.240	27.29	104	0.457	1	1.87	3.120	27.09
33	0.47	1	1.86	3.240	27.34	105	0.457	1	1.88	3.140	27.09
34	0.47	1	1.86	3.240	27.39	106	0.453	1	1.87	3.100	26.73
35	0.466	1	1.86	3.220	27.10	107	0.454	1	1.86	3.080	26.77
36	0.467	1	1.86	3.220	27.15	108	0.454	1	1.87	3.100	26.78
37	0.467	1	1.85	3.200	27.19	109	0.454	1	1.87	3.100	26.83
38	0.467	1	1.86	3.220	27.20	110	0.454	1	1.87	3.100	26.88
39	0.467	1	1.86	3.220	27.25	111	0.454	1	1.88	3.120	26.89
40	0.467	1	1.87	3.240	27.26	112	0.455	1	1.87	3.100	26.94
41	0.468	1	1.86	3.220	27.30	113	0.455	1	1.88	3.120	26.94
42	0.468	1	1.87	3.240	27.31	114	0.455	1	1.87	3.100	26.99
43	0.468	1	1.86	3.220	27.35	115	0.455	1	1.88	3.120	26.99
44	0.464	1	1.86	3.200	27.05	116	0.456	1	1.88	3.120	27.05
45	0.465	1	1.86	3.200	27.10	117	0.456	1	1.88	3.120	27.10
46	0.465	1	1.86	3.200	27.15	118	0.452	1	1.87	3.080	26.78
47	0.466	1	1.86	3.200	27.20	119	0.452	1	1.88	3.100	26.79
48	0.466	1	1.87	3.220	27.21	120	0.453	1	1.87	3.080	26.84
49	0.466	1	1.86	3.200	27.26	121	0.453	1	1.88	3.100	26.84
50	0.466	1	1.87	3.220	27.26	122	0.453	1	1.87	3.080	26.89
51	0.466	1	1.86	3.200	27.31	123	0.453	1	1.88	3.100	26.89
52	0.463	1	1.86	3.180	27.01	124	0.453	1	1.88	3.100	26.94
53	0.463	1	1.86	3.180	27.06	125	0.454	1	1.88	3.100	27.00
54	0.463	1	1.86	3.180	27.11	126	0.454	1	1.89	3.120	27.00
55	0.463	1	1.87	3.200	27.12	127	0.454	1	1.88	3.100	27.05
56	0.464	1	1.86	3.180	27.16	128	0.451	1	1.88	3.080	26.79
57	0.464	1	1.87	3.200	27.17	129	0.451	1	1.87	3.060	26.84
58	0.464	1	1.86	3.180	27.21	130	0.451	1	1.88	3.080	26.84

59	0.464	1	1.87	3.200	27.22	131	0.452	1	1.88	3.080	26.90
60	0.465	1	1.87	3.200	27.27	132	0.452	1	1.88	3.080	26.95
61	0.461	1	1.86	3.160	26.96	133	0.452	1	1.89	3.100	26.95
62	0.461	1	1.86	3.160	27.01	134	0.452	1	1.88	3.080	27.00
63	0.461	1	1.87	3.180	27.02	135	0.453	1	1.88	3.080	27.06
64	0.462	1	1.86	3.160	27.07	136	0.45	1	1.88	3.060	26.84
65	0.462	1	1.87	3.180	27.07	137	0.45	1	1.89	3.080	26.84
66	0.462	1	1.86	3.160	27.12	138	0.45	1	1.88	3.060	26.90
67	0.462	1	1.87	3.180	27.12	139	0.45	1	1.89	3.080	26.90
68	0.462	1	1.87	3.180	27.18	140	0.451	1	1.88	3.060	26.95
69	0.463	1	1.87	3.180	27.23	141	0.451	1	1.89	3.080	26.95
70	0.459	1	1.86	3.140	26.92	142	0.448	1	1.89	3.060	26.85
71	0.459	1	1.87	3.160	26.92	143	0.449	1	1.89	3.060	26.90
72	0.459	1	1.86	3.140	26.97	144	0.447	1	1.89	3.040	26.90

