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Application of Polynomial Deflection Expression in Free-Vibration Study of Thick Rectangular Plates

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Abstract:
Free-Vibration Study of Thick Rectangular Plates using Polynomial deflection expression was investigated in this study. Three different boundary conditions of rectangular plates were studied, they are; rectangular plates with opposite edges clamped and the other opposite edges having simple supports designated as CSCS, rectangular plate with a fixed support at one edge and simple support at the other three edges designated as CSSS, and rectangular plate with simple support at one edge and fixed at the other three edges designated as CCCS. A polynomial expression was used as the deflection equation to satisfy the various boundary conditions of the plate to obtain numerical values of the stiffness coefficients of the plate. These values were substituted into a simple analytical equation which yields the non-dimensional frequency parameters for the plates at any value of the span-depth ratio (a/t) and in-plane dimensions ratio (b/a). The values of the non-dimensional frequency parameter obtained from the present work when compared with the results of previous researchers on a similar subject were observed to be in good agreement. Thus, the present work offers a quick and satisfactory approach to the free-vibration analysis of thick plates.

Keywords: *Thick plates, polynomial expression, non-dimensional frequency parameter, in-plane dimensions, boundary conditions.*

I. INTRODUCTION

An increase in the use of structural thick plate elements in several engineering works has necessitated the need for a comprehensive study of the structural behavior of thick plates. Structural plate elements are occasionally subjected to loads that vary with time which could have a devastating effect on the structure. When the frequency of the time-dependent load coincides with one of the natural frequencies of the plate, a phenomenon known as resonance occurs. At resonance, very large amplitude deformations occur in the structure leading to its failure. Therefore, it is of utmost importance to carry out free vibration study on plates so as to determine these frequencies that could cause resonance in the plate structure. One of the difficult tasks in the analysis of plates is the determination of an expression for the deformed shape of the plate that will satisfy different boundary conditions of the plate. Some researchers in the past used trigonometric functions, some used exponential functions, hyperbolic functions while others used polynomial functions. In modelling of thick plates, Shear deformation theories which takes into account the effect of transverse stresses and strains are employed [1].Many researchers developed higher order shear deformation theories by involving the effects of transverse stresses and strains to improve the accuracy of their results [2]. Several researchers have in the past worked on thick plates using various methods.[3] carried out free vibration study of moderately thick plates through analytical approach by reducing the governing equations of force-displacement expression and equilibrium of forces into three partial differential equations of motion.[4] carried out vibration study of rectangular tick plates resting on elastic foundations with different boundary conditions. In their work, they used a combination of trigonometric and polynomial functions as displacement functions.[5] applied third order shear deformation theory in free vibration study of rectangular thick plates with opposite edges simply supported to obtain exact solutions for the plate. In their work, they applied Hamilton's principle to derive the equations of motion and natural boundary conditions of the plate and also, they used a combination of trigonometric and hyperbolic functions as their displacement functions. [6] derived exact characteristic equations for vibrating moderately thick rectangular plates of classical boundary conditions by

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using a combination of trigonometric and hyperbolic functions as their displacement functions. [7] obtained a simple linear equation based on higher order shear deformation theory for free vibration analysis of thick rectangular plates by making use of polynomial displacement functions. [8] developed exact displacement functions for general analysis of thick rectangular plates by carrying out a direct integration of the general governing differential equation of thick plates.[9] studied free vibration of thick rectangular plates with the following boundary conditions; one with all edges clamped and another one with adjacent edges clamped and the other adjacent edges simply supported. [10] studied the free vibration of thick rectangular plates simply supported at all edges by making use of trigonometric displacement functions. [11] applied Fourier series on first-order shear deformation theory to carry out free vibration study on a moderately thick rectangular plate. [12] studied stability and vibration analysis of thick rectangular plates by using polynomial expressions as displacement equations and shear deformation equations. [13] applied a higher-order shear deformation theory with eight unknowns in the study of vibration and buckling analysis of functionally graded plates. Buckling analysis of thick rectangular plates using polynomial displacement functions was studied by [14]. [15] used triangular elements method to carry out general analysis on stiffened plates. Bending analysis of thick rectangular plates using higher-order shear deformation theory was studied by [16]. In their work, they made use of the polynomial expression as the displacement and shear deformation functions. Post buckling study of rectangular plates was carried out by [17] using an exact method. In the present work, a polynomial deflection function derived by [8] was used to satisfy the various boundary conditions treated to obtain the stiffness values which were substituted into a simple linear equation for vibration analysis of thick plates derived by [7] to obtain the dimensionless frequency parameters for the plate.

II. ANALYTICAL EQUATION

A linear equation based on higher-order shear deformation theory for analysis of thick rectangular plates was derived by [7]. This equation was used in this work and is presented here as;

$$S_{11} + S_{12}.\left[\frac{-S_{23}.S_{31} + S_{33}.S_{21}}{{S_{32}}^2 - S_{33}.S_{22}}\right] + S_{13}.\left[\frac{-S_{23}.S_{21} + S_{22}.S_{31}}{{S_{32}}^2 - S_{33}.S_{22}}\right] = \frac{ma^4\lambda^2}{D} = \Delta^2 \quad (1)$$

Where;

$$S_{ij} = L_{ij} * {}^1/_{T_6} \quad (2)$$

$$L_{11} = H_1\left(T_1 + \frac{2T_2}{P^2} + \frac{T_3}{P^4}\right) \quad (3a)$$

$$L_{12} = -H_2\left(T_1 + \frac{T_2}{P^2}\right) \quad (3b)$$

$$L_{13} = -H_2\left(\frac{T_2}{P^2} + \frac{T_3}{P^4}\right), L_{21} = L_{12} \quad (3c)$$

$$L_{22} = T_1H_3 + \left(\frac{1-\mu}{2P^2}\right)T_2H_3 + \left(\frac{1-\mu}{2}\right) \propto^2 K_4H_4 \quad (3d)$$

$$L_{23} = \left(\frac{1+\mu}{2P^2}\right)K_2H_3, L_{31} = L_{13}, L_{32} = L_{23} \quad (3e)$$

$$L_{33} = \left(\frac{1-\mu}{2P^2}\right)T_2H_3 + \frac{T_3}{P^4}H_3 + \left(\frac{1-\mu}{2P^2}\right) \propto^2 T_5H_4 \quad (3f)$$

$$T_1 = \int_0^1\int_0^1 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 \partial R\partial Q \quad (4)$$

$$T_2 = \int_0^1\int_0^1 \left(\frac{\partial^2 h}{\partial R^2}.\frac{\partial^2 h}{\partial Q^2}\right) \partial R\partial Q \quad (5)$$

$$T_3 = \int_0^1\int_0^1 \left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \partial R\partial Q \quad (6)$$

$$T_4 = \int_0^1\int_0^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{R}}\right)^2 \partial R\partial Q \quad (7)$$

$$T_5 = \int_0^1\int_0^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{Q}}\right)^2 \partial R\partial Q \quad (8)$$

