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Comparative Analysis of 1-D, 2-D And 3-D Modeling of WASTE Stabilization Pond With Computational Fluid Dynamics

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Abstract

Waste stabilization ponds (WSP) are used extensively to provide wastewater treatment throughout the world. A review of the literature indicates that, understanding the hydraulics of waste stabilization ponds is critical to their optimization, the research in this area has been relatively limited and that there is a poor mechanistic understanding of the flow behavior that exists within these systems. This explains why there is no generally acceptable model for predicting its performance. The computational fluid dynamics (CFD) model developed in this study was extensively tested on the waste stabilization pond located in the campus of the University of Nigeria, Nsukka which was used as the field pond and also on a laboratory scale waste stabilization pond obtained from literature. Although the model may be solved by several methods, this research was limited to computational method; numerical solution using finite difference method was used in solving the one- , two- and three-dimensional partial differential equations at steady state conditions. In order to validate the quality of the model, its results were compared with the experimental data from the field and the lab-scale ponds. The results obtained were encouraging, prediction of pond performance with measured values shows that a correlation coefficient of (0.92 – 0.95) was obtained, representing an accuracy of 94% using the 3-D CFD model, an ultimate result that demonstrates that actual dispersion in the pond is three-dimensional. The 2-D model gave an accuracy of 82%. The 1-D model gave an accuracy of 73%, showing that truly dispersion in the pond is not unidirectional. The 3-D model was then used in series of investigation studies such as; effect of single inlet and outlet structures at different positions in the pond, effect of multiple inlet and outlets on the pond's performance, variation of pond performance with depth, effect of short-circuiting on pond treatment efficiency, effect of baffles on pond performance using laboratory-scale pond data and comparison with tracer studies. In all, the results agree with literature. *While it was previously concluded that a CFD model cannot always be expected to precisely predict the performance of a field pond, this work has validated its use.*

Keywords: *Stabilization pond, modeling, computational fluid dynamics, optimization, hydraulics.*

I. INTRODUCTION

Waste stabilization ponds (WSP) are cheap and effective way to treat waste water in situation where the cost of land is not a factor. Not only has it been found to be one thousand times better in destroying pathogenic bacteria and intestinal parasites than the conventional treatment plants, [1], it is also more economical, [2]. It is simple to construct, operate and maintain and it does not require any input of external energy. Although a WSP system usually requires large land area because of its long detention time which is attributable to its complete dependence on natural treatment process, it is still very suitable in several African countries and communities where land acquisition is not a problem. Besides, its efficiency depends on the availability of sunlight and high ambient temperature, which are the prevailing climatic conditions in most cases of these communities.

A. WASTE STABILIZATION PONDS (WSP)

In recent years, a rising chorus of concern has developed regarding the quality of the effluent discharged from WSPs. The basis for the concern is the algae and coli form organisms, which may be present in the effluent. The parameters used in judging the performance of WSP are bacteria rate of degradation, biochemical oxidation, dispersion, bacteria die-off rate and thermal stratification, which are influenced by temperature gradient. Many models [3];[4],[5]; [6],[7]; [8] have been proposed to describe the process of bacteria degradation. But none has been found acceptable. [9] in terms of predicting the practical performance of the WSPs. Hence, the call-in recent times has been to develop more appropriate models that will describe the process accurately [3];[5]; and [9]; [10]. Although WSP system is economical compared with the conventional treatment, no model has yet been found to describe it accurately [5];[9];[3]. WSP are becoming popular for treating wastewater, particularly in tropical and sub-tropical regions where there is an abundance of sunlight, and the ambient temperature is normally high.

B. Computational Fluid Dynamics Approach to WSPs

The term 'computational fluid dynamics', usually abbreviated to 'CFD', encompasses computer-based methods for solving the linked partial-differential equation set that governs the conservation of energy, momentum and mass in fluid flow. In order to understand the internal processes and interaction in waste stabilization ponds, the simulation of the hydrodynamics has become a tool worth studying [5]. Pond design involves several physical, hydrological, geometrical and dynamic variables to provide high hydrodynamic efficiency and maximum substrate utilization rates. Computational fluid dynamic modeling (CFD) allows the combination of these factors to predict the behavior of ponds by using different configurations. The simulation of hydrodynamic in bioreactors supported by modern computing technology is an important tool to gain an improved understanding of the process function and performance. [11]. [11] provided detailed governing dynamic equations to solving the 2D-depth integrated equations of fluid mass and momentum conservation of an incompressible fluid in two horizontal directions.

II. RESEARCH METHODOLOGY

Mathematically, a model describes a system of assumptions, equations and procedures intended to describe the performance of a prototype system. Although the model developed in this study may be solved by several methods, this research was limited to computational method; numerical solution using finite difference method was used in solving the three-dimensional partial differential equations at steady state condition and applying the Danckwerts' boundary conditions [12] and other boundary conditions obtained from the pond surface conditions.

A. Sources of data

The data requirement for the validation of the CFD model developed were obtained from literature of a full-scale field pond (WSP) located at the University of Nigeria, Nsukka, Enugu State [13] and from a published work of a laboratory-scale model [14]. The data analyzed for both the field pond and LSWSP were: : temperature (T^oC); dissolved oxygen (DO); hydrogen ion concentration (P^H); detention time (Ɵ); dispersion number (d); suspended solid (SS); algal concentration (Cs); organic loading rate (OL); faecal coliform per 100ml, the pond settling velocity (V); the maximum pond velocity under no wind (Um); the mean velocity of flow in the pond (U); biochemical oxygen demand (BOD) and chemical oxygen demand (COD).

