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Original Article

Predicting the Response of Concrete Columns to Eccentric Loading using Finite Element Analysis

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Abstract *- This study presents a numerical simulation of the behavior of reinforced concrete columns under eccentric loading using the finite element method. Three columns with different eccentricities were analyzed using Abaqus software. The results were compared with laboratory experiments to validate the accuracy of the simulations. The analysis focused on the concrete damage and cracks, force-displacement curves, and bearing and ultimate capacity of the columns. The results showed that as the eccentricity increased, the bearing capacity of the columns decreased, and the bending behavior of the columns became more pronounced. Todeschini's behavioral model was found to accurately predict the behavior of the columns when the eccentricity was low but less accurate at high eccentricity values. These findings have important implications for designing and analysing reinforced concrete columns under eccentric loading.*

Keywords *- Reinforced concrete columns, Eccentric loading, Finite element analysis, Concrete damage plasticity, Ultimate capacity.*

1. Introduction

Concrete columns are one of the most common structural elements used in construction, providing support to buildings, bridges, and other infrastructure. These columns are designed to withstand compressive forces and carry loads. However, in real-world applications, they are often subjected to eccentric loading due to various factors such as uneven distribution of load, construction errors, or seismic activity. Eccentric loading is defined as a load that is not applied through the centroid of the column's cross-sectional area, resulting in the bending of the column and inducing high stresses and strains that can lead to premature failure. The behavior of concrete columns under eccentric loading has been studied for many years. Researchers have explored different analytical, numerical, and experimental approaches to understand better the complex interactions between the load, the column, and the surrounding environment. Analytical models, such as the effective length method or the moment magnifier method, are based on theoretical assumptions and simplified equations, which are suitable for preliminary design and assessment, but may not accurately capture the nonlinear and dynamic behavior of the column under large eccentricities or high deformations.

Experimental tests, such as compression or bending tests, can provide valuable insights into the material properties, failure modes, and energy dissipation of concrete columns but are often costly, time-consuming, and limited in the number of samples and loading conditions. On the other hand, numerical simulations offer a powerful tool to model and analyze the behavior of concrete columns under different eccentric loading scenarios using advanced software such as Abaqus, ANSYS, or LS-DYNA. FEA is well-suited for studying concrete columns under eccentric loading because it allows for a more detailed and accurate representation of the complex stress and strain distributions that occur in the column under these loading conditions and can model the behavior of the column from elastic to post-failure regimes.

Also, FEA allows for a detailed and accurate representation of the column's geometry, material properties, and boundary conditions and can simulate the behavior of the column from elastic to post-failure regimes. However, FEA also requires careful calibration and validation of the material models, mesh size, and time step and can be computationally expensive for large and complex models. The behavior of concrete columns under eccentric loading is of critical importance for the safety and reliability of structures. The failure of a single column can lead to a catastrophic collapse of the entire structure, with severe consequences in terms of human lives, economic loss, and environmental impact. Therefore, the accurate prediction of the response of concrete

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columns to eccentric loads is a key factor in the design, construction, and maintenance of buildings and infrastructure.

Moreover, studying concrete columns under eccentric loading is a topic of ongoing research as new materials, geometries, and loading scenarios emerge. For example, the use of high-strength or ultra-high-performance concrete (UHPC) can significantly affect the behavior of columns under eccentric loading due to their different stress-strain curves, failure mechanisms, and ductility. Similarly, applying seismic or wind loads can create complex and dynamic eccentricities that require advanced modeling and analysis techniques. Therefore, this study aims to contribute to the current knowledge of this important topic by using the Abaqus software to model the behavior of concrete columns under eccentric loading and to provide insights into the key factors that influence the performance of these structures. This study aims to investigate the behavior of concrete columns under eccentric loading using finite element analysis with Abaqus software. The study aims to provide insights into the key factors that influence the performance of concrete columns under eccentric loading and to contribute to the current knowledge of this important topic.

2. Literature Review

In 1995 Wierzbicki and Pietruszczak investigated the behavior of reinforced concrete columns under eccentric loading using finite element analysis. The authors developed a 3D numerical model of a reinforced concrete column and validated it against experimental data. The effects of the eccentricity, column slenderness, and reinforcement ratio on the ultimate strength and deformation of the column were analyzed. The results showed that the level of eccentricity and the slenderness ratio influenced the column's failure modes. Another research by Guedes and Massuda analyzed the effect of eccentricity on the behavior of reinforced concrete columns subjected to axial load. The authors conducted a parametric study using a 3D numerical model and investigated the influence of the eccentricity, concrete strength, reinforcement ratio, and slenderness ratio on the ultimate strength and deformation of the column. The results showed that eccentricity significantly affected the failure modes and ductility of the column.