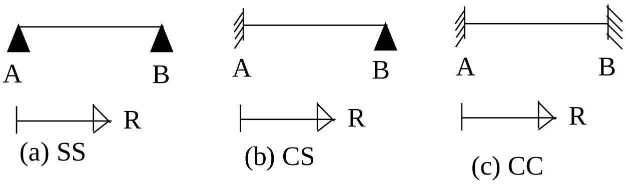
$$T_6 = \int_0^1\int_0^1 (h)^2 \partial R\partial Q \quad (9)$$

$$H_1 = 1, \quad H_2 = 0.79, \; H_3 = 0.6325, \; H_4 = 6.246 \quad (10)$$

Δ is the non-dimensional natural frequency parameter for the plate. h is the shape function that depends on the boundary condition of the plate.

III. BOUNDARY CONDITIONS

Twosupportconditions treated in this work are; simple support and clamped supports denoted as; (S) and (C) respectively. A simple beam is made up of two edges which could be any two of these supports giving rise to a total of three different beams used in this work. They are shown in Figs.1 (a, b, and c).



Where; $0 \leq R \leq 1$.

Fig. 1: Edge conditions of the orthogonal beams.

Figs.1 (a, b, and c) represent a beam having simple supports at the two edges (S-S beam), a beam with a fixed support at one end and simple support the other end (C-S beam), and a beam with fixed supports at both edges (C-C beam)respectively.

A rectangular plate is an arrangement of rectangular beams perpendicular to each other. In arranging the beams, the edge conditions of the horizontally placed beams are placed first before the edge conditions of the vertically placed beams.

The general polynomial equation for the deflection of thick rectangular plates obtained by [8] was used in this work and is presented here as;

$$w = w_x * w_y = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) * (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (11)$$

Where; w_x and w_y are the deflection equations for the horizontally placed and vertically placed beams respectively and are given as;

$$w_x = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \quad (12a)$$

$$w_y = (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (12b)$$

Differentiating Eqs. (12a) and (12b) with respect to R and Q yieldsEqs. (12c) - (12f).

$$\frac{\partial w_x}{\partial \mathrm{R}} = (a_1 + a_2 2\mathrm{R} + a_3 3R^2 + a_4 4R^3) \quad (12c)$$

$$\frac{\partial^2 w_x}{\partial \mathrm{R}^2} = (2a_2 + a_3 6\mathrm{R} + a_4 12R^2) \quad (12d)$$

$$\frac{\partial w_y}{\partial \mathrm{Q}} = (b_1 + b_2 2\mathrm{Q} + b_3 3Q^2 + b_4 4Q^3) \quad (12e)$$

$$\frac{\partial^2 w_y}{\partial \mathrm{Q}^2} = (2b_2 + b_3 6\mathrm{Q} + b_4 12Q^2) \quad (12f)$$

A. Boundary Conditions for S-S Beam

For the beam with simple supports at both edges shown in Figure (1.a), the deflection (w_x) or (w_y) and the moment $\left(\frac{\partial^2 w_x}{\partial R^2}\right)$ or $\left(\frac{\partial^2 w_y}{\partial Q^2}\right)$ at the two edges (i.e at *R=0 and R=1*) are equal to zero. Thus, we have;

$$w_x = w_y = \frac{\partial^2 w_x}{\partial R^2} = \frac{\partial^2 w_y}{\partial Q^2} = 0 \quad (13)$$

Applying Eq. (13)into Eqs. (12a), (12b), (12d) and (12f)and solving appropriately yields;

$$a_0 = 0, a_1 = a_4, a_2 = 0, a_3 = -2a_4 (14a)$$

$$b_0 = 0, b_1 = b_4, b_2 = 0, b_3 = -2b_4 (14b)$$

Substituting Eqs. (14a) and (14b) into Eqs. (12a) and (12b) respectively yields;

$$w_x = a_4(R - 2R^3 + R^4) \quad (15a)$$

$$w_y = b_4(Q - 2Q^3 + Q^4) \quad (15b).$$

B. Boundary Conditions for C-C Beam

For this beam, the deflection (w_x) or (w_y) and the slope $\left(\frac{\partial w_x}{\partial \mathrm{R}}\right)$ or $\left(\frac{\partial w_y}{\partial \mathrm{Q}}\right)$ at the two edges (i.e at *R=0 and R=1*) are equal to zero. Thus, we have;

$$w_x = w_y = \frac{\partial w_x}{\partial \mathrm{R}} = \frac{\partial w_y}{\partial \mathrm{Q}} = 0 \quad (16)$$

Applying Eq. (16) into Eqs. (12a), (12b), (12c) and (12e) and solving appropriately yields;

$$a_0 = 0;\ a_1 = 0;\ a_2 = a_4;\ a_3 = -2a_4 \quad (17a)$$

$$b_0 = 0;\ b_1 = 0;\ b_2 = b_4;\ b_3 = -2b_4 \quad (17b)$$

Substituting Eqs. (17a) and (17b) into Eqs. (12a) and (12b) respectively yields;

$$w_x = a_4(R^2 - 2R^3 + R^4) \quad (18a)$$

$$w_y = b_4(Q^2 - 2Q^3 + Q^4) \quad (18b)$$

C.Boundry Conditions for C-S Beam

For the beam with clamped support at one edge and simple support at the other edge, at the simple support (i.e at *R=1*), deflection and moment are equal to zero, while the deflection and slope at the clamped edge(i.e at *R=0*) are equal to zero. Thus;

$$w_x = w_y = \frac{\partial^2 w_x}{\partial R^2} = \frac{\partial^2 w_y}{\partial Q^2} = 0 \quad (19a)$$

$$w_x = w_y = \frac{\partial w_x}{\partial \mathrm{R}} = \frac{\partial w_y}{\partial \mathrm{Q}} = 0 \quad (19b)$$