B. Software application

Since elaborate numerical computations are involved in providing solution to the numerous partial differential equations generated, software application becomes inevitable. One of the software applications used in this study for solving the cumbersome equations generated is the MATLAB. MATLAB is a software package for high-performance numerical computation and visualization [15]. It provides an interactive environment with hundreds of built-in functions for technical computation, graphics, and animation. Best of all, it also provides easy extensibility with its own high-level programming language.

III. MODEL DERIVATION

A. The principle of conservation of mass

A mass balance can be performed on a finite segment of length Δx, as follows:

Accumulation = inflow – outflow – decay reaction

$$\frac{V\Delta C}{\Delta t} = \left(Qc(x) - DA\frac{\partial c(x)}{\partial x}\right) - \left[Q\left(C(x) + \frac{\partial c(x)}{\partial x}\Delta x\right) - DA\left(\frac{\partial c(x)}{\partial x} + \frac{\partial c}{\partial x}\frac{\partial c(x)}{\partial x}\Delta x\right) - KVC\right] \quad 4.01$$

Where; V = volume (m^3), Q = flow rate (m^3/d), C = concentration (mg/L), A = tank cross-sectional area (m^2) and K = first-order decay coefficient (d^{-1})

The dispersion terms are based on Fick's first law;

$$\text{Flux} = -D\frac{\partial c}{\partial x} \quad 4.02$$

It specifies that turbulent mixing tends to move mass from regions of high to low concentration.

The parameter D, therefore, reflects the magnitude of turbulent mixing.

By noting that; V = ADx and U = Q/A

Equation (4.01) can be simplified, thus;

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} - \frac{U\partial c}{\partial x} - Kc \quad 4.03$$

Equation (4.03) is a one-dimensional non-steady state advection-dispersion equation for non-conservative contaminants with a first-order decay rate.

At steady state, it reduces to a second – order ODE,

The material balance equation becomes;

$$Qc - DA\frac{dc}{dx} - Q(c + dc) - DA\frac{dc}{dx} + \frac{d}{dx}\frac{(DAdc)}{dx}dx - Kcd = 0 \quad 4.04$$

By simplification

$$-Qdc - DA\frac{d^2c}{dx^2}dx - Kc.Adx = 0 \quad 4.05$$

$$D\frac{d^2c}{dx^2} - U\frac{dc}{dx} - Kc = 0 \quad 4.06$$

Where; U = flow velocity

Since we are not only interested in what is happening along the x – axis, we cannot ignore what may happen on the transverse (across) axis, that is, y – axis. Similarly, we can formulate a two-dimensional equation as;

$$\frac{\partial c}{\partial t} = D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} - \frac{U\partial c}{\partial x} - \frac{V\partial c}{\partial y} - Kc \quad 4.07$$

At steady state, equation (4.07) becomes;

$$D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} - \frac{U\partial c}{\partial x} - \frac{V\partial c}{\partial y} - Kc = 0$$

Without ignoring what is taking place in the vertical axis, since we have assumed that the pond is well-mixed vertically and laterally; we extend our model to a three-dimensional one.

$$\frac{\partial c}{\partial t} = D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} - D_z\frac{\partial^2 c}{\partial z^2} - \frac{U\partial c}{\partial x} - \frac{V\partial c}{\partial y} - \frac{W\partial c}{\partial z} - Kc \qquad 4.09$$

At steady state, equation (4.09) becomes;

$$D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} - D_z\frac{\partial^2 c}{\partial z^2} - \frac{U\partial c}{\partial x} - \frac{V\partial c}{\partial y} - \frac{W\partial c}{\partial z} - Kc = 0 \qquad 4.10$$

Where,

D_x, D_y and D_z are the dispersion coefficient in the x, y and z axis respectively. U, V and W are the velocity components in the x, y and z Cartesian co-ordinate respectively.

B. The principle of conservation of momentum

The second conservation equation that is used in CFD is the momentum equation. The momentum equation is developed based on the Newton's second law of motion. Simplification of the momentum equation involves the use of the Navier-Stokes equation and is very useful for the application of the finite volume.

According to Newton's second law of motion;

$$\sum F = M.a \qquad 4.11$$

Considering the forces only in the x – direction, equation (4.11) may be written as

$$Ma_x = F_{gx} + F_{px} + F_{vx} \qquad 4.12$$

Equation (4.12) is called the Navier – Stokes equation of motion.

For complete derivation, it may be presented as:

$$\rho\left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}\right] = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + S_{mx} \qquad 4.13$$

$$\rho\left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}\right] = -\frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] + S_{my} \qquad 4.14$$

$$\rho\left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}\right] = -\frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] + S_{mz} \qquad 4.15$$

For incompressible flow, the number of unknowns is four viz; u, v, w and p.

The Navier-Stokes equation plus incompressible continuity equation are the sufficient conditions to determine the flow characteristics.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad 4.16$$

The general solution of Navier-Stokes equations has not been found as it is second order non-linear differential equation. However, the solutions have been obtained only for flow situations wherein the boundary configuration is simple and the fluid characteristics such as the density and viscosity are almost constant.