Chen, in 2007 investigated the behavior of reinforced concrete columns under eccentric loading using experimental and analytical methods. The authors conducted a series of tests on scaled-down columns with different eccentricities and reinforcement ratios and compared the results with those obtained from finite element analysis. The effects of the eccentricity and reinforcement ratio on the column's ultimate strength, deformation, and failure modes were analyzed. The results showed good agreement between the experimental and analytical data and demonstrated the importance of the reinforcement ratio in improving the ductility of the column. Also, Rao and Rajamane investigated the behavior of reinforced concrete columns under eccentric loading using experimental and analytical methods. The authors conducted a series of tests on full-scale columns with different eccentricities and concrete strengths and compared the results with those obtained from analytical models. The effects of eccentricity, concrete strength, and confinement on the ultimate strength and deformation of the column were analyzed. The results showed that the confinement level significantly affected the column's ductility and energy absorption capacity.

S. A. Ghannad et al. investigated the behavior of slender reinforced concrete columns under eccentric load using finite element analysis. The authors developed a 3D numerical model of a slender column and investigated the effects of eccentricity, slenderness ratio, and concrete strength on the behavior of the column. The results showed that the level of eccentricity significantly affected the ultimate strength and deformation of the column and that the level of slenderness and concrete strength influenced the failure modes. Silva, Barros, and Rocha, 2017, investigated the behavior of reinforced concrete columns with different cross-section geometries under eccentric loading using experimental and numerical methods. The authors conducted a series of tests on full-scale columns with circular and rectangular cross-sections and compared the results with those obtained from finite element analysis. The effects of the cross-section geometry, eccentricity, and slenderness ratio on the behavior of the column were analyzed. The results showed that the cross-section geometry significantly affected the column's ultimate strength, deformation, and failure modes.

Cheng and Lu investigated the behavior of reinforced concrete columns under axial load and eccentric loading using experimental and numerical methods. The authors conducted a series of tests on full-scale columns with different eccentricities and reinforcement ratios and compared the results with those obtained from finite element analysis. The effects of the eccentricity and reinforcement ratio on the column's ultimate strength, deformation, and energy absorption capacity were analyzed. The results showed good agreement between the experimental and numerical data, demonstrating the importance of the reinforcement ratio in improving the column's ductility and energy absorption capacity.

3. Theoretical Concepts

Reinforced concrete columns are subjected to axial loads and bending moments, which can result in complex stress states and failure modes. When a load is not applied through the centroid of the cross-section, the column is said to be under eccentric loading. This results in a bending moment being applied to the column, which can cause it to fail in bending or combined bending and axial compression (Silva et al., 2017).

The eccentricity of the load can be expressed as the distance between the cross-section's centroid and the applied load's line of action. This can be calculated using Equation 1 (Cheng & Lu, 2020):

$$e = \frac{M}{N} \times \frac{h}{2} \tag{1}$$

where e is the eccentricity, M is the applied moment, N is the axial load, and h is the height of the cross-section. The resulting bending moment can be calculated using Equation 2 (Cheng & Lu, 2020):

$$M = e \times N \tag{2}$$

The ultimate strength of a concrete column under eccentric loading can be determined using the interaction diagram, representing the maximum axial load and bending moment the column can withstand without failure. The interaction diagram can be derived using the equilibrium equations and the stress-strain relationship of concrete and steel. The stress-strain relationship of concrete is generally nonlinear and can be approximated using a parabolic curve up to the peak stress, after which it enters the post-peak stage and exhibits significant softening. The stress-strain relationship of steel is generally linear up to the yield stress. The load-carrying capacity of a concrete column under eccentric loading can be calculated using Equation 3 (Silva et al., 2017):

$$Pc = Ag \times f'c + As \times fy \tag{3}$$

where P_c is the load-carrying capacity, A_g is the gross area of the cross-section, f'_c is the compressive strength of concrete, A_s is the area of longitudinal reinforcement, and f_y is the yield strength of the reinforcement. In addition to the load-carrying capacity, the deformation capacity and ductility of the column are also important factors to consider. The ductility of the column can be determined by calculating the curvature ductility factor, which represents the ratio of the curvature at the ultimate load to the yield curvature. The curvature ductility factor can be calculated using Equation 4 (Cheng & Lu, 2020):

$$\mu_c = \kappa_u / \kappa_y \tag{4}$$

Where μ_c is the curvature ductility factor, κ_u is the ultimate curvature, and κ_y is the yield curvature.