Applying Eqs. (19a) and (19b) into Eqs. (12a) - (12f) and solving appropriately yields;

$$a_0 = 0;\ a_1 = 0;\ a_2 = 1.5\, a_4;\ \ a_3 = -2.5\, a_4 \quad (20a)$$

$$b_0 = 0;\ b_1 = 0;\ b_2 = 1.5\, b_4;\ \ b_3 = -2.5\, b_4 \quad (20b)$$

Substituting Eqs. (20a) and (20b) into Eqs. (12a) and (12b) respectively yields;

$$w_x = a_4(1.5R^2 - 2.5R^3 + R^4) \quad (21a)$$

$$w_y = b_4(1.5Q^2 - 2.5Q^3 + Q^4) \quad (21b)$$

Eqs. (21a) and (21b) can be rewritten as;

$$w_x = a_4\left(\frac{3}{2}R^2 - \frac{5}{2}R^3 + R^4\right) \quad (22a)$$

$$w_y = b_4\left(\frac{3}{2}Q^2 - \frac{5}{2}Q^3 + Q^4\right) \quad (22b)$$

IV. FREE-VIBRATION STUDY OF CSCS RECTANGULAR PLATES

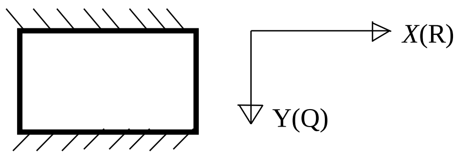


Figure 2: CSCS Rectangular Plate.

The deflection expression for this plate is a product of the deflection expression for the S-S beam (Eq. (15a))and the deflection expression for the C-C beam (Eq. (18b)) given as;

$$w = a_4(R - 2R^3 + R^4).\, b_4(Q^2 - 2Q^3 + Q^4) \quad (23)$$

Eq. (23) can be rewritten as ;

$$\mathrm{w} = A(R - 2R^3 + R^4).\,(Q^2 - 2Q^3 + Q^4) = Ah \quad (24)$$

Where;

$$h = (R - 2R^3 + R^4).\,(Q^2 - 2Q^3 + Q^4) \quad (25)$$

Where $A = a_4 b_4$is the amplitude and 'h' is the shape function for the CSCS thick plate.

Differentiating Eq. (25) concerning R and Q yields;

$$\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \quad (26a)$$

$$\frac{\partial h}{\partial Q} = (R - 2R^3 + R^4)(2Q - 6Q^2 + 4Q^3) \quad (26b)$$

$$\frac{\partial^2 h}{\partial R^2} = (-12R + 12R^2)(Q^2 - 2Q^3 + Q^4) \quad (26c)$$

$$\frac{\partial^2 h}{\partial Q^2} = (R - 2R^3 + R^4)(2 - 12Q - 12Q^2) \quad (26d)$$

Substituting Eqs. (26c) into Eqs. (4) yields;

$$T_1 = \int_0^1\int_0^1 \left(\frac{d^2h}{dR^2}\right)^2 \partial R \partial Q \quad (27a)$$

$$T_1 = \int_0^1\int_0^1 = 144(R^2 - R)^2.(Q^2 - 2Q^3 + Q^4)^2 \partial R \partial Q \quad (27b)$$

$$T_1 = \int_0^1\int_0^1 144(R^4 - 2R^3 + R^2)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\partial R \partial Q \quad (27c)$$

$$T_1 = 144\left[\frac{R^5}{5} - \frac{2R^4}{4} + \frac{R^3}{3}\right]_0^1 . \left[\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (27d)$$

$$T_1 = 144\left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right).\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) = 0.00762 \quad (27e)$$

Substituting Eqs. (26c) and (26d) into Eq. (5) yields;

$$T_2 = \int_0^1\int_0^1 \left(\frac{d^2h}{dR^2}.\frac{d^2h}{dQ^2}\right) \partial R \partial Q \quad (28a)$$

$$T_2 = \int_0^1\int_0^1 [(12R^2 - 12R)(Q - 2Q^3 + Q^4)].[(R^2 - 2R^3 + R^4)(2 - 12Q - 12Q^2)]\partial R \partial Q \quad (28b)$$

$$T_2 = \int_0^1\int_0^1 24[(R^3 - 2R^5 + R^6 - R^2 + 2R^4 - R^5)].[(Q^2 - 8Q^3 + 19Q^4 - 18Q^5 + 6Q^6)\partial R \partial Q \quad (28c)$$

$$T_2 = 24\left[\frac{R^4}{4} - \frac{2R^6}{6} + \frac{R^7}{7} - \frac{R^3}{3} + \frac{2R^5}{5} - \frac{R^6}{6}\right]_0^1 . \left[\frac{Q^3}{3} - \frac{8Q^4}{4} + \frac{19Q^5}{5} - \frac{18Q^6}{6} + \frac{6Q^7}{7}\right]_0^1 \quad (28d)$$

$$T_2 = 24\left(\frac{1}{4} - \frac{2}{6} + \frac{1}{7} - \frac{1}{3} + \frac{2}{5} - \frac{1}{6}\right).\left(\frac{1}{3} - \frac{8}{4} + \frac{19}{5} - \frac{18}{6} + \frac{6}{7}\right) = 0.009252 \quad (28e)$$

Substituting Eq. (26d) into Eq. (6) respectively yields;

$$T_3 = \int_0^1\int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 \partial R \partial Q \quad (29a)$$

$$T_3 = \int_0^1\int_0^1 (R - 2R^3 + R^4)^2.(2 - 12Q - 12Q^2)^2 \partial R \partial Q \quad (29a)$$

$$T_3 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8).(4 - 48Q + 192Q^2 - 288Q^3 + 144Q^4)\partial R \partial Q \quad (29b)$$

$$T_3 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1 . \left[\frac{4Q}{1} - \frac{48Q^2}{2} + \frac{192Q^3}{3} - \frac{288Q^4}{4} + \frac{144Q^5}{5}\right]_0^1 \quad (29c)$$

$$T_3 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{4}{1} - \frac{48}{2} + \frac{192}{3} - \frac{288}{4} + \frac{144}{5}\right) = 0.039365 \quad (29d)$$

Substituting Eq. (26a)into Eq. (7) yields;

$$T_4 = \int_0^1\int_0^1 \left(\frac{dh}{dR}\right)^2 \partial R \partial Q \quad (30a)$$

$$T_4 = \int_0^1\int_0^1 (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6).(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\partial R \partial Q \quad (30b)$$

$$T_4 = \left[R - \frac{12R^3}{3} + \frac{8R^4}{4} + \frac{36R^5}{5} - \frac{48R^6}{6} + \frac{16R^7}{7}\right]_0^1 . \left[\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (30c)$$

$$T_4 = \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7}\right).\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) = 0.000771 \quad (30d)$$