By applying the boundary configuration, equations (4.13), (4.14), (4.15) and (4.16) are solved simultaneously using the finite difference method to determine the fluid velocities in the X, Y and Z directions in the CFD model.

C. Finite difference solution of the 3-D equation

$$Dx\frac{\partial^2 C}{\partial x^2} + D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2} - \frac{U\partial C}{\partial x} - \frac{V\partial C}{\partial y} - \frac{W\partial C}{\partial z} - KC = 0 \qquad 4.17$$

Writing the finite difference scheme,

$$\frac{\partial^2 C}{\partial x^2} = \frac{C_{i+1,j,k} - 2C_{i,j,k} + C_{i-1,j,k}}{h^2}, \qquad \frac{\partial^2 C}{\partial y^2} = \frac{C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}}{p^2},$$

$$\frac{\partial^2 C}{\partial z^2} = \frac{C_{i,j,k+1} - 2C_{i,j,k} + C_{i,j,k-1}}{l^2}$$

$$\frac{\partial C}{\partial x} = \frac{C_{i+1,j,k} - 2C_{i,j,k} + C_{i-1,j,k}}{2h}, \qquad \frac{\partial C}{\partial y} = \frac{C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}}{2p},$$

$$\frac{\partial C}{\partial z} = \frac{C_{i,j,k+1} - 2C_{i,j,k} + C_{i,j,k-1}}{2l}$$

$$Dx\left[\frac{C_{i+1,j,k} - 2C_{i,j,k} + C_{i-1,j,k}}{h^2}\right] + Dy\left[\frac{C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}}{p^2}\right] + Dz\left[\frac{C_{i,j,k+1} - 2C_{i,j,k} + C_{i,j,k-1}}{l^2}\right] - U\left[\frac{C_{i+1,j,k} - 2C_{i,j,k} + C_{i-1,j,k}}{2h}\right] - V\left[\frac{C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}}{2p}\right] - W\left[\frac{C_{i,j,k+1} - 2C_{i,j,k} + C_{i,j,k-1}}{2l}\right] - C_{i,j,k} = 0$$

Rearranging the expression, it becomes;

$$\left(\frac{Dx}{h^2} - \frac{U}{2h}\right)C_{i+1,j,k} + \left(\frac{Dx}{h^2} + \frac{U}{2h}\right)C_{i-1,j,k} + \left(\frac{Dy}{p^2} - \frac{V}{2p}\right)C_{i,j+1,k} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1,k} + \left(\frac{Dz}{l^2} - \frac{W}{2l}\right)C_{i,j,k+1} + \left(\frac{Dz}{l^2} + \frac{W}{2l}\right)C_{i,j,k-1} - \left(\frac{2Dx}{h^2} + \frac{2Dy}{p^2} + \frac{2Dz}{l^2} + K\right)C_{i,j,k} = 0$$

4.18

Boundary conditions;

$$UC = UC_o - D\frac{\partial C}{\partial x}, \quad x = 0 \quad \text{(Inlet)} \quad \text{(Danckwert, 1957)}$$

$$\frac{\partial C}{\partial x} = 0, x = L \quad \text{(Outlet) ends}$$

$$\frac{\partial C}{\partial x} = 0, y = 0, B, \qquad \frac{\partial C}{\partial x}\ \frac{\partial C}{\partial z} = 0, \qquad 0 \le Z \le d$$

At the inlet where x = 0, the term $C_{i-1,j,k}$ outside the scheme was obtained

$$\left(\frac{Dx}{h^2} - \frac{U}{2h}\right)C_{i+1,j,k} + \left(\frac{Dx}{h^2} + \frac{U}{2h}\right)C_{i-1,j,k} + \left(\frac{Dy}{p^2} - \frac{V}{2p}\right)C_{i,j+1,k} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1,k} + \left(\frac{Dz}{l^2} - \frac{W}{2l}\right)C_{i,j,k+1} + \left(\frac{Dz}{l^2} + \frac{W}{2l}\right)C_{i,j,k-1} - \left(\frac{2Dx}{h^2} + \frac{2Dy}{p^2} + \frac{2Dz}{l^2} + K\right)C_o = 0 \qquad 4.19$$

For the boundary condition simplified as below, we obtained that

$C_{i+1,j,k} \approx C_{i,j,k+1}$ and $C_{i-1,j,k} \approx C_{i,j,k-1}$

This simplifies the above equation to give;

$$\left(\frac{Dx}{h^2} + \frac{U}{2h} + \frac{Dz}{l^2} + \frac{W}{2l}\right)C_{i-1,j,k} - \left(\frac{2Dx}{h^2} + \frac{2Dy}{p^2} + \frac{2Dz}{l^2} + K\right)C_o + \left(\frac{Dx}{h^2} + \frac{Dz}{l^2} - \frac{U}{2h} - \frac{W}{2l}\right)C_{i+1,j,k} + \left(\frac{Dy}{p^2} - \frac{V}{2p}\right)C_{i,j+1,k} + \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1,k} = 0 \qquad 4.20$$

By involving the boundary condition for the inlet;

$$UC_{in} = UC_o - Dx\frac{\partial C}{\partial x}$$

A finite divided difference can be substituted for the derivative, where C_o = concentration at x = 0. Thus