In this study, the Todeschini method is used for modeling the nonlinear compressive strength of concrete in finite element analysis (FEA). This method involves defining the stress-strain relationship for the concrete using an analytical function, such as a polynomial or a hyperbolic function. The function coefficients are calibrated using experimental data, such as compression tests on concrete specimens (Figure 1).
In the relationship shown in Figure 1, f'c is compressive strength, ε is strain corresponding to stress, and ε0 is strain corresponding to the maximum strength.

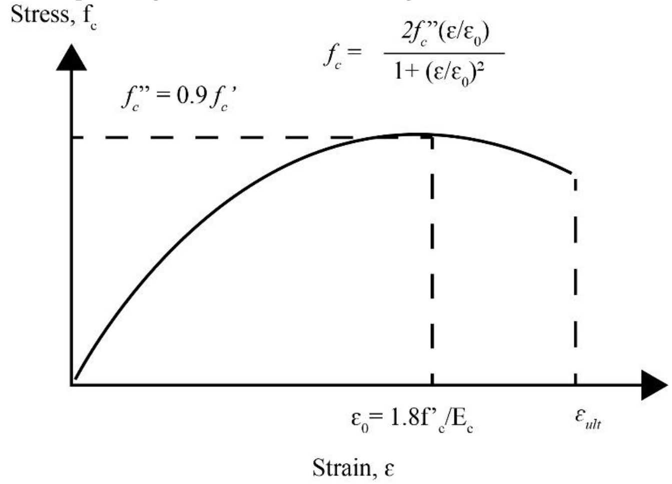

Fig. 1 Todeschini compressive strength model

Using the Todeschini method to model the compressive strength of concrete, the behavior of the concrete column under loading can be more accurately predicted in FEA software. This is because the Todeschini method takes into account the nonlinear behavior of the concrete, which is important for the accurate prediction of the column and its crack propagation.

4. Finite Element Simulation of RC Column using ABAQUS

The numerical modeling section of this study involved using the Abaqus software to simulate the behavior of concrete columns under eccentric loading. The Abaqus software is a powerful finite element analysis tool that allows for creating detailed models of complex structures and simulating their behavior under various loading conditions. In this study, we used the Todeschini method to model the nonlinear compressive strength of the concrete in the Abaqus software, and we incorporated other important material properties, such as the tensile and shear strengths of the concrete and the reinforcing steel, into the model. We also took into account the effects of concrete cracking and the bond between the concrete and the steel reinforcement. To better evaluate the results of this numerical modelling, we used a laboratory study conducted by Jin et al. (2016) on the size effect in eccentrically loaded stocky reinforced concrete (RC) columns (figure 2). This study provided experimental data on the behavior of concrete columns under eccentric loading, which we could compare with the results of numerical simulations. This allowed us to validate the accuracy of the modeling approach and to identify areas where further improvement may be necessary. In their tests, a total of 30 RC columns were cast from the same batch of concrete to minimize the statistical scatter of the results, which ensured the material properties of all the specimens were virtually the same. A local ready-mix company supplied the concrete. The maximum fine aggregate size was 5 mm, and the maximum coarse aggregate size was 30 mm. The compressive and splitting tensile strengths, determined by standard plain concrete cube samples sized 150 × 150 × 150 mm, were 30.7 and 2.4 MPa, respectively. The elastic modulus of concrete measured was 30 GPa. Longitudinal reinforcement in the RC columns was made from ribbed bars with an average yield strength of 335 MPa. Transversal reinforcement, only designed for suppressing local damage from happening, in the corbel was made from smooth bars with an average yield strength of 235 MPa. Figure 2 shows the longitudinal and transverse rebars of the columns.

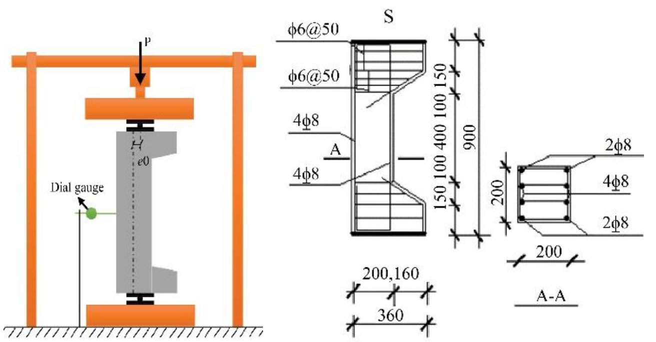

Fig. 2 Loading arrangement and reinforcement detail of columns –Jin et al. (2016)