Substituting Eq. (26b) into Eq. (8) yields;

$$T_5 = \int_0^1\int_0^1 \left(\frac{dh}{dQ}\right)^2 \partial R \partial Q \quad (31a)$$

$$T_5 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)(4Q^2 - 24Q^3 + 52Q^4 - 48Q^5 + 16Q^6)\partial R \partial Q \quad (31b)$$

$$T_5 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1 . \left[\frac{4Q^3}{3} - \frac{24Q^4}{4} + \frac{52Q^5}{5} - \frac{48Q^6}{6} + \frac{16Q^7}{7}\right]_0^1 \quad (31c)$$

$$T_5 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7}\right) = 0.000937 \quad (31d)$$

Substituting Eq. (25) into Eq. (9) yields;

$$T_6 = \int_0^1\int_0^1 (\mathrm{h})^2 \partial R \partial Q \quad (32a)$$

$$T_6 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8).(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\partial R \partial Q \quad (32b)$$

$$T_6 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1 . \left[\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (32c)$$

$$T_6 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right) = 0.000078 \quad (32d)$$

Substituting Eqs. (27e), (28e), (29d), (30d), (31d), (32d) and (10) into Eq. (3) yields the values for Eq. (3). Substituting the values of Eq. (3) into Eqs. (2) and (1) yields the non-dimensional natural frequency parameters(Δ) for the CSCS plate at any value of the span-depth ratio (a/t) and in-plane dimensions ratio (b/a) as shown in Table 1. The values of the non-dimensional natural frequency parameter (Δ) were plotted against the span-depth ratio (a/t) at in-plane dimensions ratio (b/a) = 1, for the results obtained from the present study and the works of [6] and presented in Figure 5.

V. FREE-VIBRATION STUDY OF CSSS RECTANGULAR PLATES

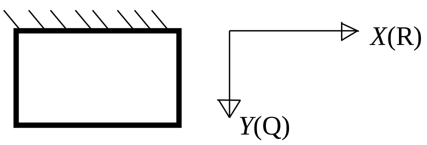


Figure 3: CSSS Rectangular Plate.

The deflection expression for this plate is a product of the deflection expression for S-S beam (Eq. (15a)) and the deflection expression for C-S beam (Eq. (22b)) given as;

$$\mathrm{w} = Ah = A(R - 2R^3 + R^4).\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right) \quad (33)$$

Where;

$$h = (R - 2R^3 + R^4).\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right) \quad (34)$$

Where $A = a_4 b_4$is the amplitude and 'h' is the shape function for CSSS thick plate.

Differentiating Eq. (34) concerning R and Q yields;

$$\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right) \quad (35a)$$

$$\frac{\partial h}{\partial Q} = (R - 2R^3 + R^4)\left(3Q - \frac{15Q^2}{2} + 4Q^3\right) \quad (35b)$$

$$\frac{\partial^2 h}{\partial R^2} = 12(R^2 - R)\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right) \quad (35c)$$

$$\frac{\partial^2 h}{\partial Q^2} = (R - 2R^3 + R^4)(3 - 15Q + 12Q^2) \quad (35d)$$

Substituting Eq. (35c) into Eq. (4) yields;

$$T_1 = \int_0^1\int_0^1 \left(\frac{d^2 h}{dR^2}\right)^2 \partial R \partial Q \quad (36a)$$

$$T_1 = \int_0^1\int_0^1 \left[144(R^2 - R)^2.\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right)^2\right]\partial R \partial Q \quad (36b)$$

$$T_1 = 144\int_0^1\int_0^1 \left[(R^4 - 2R^3 + R^2)\left(\frac{9Q^4}{4} - \frac{30Q^5}{4} + \frac{6Q^6}{2} + \frac{25Q^6}{4} - 5Q^7 + Q^8\right)\right]\partial R \partial Q \quad (36c)$$

$$T_1 = 144\left[\frac{R^5}{5} - \frac{2R^4}{4} + \frac{R^3}{3}\right]_0^1 . \left[\frac{9Q^5}{20} - \frac{30Q^6}{24} + \frac{6Q^7}{14} + \frac{25Q^7}{28} - \frac{5Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (36d)$$

$$T_1 = 144\left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right).\left(\frac{9}{20} - \frac{30}{24} + \frac{6}{14} + \frac{25}{28} - \frac{5}{8} + \frac{1}{9}\right) = 0.036192 \quad (36e)$$

Substituting Eqs. (35c) and (35d) into Eq. (5) yields;

$$T_2 = \int_0^1\int_0^1 \left(\frac{d^2 h}{dR^2}.\frac{d^2 h}{dQ^2}\right)\partial R \partial Q \quad (37a)$$

$$T_2 = \int_0^1\int_0^1 [12(R^2 - R)(R - 2R^3 + R^4)].\left[(3 - 15Q + 12Q^2)\left(\frac{3Q^2}{2} - \frac{5Q^3}{2} + Q^4\right)\right]\partial R \partial Q \quad (37b)$$

$$T_2 = 12\int_0^1\int_0^1 [(R^3 - 3R^5 + R^6 - R^2 + 2R^4)].\left[\left(\frac{9Q^2}{2} - \frac{60Q^3}{2} + \frac{117Q^4}{2} - \frac{90Q^5}{2} + 12Q^6\right)\right]\partial R\partial Q \quad (37c)$$

$$T_2 = 12\left[\frac{R^4}{4} - \frac{3R^6}{6} + \frac{R^7}{7} - \frac{R^3}{3} + \frac{2R^5}{5}\right]_0^1.\left[\frac{9Q^3}{6} - \frac{60Q^4}{8} + \frac{117Q^5}{10} - \frac{90Q^6}{12} + \frac{12Q^7}{7}\right]_0^1 (3.350f) \quad (37d)$$

$$T_2 = 12\left(\frac{1}{4} - \frac{3}{6} + \frac{1}{7} - \frac{1}{3} + \frac{2}{5}\right)\left(\frac{9}{6} - \frac{60}{8} + \frac{117}{10} - \frac{90}{12} + \frac{12}{7}\right) = 0.041633 \quad (37e)$$

Substituting Eq. (35d) into Eq. (6) yields;

$$T_3 = \int_0^1\int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 \partial R\partial Q \quad (38a)$$

$$T_3 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8).(9 - 90Q + 297Q^2 - 360Q^3 + 144Q^4)\partial R\partial Q \quad (38b)$$

$$T_3 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1.\left[\frac{9Q}{1} - \frac{90Q^2}{2} + \frac{297Q^3}{3} - \frac{360Q^4}{4} + \frac{144Q^5}{5}\right]_0^1 \quad (38c)$$

$$T_3 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{9}{1} - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5}\right) = 0.088571 \quad (38d)$$