$UC_{in} = UC_o - Dx\left(\frac{C_{i+1,j,k} - C_{i-1,j,k}}{2h}\right)$, which can be solved for

$C_{i-1,j,k} = \frac{2hUC_{in}}{Dx} - \frac{2hUC_o}{Dx} + C_{i+1,j,k}$, substitute in equation (4.20)

$$\left(\frac{2Dx}{h^2} + \frac{2Dy}{p^2} + \frac{2Dz}{l^2} + K + \frac{2U}{h} + \frac{U^2}{Dx} + \frac{2DzhU}{l^2Dx} + \frac{WhU}{lDx}\right)C_o - 2\left(\frac{Dx}{h^2} + \frac{Dz}{l^2}\right)C_{i+1,j,k} - \left(\frac{Dy}{p^2} + \frac{V}{2p}\right)C_{i,j-1,k} - \left(\frac{Dy}{p^2} - \frac{V}{2p}\right)C_{i,j+1,k} = \left(\frac{2U}{h} + \frac{U^2}{Dx} + \frac{2DzhU}{l^2Dx} + \frac{WhU}{lDx}\right)C_{in} \qquad 4.21$$

For the outlet, the slope must be zero, that is;

$$\frac{\partial C}{\partial x} = 0, x = L$$

A finite divided difference can be written as: $\frac{C_{i+1,j,k}-C_{i-1,j,k}}{2h}=0$

It implies that $C_{i+1,j,k}=C_{i-1,j,k}$, Substitute this result in equation (4.18).

Also, $\frac{\partial C}{\partial x}=\frac{\partial C}{\partial z}=0,\quad 0\le Z\le d$

The finite divided difference can be written thus

$\frac{C_{i+1,j,k}-C_{i-1,j,k}}{2h}=\frac{C_{i,j,k+1}-C_{i,j,k-1}}{2l}=0$, This implies that $C_{i+1,j,k}=C_{i-1,j,k}$ and $C_{i,j,k+1}=C_{i,j,k-1}$

Inspection of this equation leads us to conclude that $C_{i+1,j,k}\equiv C_{i,j,k+1}$ And $C_{i-1,j,k}\equiv C_{i,j,k-1}$

By multiplying the above expression with a coefficient as a function of dispersion, velocity and mesh size in that direction, an approximate value can be obtained. That is.

$C_{i+1,j,k}\approx C_{i,j,k+1}$ And $C_{i-1,j,k}\approx C_{i,j,k-1}$

Substitute this result in equation (4.18) for $0\le Z\le d$ representing the upper and lower layers of the pond. Therefore, at the pond outlet

$$2\left(\frac{Dx}{h^2}\right)C_{i-1,j,k}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1,k}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1,k}+2\left(\frac{Dx}{l^2}\right)C_{i-1,j,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.22$$

By further simplification

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i-1,j,k}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1,k}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.23$$

At the sides of the pond: $\frac{\partial C}{\partial y}=0,\quad y=0,B$

The divided difference is written as; $\frac{C_{i,j+1,k}-C_{i,j-1,k}}{2p}=0$

Hence, $C_{i,j+1,k}=C_{i,j-1,k}$

Substitute in equation (4.18), for y = 0

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}+\frac{Dz}{l^2}-\frac{W}{2l}\right)C_{i+1,j,k}+\left(\frac{Dx}{h^2}+\frac{U}{2h}+\frac{Dz}{l^2}+\frac{W}{2l}\right)C_{i-1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.24$$

For y = B

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}+\frac{Dz}{l^2}-\frac{W}{2l}\right)C_{i+1,j,k}+\left(\frac{Dx}{h^2}+\frac{U}{2h}+\frac{Dz}{l^2}+\frac{W}{2l}\right)C_{i-1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.25$$

At the pond outlet, for which y = 0, the equation can be written as

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i-1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.26$$

At the pond outlet, for which y = B, the equation is

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i-1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.27$$

The general equation may be written for each of the system's nodes within the pond as:

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}+\frac{Dz}{l^2}-\frac{W}{2l}\right)C_{i+1,j,k}+\left(\frac{Dx}{h^2}+\frac{U}{2h}+\frac{Dz}{l^2}+\frac{W}{2l}\right)C_{i-1,j,k}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1,k}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0$$

Applying the boundary condition;

$\frac{\partial C}{\partial x}=0,\quad x=0,\quad 0<y<B$

The divided difference is written as; $\frac{C_{i+1,j,k}-C_{i-1,j,k}}{2h}=0$

Hence, $C_{i+1,j,k}=C_{i-1,j,k}$

Substituting in equation (4.18), yields

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i+1,j,k}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1,k}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0$$

At the pond edge (x = 0) for which y = 0, $C_{i,j+1,k}=C_{i,j-1,k}$

Substitute in equation (4.29)

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i+1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.30$$

At the pond edge (x = 0) for which y = b

$$2\left(\frac{Dx}{h^2}+\frac{Dz}{l^2}\right)C_{i+1,j,k}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1,k}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+\frac{2Dz}{l^2}+K\right)C_{i,j,k}=0 \qquad 4.31$$

Equations (4.21), (4.23), (4.24), (4.25), (4.26), (4.27), (4.28), (4.29), (4.30) and (4.31) are then applied at corresponding nodes within the system; the numerous equations generated are solved simultaneously to determine the variation of concentrations within the pond.