V. CONCLUSION AND RECOMMENDATION

The trial mixes error method has not been efficient because of the associated complexity in identifying optimum mix proportion, especially when more components are involved as in the case of partial replacement of aggregate with Expanded Polystyrene beads. Therefore the use of a mathematical model has proved to be more efficient and accurate as shown in this research. Although, as shown in this study, the highest compressive strength results obtained from the component proportion model of 27.55 N/mm^{2}seemslightly lesser to that obtained from the pseudo component model with a compressive strength of 27.920 N/mm^2. With these models, the mix proportions for the desired lightweight concrete performance can easily be replicated without any further trial mixes. However, it is recommended that further studies should be carried out with larger mix ratios, in other to ascertain the best optimized compressive strength for polystyrene lightweight concrete that can achieve a result of 40 N/mm^2 and above.

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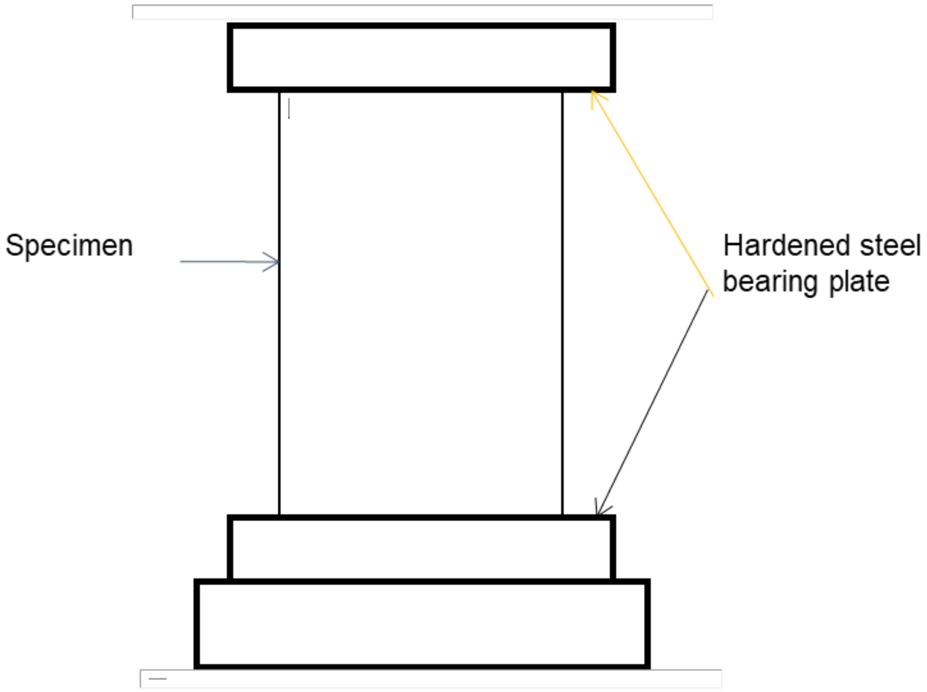
Appendix 1a: Uniaxial compressive strength test of concrete.



Appendix 1b: Determination of the compressive strength of EPS concrete

Appendix 2a: Actual (Zi) and Pseudo (xi)components for Scheffe's (4, 2) Simplex Lattice

S/N	X_1	X_2	X_3	X_4	Response	R_1	R_2	R_3	R_4
1.	1	0	0	0	Y_1	0.45	0.50	0.46	0.44
2.	0	1	0	0	Y_2	1	1	1	1
3.	0	0	1	0	Y_3	1.5	2.0	2.5	3.0
4.	0	0	0	1				5.0	6.0
5.	½	½	0	0		`		2.75	3.5
6.	½	0	½	0				2.0	5.0
7.	½	0	0	½	Y_{14}	0.445	1	2.25	4.5
8.	0	½	½	0	Y_{23}	0.48	1	2.25	4.5
9.	0	½	0	½	Y_{24}	0.47	1	2.5	4.5
10.	0	0	½	½	Y_{34}	0.45	1	2.75	5.5

Appendix 2b: Control Points Actual (xi) and Pseudo (zi)components for Scheffe's (4, 2) Simplex Lattice