Substituting Eq. (35a) into Eq. (7) yields;

$$T_4 = \int_0^1\int_0^1 \left(\frac{dh}{dR}\right)^2 \partial R\partial Q \quad (39a)$$

$$T_4 = \int_0^1\int_0^1 (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)\left(\frac{9Q^4}{4} - \frac{15Q^5}{2} + \frac{37Q^6}{4} - 5Q^7 + Q^8\right)\partial R\partial Q \quad (39b)$$

$$T_4 = \left[R - \frac{12R^3}{3} + \frac{8R^4}{4} + \frac{36R^5}{5} - \frac{48R^6}{6} + \frac{16R^7}{7}\right]_0^1.\left[\frac{9Q^5}{20} - \frac{15Q^6}{12} + \frac{37Q^7}{28} - \frac{5Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (39c)$$

$$T_4 = \left(1 - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7}\right).\left(\frac{9}{20} - \frac{15}{12} + \frac{37}{28} - \frac{5}{8} + \frac{1}{9}\right) = 0.003662 \quad (39d)$$

Substituting Eq. (35b), into Eq. (8) yields;

$$T_5 = \int_0^1\int_0^1 \left(\frac{dh}{dQ}\right)^2 \partial R\partial Q \quad (40a)$$

$$T_5 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)\left(9Q^2 - 45Q^3 + \frac{321Q^4}{4} - 60Q^5 + 16Q^6\right)\partial R\partial Q \quad (40b)$$

$$T_5 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1.\left[\frac{9Q^3}{3} - \frac{45Q^4}{4} + \frac{321Q^5}{20} - \frac{60Q^6}{6} + \frac{16Q^7}{7}\right]_0^1 \quad (40c)$$

$$T_5 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{9}{3} - \frac{45}{4} + \frac{321}{20} - \frac{60}{6} + \frac{16}{7}\right) = 0.004218 \quad (40d)$$

Substituting Eq. (34) into Eq. (9) yields;

$$T_6 = \int_0^1\int_0^1 (\mathrm{h})^2\partial R\partial Q \quad (41a)$$

$$T_6 = \int_0^1\int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8).\left(\frac{9Q^4}{4} - \frac{15Q^5}{2} + \frac{37Q^6}{4} - 5Q^7 + Q^8\right)\partial R\partial Q \quad (41b)$$

$$T_6 = \left[\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right]_0^1.\left[\frac{9Q^5}{20} - \frac{15Q^6}{12} + \frac{37Q^7}{28} - \frac{5Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (41c)$$

$$T_6 = \left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right).\left(\frac{9}{20} - \frac{15}{12} + \frac{37}{28} - \frac{5}{8} + \frac{1}{9}\right) = 0.000371 \quad (41d)$$

Substituting Eqs. (36e), (37e), (38d), (39d), (40d), (41d) and (10) into Eq. (3) yields the values for Eq. (3). Substituting the values of Eq. (3) into Eqs. (2) and (1) yields the non-dimensional natural frequency parameters (Δ) for the CSSS plate at any value of the span-depth ratio (a/t) and planer span ratio (b/a) as shown in Table 2.

VI. FREE-VIBRATION STUDY OF CCCS RECTANGULAR PLATES

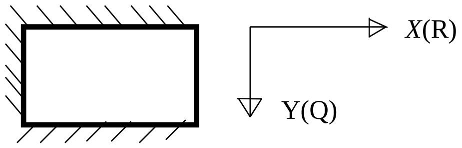


Figure 4: CCCS Rectangular Plate.

The deflection expression for this plate is a product of the deflection expression for the C-S beam (Eq. (22a)) and the deflection expression for the C-C beam (Eq. (18b)) given as;

$$w = a_4\left(\frac{3}{2}R^2 - \frac{5}{2}R^3 + R^4\right).b_4(Q^2 - 2Q^3 + Q^4) \quad (42)$$

Equation (3.410) can be rewritten as Equation (3.411).

$$\mathrm{w} = Ah = A\left(\frac{3}{2}R^2 - \frac{5}{2}R^3 + R^4\right).(Q^2 - 2Q^3 + Q^4) \quad (43)$$

Where;

$$h = \left(\frac{3}{2}R^2 - \frac{5}{2}R^3 + R^4\right).(Q^2 - 2Q^3 + Q^4)(44)$$

Where $A = a_4 b_4$ is the amplitude and 'h' is the shape function for CCCS thick plate.

Differentiating Eq. (44) concerning R and Q yield;

$$\frac{\partial h}{\partial R} = \left(3R - \frac{15R^2}{2} + 4R^3\right).(Q^2 - 2Q^3 + Q^4) \quad (45a)$$

$$\frac{\partial h}{\partial Q} = \left(\frac{3R^2}{2} - \frac{5R^3}{2} + R^4\right)(2Q - 6Q^2 + 4Q^3) \quad (45b)$$

$$\frac{\partial^2 h}{\partial R^2} = (3 - 15R + 12R^2).(Q^2 - 2Q^3 + Q^4) \quad (45c)$$

$$\frac{\partial^2 h}{\partial Q^2} = \left(\frac{3R^2}{2} - \frac{5R^3}{2} + R^4\right)(2 - 12Q + 12Q^2) \quad (45d)$$

Substituting Eq. (45c) into Eq. (4) yields;

$$T_1 = \int_0^1\int_0^1 \left(\frac{d^2h}{dR^2}\right)^2 \partial R\partial Q \quad (46a)$$

$$T_1 = \int_0^1\int_0^1 (9 - 90R + 297R^2 - 360R^3 + 144R^4)(Q^4 - 4Q^5 + 2Q^6 + 4Q^6 - 4Q^7 + Q^8)\partial R\partial Q \quad (46b)$$

$$T_1 = \left[9R - \frac{90R^2}{2} + \frac{297R^3}{3} - \frac{360R^4}{4} + \frac{144R^5}{5}\right]_0^1 . \left[\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{2Q^7}{7} + \frac{4Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9}\right]_0^1 \quad (46c)$$

$$T_1 = \left(9 - \frac{90}{2} + \frac{297}{3} - \frac{360}{4} + \frac{144}{5}\right).\left(\frac{1}{5} - \frac{4}{6} + \frac{2}{7} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) = 0.002857 \quad (46d)$$