D. Finite difference solution of the 2-D equation

$$Dx\frac{\partial^2 C}{\partial x^2}+Dy\frac{\partial^2 C}{\partial y^2}-U\frac{\partial C}{\partial x}-\frac{V\partial C}{\partial y}-KC=0 \qquad 4.32$$

Writing the finite difference scheme

$\frac{\partial^2 C}{\partial x^2}=\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}}{h^2}$, $\frac{\partial^2 C}{\partial y^2}=\frac{C_{i,j+1}-2C_{i,j}+C_{i,j-1}}{p^2}$, $\frac{\partial C}{\partial x}=\frac{C_{i+1,j}-C_{i-1,j}}{2h}$, $\frac{\partial C}{\partial y}=\frac{C_{i,j+1}-C_{i,j-1}}{2p}$

$$Dx\left[\frac{C_{i+1j}-2C_{ij}+C_{i-1j}}{h^2}\right]+Dy\left[\frac{C_{ij+1}-2C_{ij}+C_{ij-1}}{p^2}\right]-U\left[\frac{C_{i+1j}-C_{i-1j}}{2h}\right]-V\left[\frac{C_{ij+1}-C_{ij-1}}{2p}\right]-KC_{ij}=0$$

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{i+1j}+\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{i-1j}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{ij+1}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{ij-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{ij}=0 \qquad 4.33$$

Boundary conditions: $UC_{in}=UC_o-D\frac{\partial C}{\partial x}, x=0$ (Inlets) $\frac{\partial C}{\partial x}=0,\ x=0,\ 0<y<B$ And $\frac{\partial C}{\partial x}=0,\ x=L$ (Outlet end), $\frac{\partial C}{\partial y}=0,\ y=0,B$

At the inlet where x = 0, the term $C_{i-1,j}$ outside the scheme was obtained

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{i+1,j}+\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{i-1,j}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{ij-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_0=0 \qquad 4.34$$

By invoking the boundary condition for the inlet; $UC_{in}=UC_o-D\frac{\partial C}{\partial x}$

A finite divide difference can be substituted for the derivative, where Co = concentration at x = 0 at the inlet position. Thus, $UC_{in}=UC_o-Dx\left(\frac{C_{i+1,j}-C_{i-1,j}}{2h}\right)$, which can be solved to give

$C_{i-1,j}=\frac{2hu}{Dx}C_{in}-\frac{2hu}{Dx}C_o+C_{i+1,j}$ Substitute in equation (4.34)

$$\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K+\frac{2U}{h}+\frac{U^2}{Dx}\right)C_0-\left(\frac{2Dx}{h^2}\right)C_{i+1,j}-\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1}-\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1}=\left(\frac{2U}{h}+\frac{U^2}{Dx}\right)C_{in} \qquad 4.35$$

$\frac{\partial C}{\partial x}=0,\ x=0,\quad 0<y<B$, $\frac{C_{i+1,j}-C_{i-1,j}}{2h}=0$, It implies that, $C_{i+1,j}=C_{i-1,j}$

Substitute in equation (4.33)

$$\left(\frac{2Dx}{h^2}\right)C_{i+1,j}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.36$$

At the edge of the pond where x = 0, y = 0; $\frac{\partial C}{\partial y}=0$, $\frac{C_{i,j+1}-C_{i,j-1}}{2p}=0$

Hence $C_{i,j+1}=C_{i,j-1}$ Substitute in equation (4.36)

$$\left(\frac{2Dx}{h^2}\right)C_{i+1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.37$$

At the edge of the pond where x = 0, y = B

$$\left(\frac{2Dx}{h^2}\right)C_{i+1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.38$$

For the outlet, the slope must be zero, that is; $\frac{\partial C}{\partial x}=0, x=L,\quad 0<y<B$

The finite divided difference will yield, $C_{i+1,j}=C_{i-1,j}$

Substitute in equation (4.33)

$$\left(\frac{2Dx}{h^2}\right)C_{i-1,j}+\left(\frac{Dy}{p^2}-\frac{V}{2p}\right)C_{i,j+1}+\left(\frac{Dy}{p^2}+\frac{V}{2p}\right)C_{i,j-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.39$$

At the pond outlet ends, for which y = 0, the equation can be written as

$$\left(\frac{2Dx}{h^2}\right)C_{i-1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.40$$

At the pond outlet ends for which y = B, the equation becomes

$$\left(\frac{2Dx}{h^2}\right)C_{i-1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.41$$

At the sides of the pond, $\frac{\partial C}{\partial y}=0,\ \ y=0,B$. It implies as before $C_{i,j+1}=C_{i,j-1}$

Substitute in equation (4.33) for y = 0

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{i+1,j}+\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{i-1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j+1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad (4.42)$$

For y = B

$$\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{i+1,j}+\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{i-1,j}+\left(\frac{2Dy}{p^2}\right)C_{i,j-1}-\left(\frac{2Dx}{h^2}+\frac{2Dy}{p^2}+K\right)C_{i,j}=0 \qquad 4.43$$

The general equation for the system's nodes within the pond may be written using equation (4.33) above.