S/N	X_1	X_2	X_3	X_4	Response	R_1	R_2	R_3	R_4
11.	½	1/4	¼	0	C_1	0.465	1	1.88	3.75
12.	1/4	1/4	¼	1/4	C_2	0.463	1	2.25	4.5
13.	0	1/4	0	3/4	C_3	0.46	1	2.63	5.5
14.	½	0	¼	1/4	C_4	0.48	1	2.13	4.25
15.	½	1/4	0	1/4	C_5	0.46	1	2.0	4.0
16.	0	1/4	¾	0	C_6	0.47	1	2.38	4.75
17.	0	½	¼	1/4	C_7	0.475	1	2.13	4.75
18.	1/4	1/8	½	1/8	C_8	0.46	1	2.25	4.50
19.	1/4	1/4	0	½	C_9	0.458	1	2.38	4.75
20.	1/8	1/8	¼	½	C_{10}	0.454	1	2.56	5.13

APPENDIX 3a:

Test of Hypothesis for the Pseudo Component Model using the t-Test

$$t = \frac{\dot{X}-\dot{Y}}{\sqrt{\frac{\sigma X^2}{Nx}+\frac{\sigma Y^2}{Ny}}}$$

Where;

t = t-test; X = Laboratory compressive strength; Y = Pseudo component model compressive strength

σX^2 = Variance of X; σY^2 = Variance of Y; Nx = Sample size of X; Ny = Sample size of Y

S/N	Laboratory Tensile Strength (X)	Model Tensile Strength (Y)	$(X-\dot{X})$	$(Y-\dot{Y})$	$(X-\dot{X})^2$	$(Y-\dot{Y})^2$
Total	453.32	447.251	22.666	22.36255	332.498	355.962
Mean	22.666	22.36255		Variance	17.500	18.735

$$t = \frac{22.666-22.363}{\sqrt{\frac{17.500^2}{20}+\frac{18.735^2}{20}}}$$

$$t = \frac{0.303}{\sqrt{\frac{306.25}{20}+\frac{351}{20}}} \quad = \quad t = \frac{0.303}{\sqrt{15.313+17.55}}$$

$$t = \frac{0.303}{\sqrt{15.313 + 17.55}} = t = \frac{0.303}{\sqrt{32.863}}$$

$$t = \frac{0.303}{5.733}$$

Calculated t = 0.053

DF = 40 -2 = 38 at 95% confidence level

Table Value = 2.04

APPENDIX 3b:

Test of Hypothesis for the Component Proportion Model using the t-Test

$$t = \frac{\dot{X} - \dot{Y}}{\sqrt{\frac{\sigma X^2}{Nx} + \frac{\sigma Y^2}{Ny}}}$$

Where;

t = t-test; X = Laboratory tensile strength; Y = Pseudo component model tensile strength

σX^2 = Variance of X; σY^2 = Variance of Y; Nx = Sample size of X; Ny = Sample size of Y

S/N	Laboratory Tensile Strength (X)	Model Tensile Strength (Y)	$(X-\dot{X})$	$(Y-\dot{Y})$	$(X-\dot{X})^2$	$(Y-\dot{Y})^2$
Total	453.32	448.62	22.666	22.431	332.498	191.830
Mean	22.666	22.431		Variance	17.500	10.096

$$t = \frac{22.666 - 22.431}{\sqrt{\frac{17.500^2}{20} + \frac{10.096^2}{20}}}$$

$$t = \frac{0.235}{\sqrt{\frac{306.25}{20} + \frac{101.929}{20}}} = t = \frac{0.235}{\sqrt{15.313 + 5.096}}$$

$$t = \frac{0.235}{\sqrt{20.409}} = t = \frac{0.235}{4.518}$$

Calculated t = 0.052

DF = 40 -2 = 38 at 95% confidence level

Table Value = 2.04

Appendix 3: Scheffe’s {4,2} lattice simplex matrix and laboratory compressive strength data.