Substituting Eqs. (45c) and (45d) into Eq. (5) yields;

$$T_2 = \int_0^1\int_0^1 \left(\frac{d^2h}{dR^2}.\frac{d^2h}{dQ^2}\right)\partial R\partial Q \quad (47a)$$

$$T_2 = \int_0^1\int_0^1 6\left[\left(\frac{3R^2}{2} - \frac{20R^3}{2} + \frac{39R^4}{2} - \frac{30R^5}{2} + 4R^6\right)\right].(Q^2 - 8Q^3 + 19Q^4 - 18Q^5 + 6Q^6)\partial R\partial Q \quad (47b)$$

$$T_2 = 6\left[\frac{3R^3}{6} - \frac{20R^4}{8} + \frac{39R^5}{10} - \frac{30R^6}{12} + \frac{4R^7}{7}\right]_0^1 . \left[\frac{Q^3}{3} - \frac{8Q^4}{4} + \frac{19Q^5}{5} - \frac{18Q^6}{6} + \frac{6Q^7}{7}\right]_0^1 \quad (47c)$$

$$T_2 = 6\left(\frac{3}{6} - \frac{20}{8} + \frac{39}{10} - \frac{30}{12} + \frac{4}{7}\right)\left(\frac{1}{3} - \frac{8}{4} + \frac{19}{5} - \frac{18}{6} + \frac{6}{7}\right) = 0.001633 \quad (47d)$$

Substituting Eq. (45d) into Eq. (6) yields;

$$T_3 = \int_0^1\int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 \partial R\partial Q \quad (48a)$$

$$T_3 = \int_0^1\int_0^1 \left(\frac{9R^4}{4} - \frac{30R^5}{4} + \frac{6R^6}{2} + \frac{25R^6}{4} - 5R^7 + \mathrm{R}^8\right).(4 - 48Q + 48Q^2 + 144Q^2 - 288Q^3 + 144Q^4)\partial R\partial Q \quad (48b)$$

$$T_3 = \left[\frac{9R^5}{20} - \frac{30R^6}{24} + \frac{6R^7}{14} + \frac{25R^7}{28} - \frac{5R^8}{8} + \frac{R^9}{9}\right]_0^1 . \left[\frac{4Q}{1} - \frac{48Q^2}{2} + \frac{48Q^3}{3} + \frac{144Q^3}{3} - \frac{288Q^4}{4} + \frac{144Q^5}{5}\right]_0^1 \quad (48c)$$

$$T_3=\left(\frac{9}{20}-\frac{30}{24}+\frac{6}{14}+\frac{25}{28}-\frac{5}{8}+\frac{1}{9}\right).\left(\frac{4}{1}-\frac{48}{2}+\frac{48}{3}+\frac{144}{3}-\frac{288}{4}+\frac{144}{5}\right)=0.006032\quad(48d)$$

Substituting Eq. (45a) into Eq. (7) yields;

$$T_4=\int_0^1\int_0^1\left(\frac{dh}{dR}\right)^2\partial R\partial Q\quad(49a)$$

$$T_4=\int_0^1\int_0^1\left(9R^2-45R^3+\frac{321R^4}{4}-60R^5+16R^6\right).(Q^4-4Q^5+6Q^6-4Q^7+Q^8)\partial R\partial Q\quad(49b)$$

$$T_4=\left[\frac{9R^3}{3}-\frac{45R^4}{4}+\frac{321R^5}{20}-\frac{60R^6}{6}+\frac{16R^7}{7}\right]_0^1.\left[\frac{Q^5}{5}-\frac{4Q^6}{6}+\frac{6Q^7}{7}-\frac{4Q^8}{8}+\frac{Q^9}{9}\right]_0^1\,(49c)$$

$$T_4=\left(\frac{9}{3}-\frac{45}{4}+\frac{321}{20}-\frac{60}{6}+\frac{16}{7}\right).\left(\frac{1}{5}-\frac{4}{6}+\frac{6}{7}-\frac{4}{8}+\frac{1}{9}\right)=0.000136(49d)$$

Substituting Eq. (45b) into Eq. (8) yields;

$$T_5=\int_0^1\int_0^1\left(\frac{dh}{dQ}\right)^2\partial R\partial Q\,(50a)$$

$$T_5=\int_0^1\int_0^1\left(\frac{9R^4}{4}-\frac{30R^5}{4}+\frac{6R^6}{2}+\frac{25R^6}{4}-5R^7+R^8\right)(4Q^2-24Q^3+52Q^4-48Q^5+16Q^6)\partial R\partial Q\quad(50b)$$

$$T_5=\left[\frac{9R^5}{20}-\frac{30R^6}{24}+\frac{6R^7}{14}+\frac{25R^7}{28}-\frac{5R^8}{8}+\frac{R^9}{9}\right]_0^1.\left[\frac{4Q^3}{3}-\frac{24Q^4}{4}+\frac{52Q^5}{5}-\frac{48Q^6}{6}+\frac{16Q^7}{7}\right]_0^1\quad(50c)$$

$$T_5=\left(\frac{9}{20}-\frac{30}{24}+\frac{6}{14}+\frac{25}{28}-\frac{5}{8}+\frac{1}{9}\right).\left(\frac{4}{3}-\frac{24}{4}+\frac{52}{5}-\frac{48}{6}+\frac{16}{7}\right)=0.000144\quad(50d)$$

Substituting Eq. (45b) into Eq. (8) yields;

$$T_6=\int_0^1\int_0^1(\text{h})^2\partial R\partial Q\quad(51a)$$

$$T_6=\int_0^1\int_0^1\left(\frac{9R^4}{4}-\frac{15R^5}{2}+\frac{37R^6}{4}-5R^7+R^8\right).(Q^4-4Q^5+6Q^6-4Q^7+Q^8)\partial R\partial Q\quad(51b)$$

$$T_6=\left[\frac{9R^5}{20}-\frac{15R^6}{12}+\frac{37R^7}{28}-\frac{5R^8}{8}+\frac{R^9}{9}\right]_0^1.\left[\frac{Q^5}{5}-\frac{4Q^6}{6}+\frac{6Q^7}{7}-\frac{4Q^8}{8}+\frac{Q^9}{9}\right]_0^1\,(51c)$$

$$T_6=\left(\frac{9}{20}-\frac{15}{12}+\frac{37}{28}-\frac{5}{8}+\frac{1}{9}\right).\left(\frac{1}{5}-\frac{4}{6}+\frac{6}{7}-\frac{4}{8}+\frac{1}{9}\right)=0.000012(51d)$$

Substituting Eqs. (46d), (47d), (48d), (49d), (50d), (51d) and (10) into Eq. (3) yields the values for Eq. (3). Substituting the values of Eq. (3) into Eqs. (2) and (1) yields the non-dimensional natural frequency parameters (Δ) for the CCCS plate at any value of the span-depth ratio (a/t) and planer span ratio (b/a) as shown in Table 3.

VII. RESULTS AND DISCUSSIONS.

Table 1: Non–Dimensional Natural Frequencies of CSCS Thick Plate.