E . difference solution of the 1-D equation

$$Dx\frac{d^2C}{dx^2}-\frac{UdC}{dx}-KC=0$$

The finite difference scheme can be written as

$\frac{d^2C}{dx^2}=\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}}{h^2}$, $\frac{dC}{dx}=\frac{C_{i+1}-C_{i-1}}{2h}$ Hence,

$$Dx\left(\frac{C_{i+1,j}-2C_{i,j}+C_{i-1,j}}{h^2}\right)-U\left(\frac{C_{i+1}-C_{i-1}}{2h}\right)-KC_i=0$$

Rearranging the equation;

$$\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{i-1}+\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{i+1}-\left(\frac{2Dx}{h^2}+K\right)C_i=0 \qquad 4.45$$

Boundary conditions: $UC_{in}=UC_0-D\frac{\partial C}{\partial x}, x=0$ (Inlet)

$\frac{\partial C}{\partial x}=0,\qquad x=L,\qquad$ (outlet)

At the inlet where x = 0, the term C_{i-1} outside the scheme was obtained

$$\left(\frac{Dx}{h^2}+\frac{U}{2h}\right)C_{-1}+\left(\frac{Dx}{h^2}-\frac{U}{2h}\right)C_{+1}-\left(\frac{2Dx}{h^2}+K\right)C_0=0 \qquad 4.46$$

By invoking the boundary condition for the inlet;

$$UC_{in}=UC_0-Dx\frac{\partial C}{\partial x}$$

A finite difference scheme can be substituted for the derivative, where C_0 = concentration at x = 0. Thus

$UC_{in}=UC_0-Dx\left(\frac{C_{i+1}-C_{i-1}}{2h}\right)$ This can be solved for

$C_{i-1}=\frac{2hU}{Dx}C_{in}-\frac{2hU}{Dx}C_0+C_{i-1}$ Substitute in equation 4.46

$$\left(\frac{2Dx}{h^2}+K+\frac{2U}{h}+\frac{U^2}{Dx}\right)C_0-\left(\frac{2Dx}{h^2}\right)C_{i+1}=\left(\frac{2U}{h}+\frac{U^2}{Dx}\right)C_{in} \qquad 4.47$$

At the outlet, the slope must be zero, that is; $\frac{\partial C}{\partial x}=0, x=L$ a finite divided difference can be written as: $\frac{C_{i+1}-C_{i-1}}{2h}=0$

Hence, $C_{i+1}=C_{i-1}$

Substitute in equation (4.45)

$$\left(\frac{2Dx}{h^2}\right)C_{i-1}-\left(\frac{2Dx}{h^2}+K\right)C_i=0 \qquad 4.48$$

The general equation for the system nodes along the pond may be written using equation (4.45) above.

IV. RESULTS AND DISCUSSIONS

A. Comparisons of prediction of pond performance in 3-D, 2-D and 1-D CFD model with measured values.

These comparisons were made at different depth of the pond and at varying inlets and outlets positions. The following figures (1 – 5) demonstrate it appropriately.

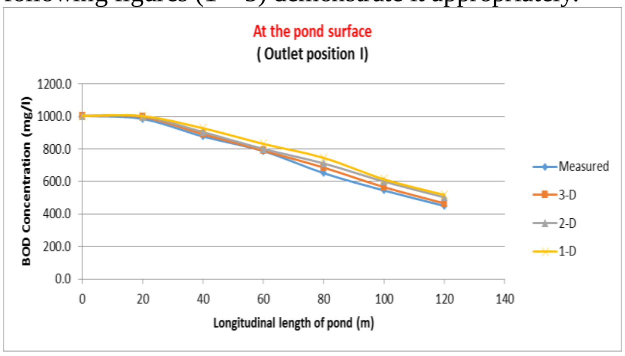

Fig. 1: Comparison between measured value and CFD models using inlet position I.

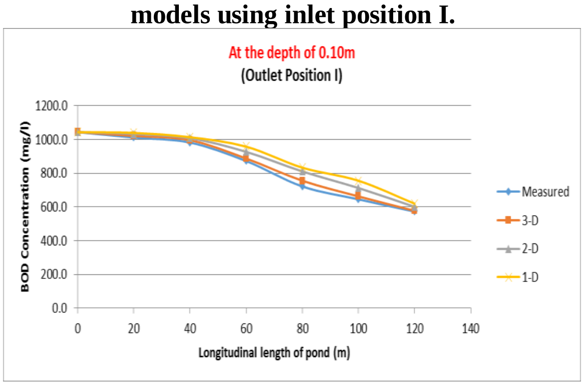

Fig. 2: Comparison between measured value and CFD models using inlet position I.

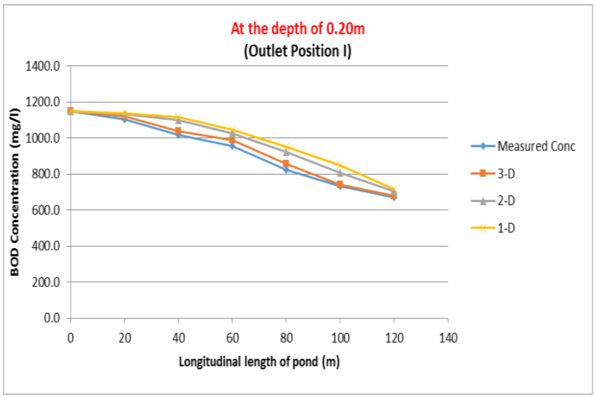

Fig. 3: Comparison between measured value and CFD models using inlet position I.