S/N	X_1	X_2	X_3	X_4	X1 * X2	X1 * X3	X1 * X4	X2 * X3	X2 * X4	X3 * X4	R1	R2	R3	R4	Z_1	Z_2	Z_3	Z_4	Z1 * Z2	Z1 * Z3	Z1 * Z4	Z2 * Z3	Z2 * Z4	Z3 * Z4	Compressive Strength
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.45	1.00	1.50	2.00	0.091	0.202	0.303	0.404	0.018	0.028	0.037	0.061	0.082	0.122	31.09
2	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.50	1.00	2.00	4.00	0.067	0.133	0.267	0.533	0.009	0.018	0.036	0.036	0.071	0.142	26.31
3	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.46	1.00	2.50	5.00	0.051	0.112	0.279	0.558	0.006	0.014	0.029	0.031	0.062	0.156	18.79
4	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.44	1.00	3.00	6.00	0.042	0.096	0.287	0.575	0.004	0.012	0.024	0.028	0.055	0.165	16.00
5	0.500	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.48	1.00	1.75	3.00	0.076	0.161	0.281	0.482	0.012	0.021	0.037	0.045	0.077	0.135	26.04
6	0.500	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.46	1.00	2.00	3.50	0.065	0.144	0.288	0.503	0.009	0.019	0.033	0.041	0.072	0.145	24.90
7	0.500	0.000	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.45	1.00	2.25	4.00	0.058	0.130	0.292	0.520	0.008	0.017	0.030	0.038	0.068	0.152	25.46
8	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.48	1.00	2.25	4.50	0.058	0.122	0.273	0.547	0.007	0.016	0.032	0.033	0.066	0.149	25.14
9	0.000	0.500	0.000	0.500	0.000	0.000	0.000	0.000	0.250	0.000	0.47	1.00	2.50	5.00	0.052	0.111	0.279	0.557	0.006	0.015	0.029	0.031	0.062	0.155	25.53
10	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.250	0.45	1.00	2.75	5.50	0.046	0.103	0.284	0.567	0.005	0.013	0.026	0.029	0.058	0.161	16.54
11	0.500	0.250	0.250	0.000	0.125	0.125	0.000	0.063	0.000	0.000	0.47	1.00	1.88	3.25	0.071	0.152	0.285	0.493	0.011	0.020	0.035	0.043	0.075	0.140	28.01
12	0.250	0.250	0.250	0.250	0.063	0.063	0.063	0.063	0.063	0.063	0.46	1.00	2.25	4.25	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.035	0.067	0.151	25.20
13	0.000	0.250	0.000	0.750	0.000	0.000	0.000	0.000	0.188	0.000	0.46	1.00	2.75	5.50	0.047	0.103	0.283	0.567	0.005	0.013	0.027	0.029	0.058	0.161	16.49
14	0.500	0.000	0.250	0.250	0.000	0.125	0.125	0.000	0.000	0.063	0.45	1.00	2.13	3.75	0.061	0.137	0.290	0.512	0.008	0.018	0.031	0.040	0.070	0.149	25.72
15	0.500	0.250	0.000	0.250	0.125	0.000	0.125	0.000	0.063	0.000	0.46	1.00	2.00	3.50	0.066	0.144	0.287	0.503	0.009	0.019	0.033	0.041	0.072	0.145	26.86
16	0.000	0.250	0.750	0.000	0.000	0.000	0.000	0.188	0.000	0.000	0.47	1.00	2.38	4.75	0.055	0.116	0.276	0.553	0.006	0.015	0.030	0.032	0.064	0.153	22.16
17	0.000	0.250	0.250	0.250	0.000	0.000	0.000	0.063	0.063	0.063	0.35	0.75	1.88	3.75	0.052	0.112	0.279	0.558	0.006	0.015	0.029	0.031	0.062	0.155	25.06
18	0.250	0.125	0.500	0.125	0.031	0.125	0.031	0.063	0.016	0.063	0.46	1.00	2.25	4.25	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.036	0.067	0.151	25.26
19	0.250	0.250	0.000	0.500	0.063	0.000	0.125	0.000	0.125	0.000	0.46	1.00	2.38	4.50	0.055	0.120	0.285	0.540	0.007	0.016	0.030	0.034	0.065	0.154	22.76
20	0.125	0.125	0.250	0.500	0.016	0.031	0.063	0.031	0.063	0.125	0.45	1.00	2.56	5.00	0.050	0.111	0.284	0.555	0.006	0.014	0.028	0.032	0.062	0.158	16.96

Source: Author’s research work