$\propto = {}^{a}/_{t}$	$^{b}/_{a} = 1.0$	$^{b}/_{a} = 1.2$	$^{b}/_{a} = 1.5$	$^{b}/_{a} = 1.8$	$^{b}/_{a} = 2.0$	$^{b}/_{a} = 2.2$	$^{b}/_{a} = 2.4$	$^{b}/_{a} = 2.5$
	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$							
	Δ							
5	23.7305	19.3321	15.5849	13.5416	12.6743	12.0450	11.5762	11.3860
6.67	25.6223	20.5275	16.3018	14.0486	13.1046	12.4245	11.9207	11.7170
10	27.3170	21.5442	16.8833	14.4493	13.4409	12.7188	12.1864	11.9718
15	28.1992	22.0534	17.1650	14.6398	13.5997	12.8571	12.3109	12.0909
20	28.5306	22.2411	17.2672	14.7085	13.6567	12.9066	12.3553	12.1335
25	28.6884	22.3298	17.3152	14.7406	13.6833	12.9297	12.3761	12.1533
30	28.7752	22.3785	17.3415	14.7581	13.6979	12.9423	12.3874	12.1642
35	28.8280	22.4080	17.3574	14.7687	13.7066	12.9500	12.3943	12.1707
40	28.8625	22.4272	17.3677	14.7756	13.7124	12.9549	12.3987	12.1750
45	28.8862	22.4405	17.3748	14.7804	13.7163	12.9583	12.4018	12.1779
50	28.9031	22.4499	17.3799	14.7837	13.7191	12.9608	12.4040	12.1800
55	28.9157	22.4569	17.3837	14.7863	13.7212	12.9626	12.4056	12.1815
60	28.9253	22.4623	17.3866	14.7882	13.7227	12.9639	12.4068	12.1827
65	28.9328	22.4664	17.3888	14.7896	13.7240	12.9650	12.4078	12.1836
70	28.9387	22.4697	17.3906	14.7908	13.7250	12.9658	12.4085	12.1843
75	28.9435	22.4724	17.3920	14.7918	13.7257	12.9665	12.4091	12.1849
80	28.9474	22.4746	17.3932	14.7926	13.7264	12.9671	12.4096	12.1854
85	28.9507	22.4764	17.3941	14.7932	13.7269	12.9676	12.4101	12.1858
90	28.9534	22.4779	17.3949	14.7937	13.7274	12.9679	12.4104	12.1861
95	28.9557	22.4792	17.3956	14.7942	13.7278	12.9683	12.4107	12.1864
100	28.9577	22.4803	17.3962	14.7946	13.7281	12.9686	12.4110	12.1867

Table 2: Non–Dimensional Natural Frequencies of CSSS Thick Plate.

$\propto = {}^{a}/_{t}$	$^{b}/_{a} = 1.0$	$^{b}/_{a} = 1.2$	$^{b}/_{a} = 1.5$	$^{b}/_{a} = 1.8$	$^{b}/_{a} = 2.0$	$^{b}/_{a} = 2.2$	$^{b}/_{a} = 2.4$	$^{b}/_{a} = 2.5$
	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$							
	Δ							
5	20.6969	17.2290	14.3377	12.7687	12.0975	11.6054	11.2346	11.0826
6.67	21.8525	18.0111	14.8639	13.1783	12.4623	11.9394	11.5463	11.3856
10	22.8101	18.6405	15.2771	13.4960	12.7438	12.1960	11.7852	11.6175
15	23.2806	18.9436	15.4729	13.6452	12.8755	12.3158	11.8966	11.7255
20	23.4526	19.0534	15.5433	13.6986	12.9226	12.3586	11.9363	11.7640
25	23.5336	19.1049	15.5762	13.7236	12.9446	12.3786	11.9549	11.7820
30	23.5779	19.1330	15.5941	13.7372	12.9566	12.3895	11.9650	11.7918
35	23.6048	19.1501	15.6050	13.7454	12.9638	12.3960	11.9711	11.7977
40	23.6222	19.1611	15.6121	13.7507	12.9685	12.4003	11.9750	11.8016
45	23.6342	19.1687	15.6169	13.7544	12.9718	12.4032	11.9778	11.8042
50	23.6429	19.1742	15.6204	13.7570	12.9741	12.4053	11.9797	11.8061
55	23.6492	19.1782	15.6230	13.7590	12.9758	12.4069	11.9812	11.8075
60	23.6541	19.1813	15.6249	13.7605	12.9771	12.4081	11.9823	11.8085
65	23.6578	19.1837	15.6264	13.7616	12.9781	12.4090	11.9831	11.8094
70	23.6608	19.1856	15.6276	13.7625	12.9789	12.4097	11.9838	11.8100
75	23.6633	19.1871	15.6286	13.7633	12.9795	12.4103	11.9843	11.8106
80	23.6652	19.1884	15.6294	13.7639	12.9801	12.4108	11.9848	11.8110
85	23.6669	19.1894	15.6301	13.7644	12.9805	12.4112	11.9852	11.8114
90	23.6683	19.1903	15.6306	13.7648	12.9809	12.4115	11.9855	11.8117
95	23.6694	19.1910	15.6311	13.7652	12.9812	12.4118	11.9857	11.8119
100	23.6704	19.1916	15.6315	13.7655	12.9815	12.4121	11.9860	11.8121

Table 3: Non–Dimensional Natural Frequencies of CCCS Thick Plate.

$\propto = a/t$	$b/a = 1.0$	$b/a = 1.2$	$b/a = 1.5$	$b/a = 1.8$	$b/a = 2.0$	$b/a = 2.2$	$b/a = 2.4$	$b/a = 2.5$
	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$							
	Δ							
5	25.8748	21.9100	18.6787	17.0017	16.3138	15.8249	15.4670	15.3233
6.67	28.0174	23.3997	19.7365	17.8777	17.1247	16.5932	16.2058	16.0508
10	29.9403	24.6771	20.6130	18.5932	17.7839	17.2158	16.8034	16.6388
15	30.9428	25.3205	21.0437	18.9413	18.1036	17.5170	17.0922	16.9227
20	31.3197	25.5584	21.2011	19.0679	18.2196	17.6264	17.1969	17.0257
25	31.4991	25.6709	21.2752	19.1274	18.2742	17.6777	17.2461	17.0740
30	31.5980	25.7326	21.3157	19.1600	18.3040	17.7057	17.2729	17.1004
35	31.6581	25.7701	21.3403	19.1797	18.3220	17.7227	17.2892	17.1164
40	31.6973	25.7945	21.3563	19.1925	18.3338	17.7338	17.2998	17.1268
45	31.7242	25.8113	21.3673	19.2013	18.3418	17.7414	17.3071	17.1340
50	31.7435	25.8233	21.3752	19.2076	18.3476	17.7468	17.3123	17.1391
55	31.7579	25.8322	21.3810	19.2123	18.3519	17.7508	17.3161	17.1429
60	31.7688	25.8390	21.3854	19.2158	18.3552	17.7539	17.3191	17.1458
65	31.7773	25.8442	21.3889	19.2186	18.3577	17.7563	17.3213	17.1480
70	31.7840	25.8484	21.3916	19.2208	18.3597	17.7582	17.3232	17.1498
75	31.7895	25.8518	21.3939	19.2226	18.3613	17.7597	17.3246	17.1512
80	31.7939	25.8546	21.3957	19.2240	18.3627	17.7609	17.3258	17.1524
85	31.7976	25.8569	21.3972	19.2252	18.3638	17.7620	17.3268	17.1534
90	31.8007	25.8588	21.3984	19.2263	18.3647	17.7629	17.3276	17.1542
95	31.8033	25.8604	21.3995	19.2271	18.3655	17.7636	17.3283	17.1549
100	31.8056	25.8618	21.4004	19.2278	18.3661	17.7642	17.3289	17.1555