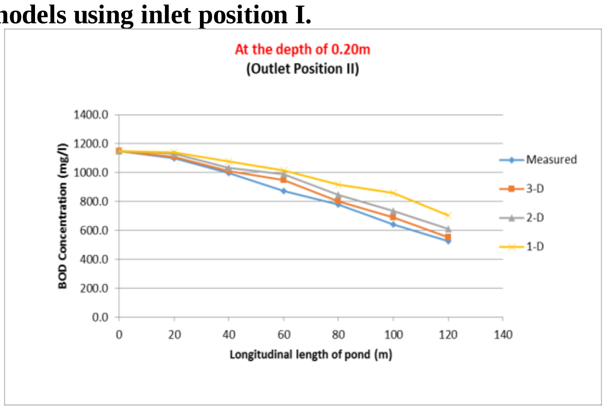

Fig. 4: Comparison between measured value and CFD models using inlet position I.

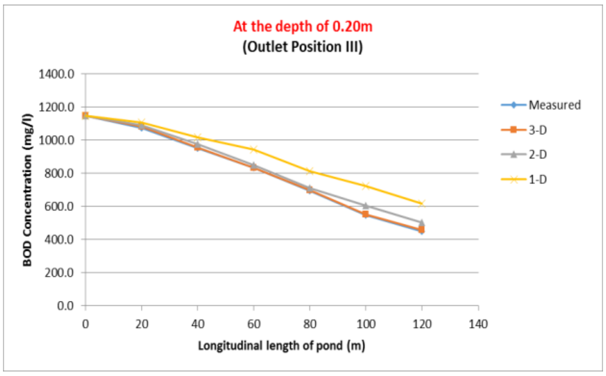

Fig. 5: Comparison between measured value and CFD models using inlet position I.

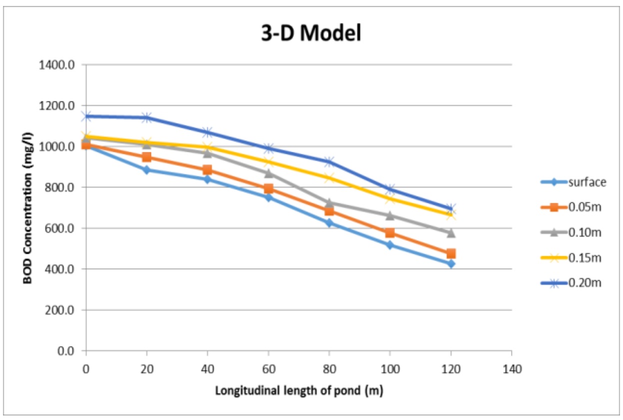

Fig. 6: Variation of BOD Concentration with depth of pond along longitudinal length of pond.

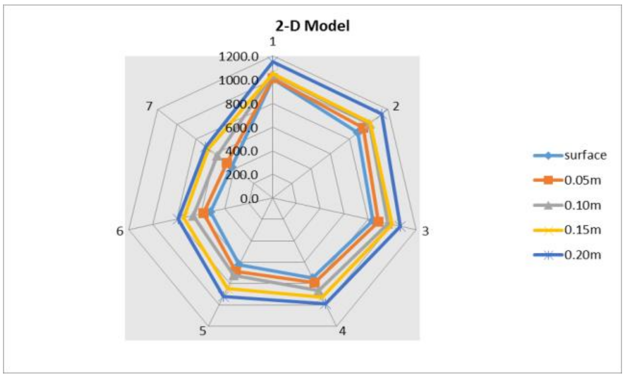

Fig. 7: Variation of BOD Concentration with depth of pond

B. Effect of single inlet and outlet positions on pond performance

As demonstrated previously, position of the inlet and outlet structures affects the pond's treatment efficiency. Simulation using the model was performed. The result obtained shows variance with respect to inlet/outlet positions used, and this prove the fact.

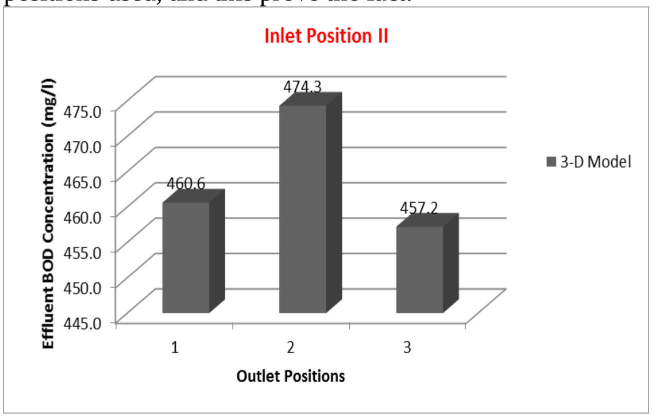

Fig. 8: Effect of single inlet/outlet position on WSP performance.

C. Effect of multiple inlet and outlets on pond performance

Multiple inlets and/or outlets also adversely affect the pond's performance. In most cases, it leads to short-circuiting in the pond. A situation whereby the detention time of fluid particle in the pond is shortened due to flow conditions (in this case, due to excessive inflow and/or outflow).