Table 4: Comparison of the Non-DimensionalFundamental Natural Frequencies from The Present Study with that of [6] for CSCS Thick Plates.

b/a	a/t	$\lambda = \frac{\Delta}{a^2}\sqrt{\frac{D}{m}}$		% Difference. $\frac{(P.S - H.A) * 100}{P.S}$
		Δ		
		Present Study (P.S.)	**Hashemi and Arsanjani, (2004) (H.A) [6]**	
1	**100**	**28.9577**	**28.9250**	**0.11**
	20	28.5306	28.3324	**0.69**
	10	27.3170	26.7369	**2.12**
	6.67	25.6223	24.6627	**3.75**
	5	23.7305	22.5099	**5.14**
1.5	100	17.3962	17.3650	**0.18**
	20	17.2672	17.1780	**0.52**
	10	16.8833	16.6455	**1.41**
	6.67	16.3018	15.8866	**2.55**
	5	15.5849	15.0147	**3.66**
2	100	13.7281	13.6815	**0.34**
	20	13.6567	13.5802	**0.56**
	10	13.4409	13.2843	**1.17**
	6.67	13.1046	12.8449	**1.98**
	5	12.6743	12.3152	**2.83**
2.5	100	12.1867	11.4464	**6.07**
	20	12.1335	11.3906	**6.12**
	10	11.9718	11.2260	**6.23**
	6.67	11.7170	10.9617	**6.45**
	5	11.3860	10.6307	**6.63**

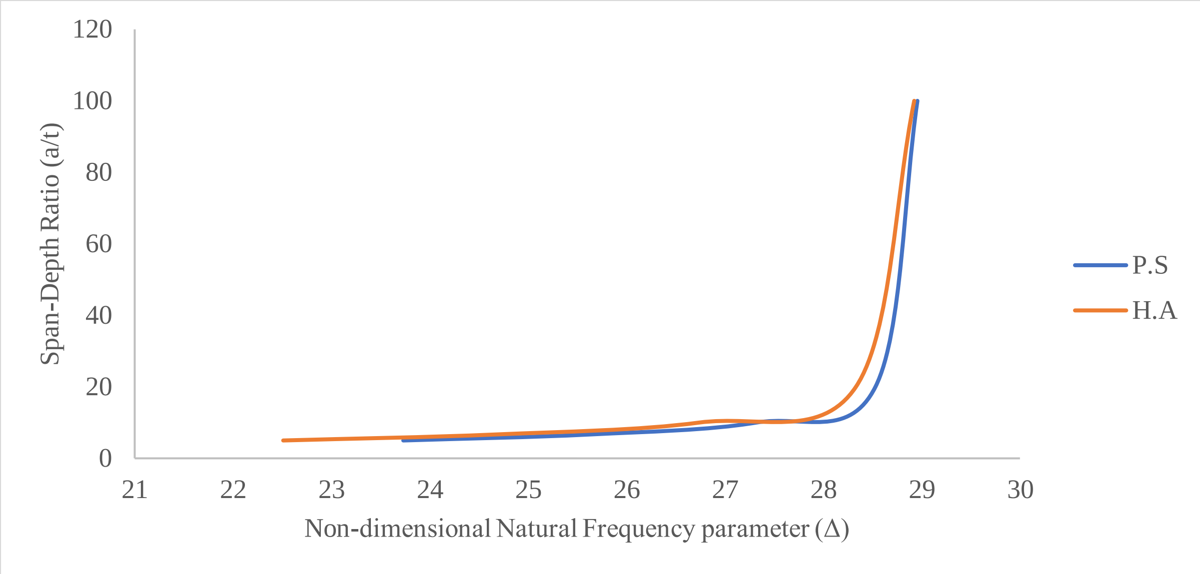


Figure 5: Present Study Result Compared with the results of [6] (H.A), for Free Vibration Analysis of CSCS Thick Plates for Aspect Ratio, P = 1.0

A thorough study of Tables (1) – (3) reveals that at the same value of the ratio of the in-plane dimensions (b/a), values of Δ increases as the span – depth ratio (a/t) increases. This implies that the effect of vibratory load on the plate increases with an increase in the span – depth ratio.

At the same value of the span-depth ratio (a/t), as the value of the ratio of the in-plane dimensions (b/a) increases, there is a decrease in the value of the non-dimensional frequency parameter (Δ)with its maximum value occurring at (b/a = 1, that is, the Square plate) and its minimum value occurring at (b/a) = 2.5. This implies that the ability of the plate to resist vibratory load decreases as the ratio of the in-plane dimensions (b/a) increases.

A Study of Table 4 where the results of CSCS thick plate analysis from the present study were compared with the results of [6]at different span – depth ratios (a/t) and in-plane dimensions ratio (b/a) shows the percentage difference ranging from a maximum value of 6.63 to a minimum value of 0.11 which are quite negligible and acceptable in statistics as being close. More so, it is quite evident from Fig.5 that the results from the present study are in good agreement with the results of [6]. Thus, the present study provides a good solution for the free vibration analysis of thick plates.

VIII. CONCLUSIONS

From the present study, the following conclusions could be drawn;

- The polynomial deflection function could easily satisfy the various boundary conditions.
- This implies that the effect of vibratory load on the plate increases with an increase in the span – depth ratio.
- The ability of the plate to resist vibratory load decreases as the ratio of the in-plane dimensions (b/a) increases.
- The results from the present study are in good agreement with the results of previous researchers.
- The simple linear equation for vibration analysis of thick plates produces efficient results.

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