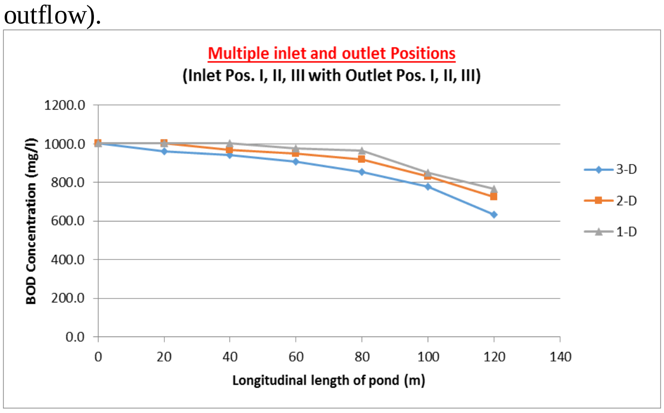

Fig. 9: Effect of multiple inlet/outlets on WSP performance.

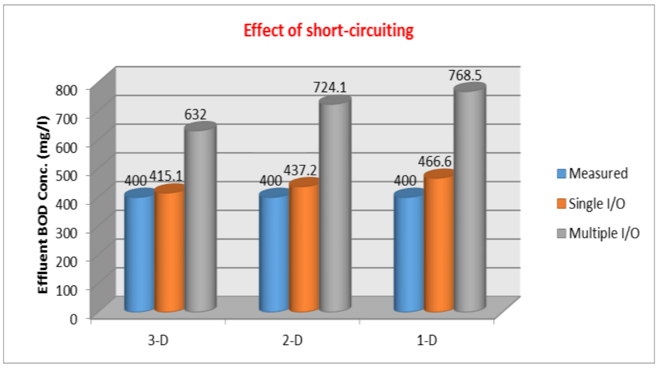

Fig. 10: Effect of short-circuiting on pond performance

C. Prediction of short-circuiting from flow circulation.

Figure:10 above shows the effect of short-circuiting on the treatment efficiency of WSP. The effluent values for single inlet and outlet was smaller than that of the multiple inlet/outlet and in agreement with measured values, this indicates that flow circulation was normal. On the other hand, effluent BOD concentration obtained from the multiple case of inlet and outlet shows a wide variation from the measured values, indicating that there is a problem with flow circulation. This is nothing but the presence of short-circuiting due to excessive inflow and outflow.

D. Comparisons between tracer studies and CFD predictions

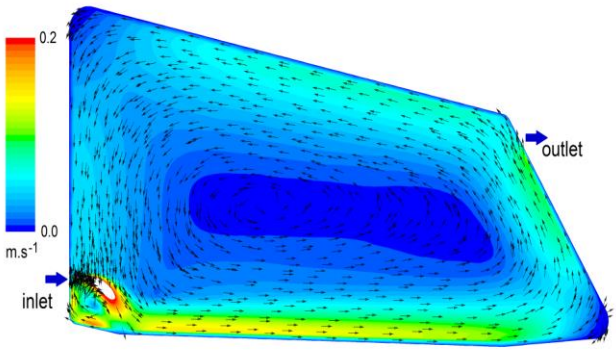

Fig11: Sequential plots of the simulated transient tracer movement in pond. Colors at the top of the scale indicate higher tracer concentration.

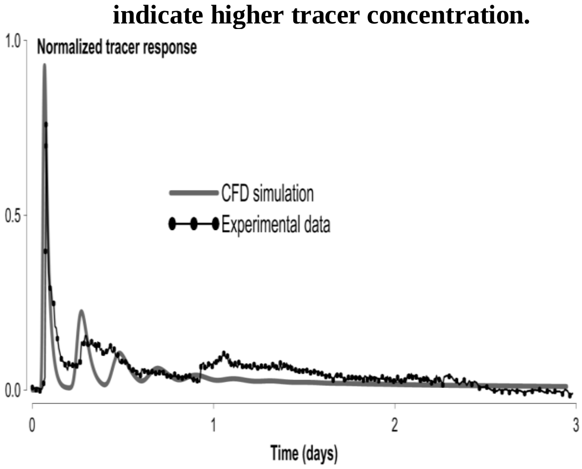

Fig. 12: Comparison of the RTD curves obtained from the tracer experiment and the CFD model.

Figure 12 shows a reasonable agreement between the RTD curves from the tracer study and the CFD model prediction. The CFD model was able to capture the magnitude and timing of the first peak reasonably well, but the subsequent peaks are less clear in the experimental results even though there seems to be one between 0.25 d and 0.50 d. The disagreement could be due to the simplifying assumptions in the CFD model. Although the match is not perfect over the entire RTD curves, the validation results of the CFD model are fairly good.

V. CONCLUSIONS

Modeling waste stabilization pond is somewhat a daunting course due to the complexity involve in understanding its hydraulics. The results obtained were encouraging, prediction of pond performance with measured values shows that an accuracy of 94% was obtained using the 3-D CFD model, an ultimate result that shows that actual dispersion in the pond is three-dimensional. Although the 2-D model gave a reasonable results, as an average accuracy of 82% was recorded, but did not give a true picture of what the hydrodynamics is, in the pond. The 1-D model gave an accuracy of 73%, showing that truly dispersion in the pond is not unidirectional. The 3-D model was then used in series of investigation studies such as; effect of single inlet and outlet structures at different positions in the pond, effect of multiple inlet and outlets, variation of pond performance with depth, effect of short-circuiting on pond treatment efficiency, effect of baffles on pond performance using laboratory-scale pond data and comparison with tracer studies. In all, the results were satisfactory.

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