A total of 3 geometrically similar RC columns sized from 200 × 200 × 900 mm was chosen for numerical modeling with the eccentricities of $0.1h_0$, $0.25h_0$ and $0.9h_0$ (h0 denotes the effective cross-sectional height). In this study, we used the Todeschini method to model the nonlinear compressive strength of the concrete in the Abaqus software, and we incorporated other important material properties, such as the tensile and shear strengths of the concrete and the reinforcing steel, into the model. We also took into account the effects of concrete cracking and the bond between the concrete and the steel reinforcement. Figure 4 shows the stress-strain curve defined as the non-linear compressive behavior of concrete in Abaqus. The material properties of the concrete and the reinforcement should also be specified in the model. The concrete can be modeled using either an elastic-plastic material model or a more sophisticated model that includes nonlinear behavior such as cracking and crushing. The reinforcement can be modeled using either a linear-elastic or nonlinear material model, depending on the level of accuracy required.

Concrete Damage Plasticity (CDP) is a widely used failure criterion for simulating the behavior of concrete under complex loading conditions in finite element analysis (FEA) software. CDP is a continuum damage model that considers the nonlinear behavior of concrete, including cracking and crushing. CDP defines concrete failure as a combination of plastic deformation and damage accumulation (Han et al., 2020). The model accounts for the degradation of the elastic modulus, tensile strength, and compressive strength of concrete due to micro-cracks development and the propagation of cracks under loading. The CDP criterion also considers the evolution of crack patterns in concrete, including the opening and closing of cracks, as well as the frictional sliding of the crack surfaces (Figure 3).

In this research, we used the Concrete Damage Plasticity (CDP) criterion to model the behavior of concrete columns under eccentric loading. By incorporating the CDP criterion into finite element analysis (FEA) simulations, we could capture the complex behavior of concrete under varying load conditions and simulate the formation and propagation of cracks in the concrete. This allowed us to more accurately predict the response of the column to eccentric loading and evaluate its ultimate capacity and failure mode (Lee et al., 2017). Table 1 shows the values defined in the ABAQUS for the main parameters of this criterion.

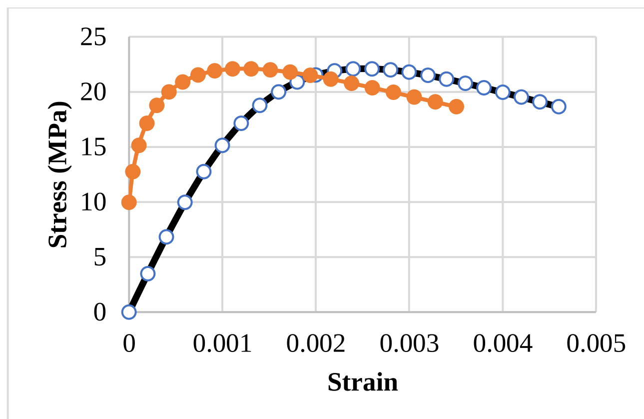

Fig. 3 Concrete compressive stress-strain behavior (Inelastic strain shifted)

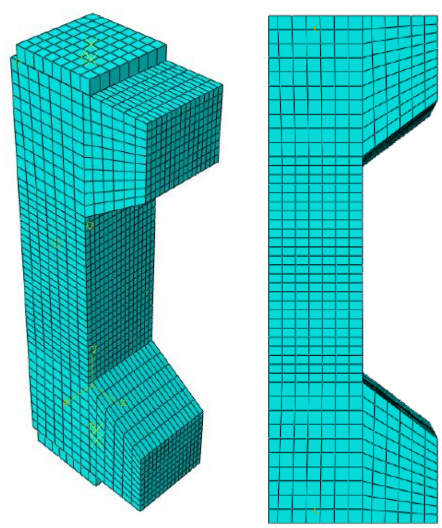

Fig. 4 Concrete column geometry - Meshed for FEA

Table 1. Concrete damage plasticity parameters

Dilation Angle	Eccentricity	Fb0/fc0	K	Viscosity Parameter
38	0.1	1.16	0.667	0.02

In this study, we used a solid element in the meshing module of the Abaqus software to represent the concrete material in the numerical simulations. The solid element is a type of finite element used to represent three-dimensional objects (C3D8R). The solid element is particularly useful for simulating the behavior of homogenous and isotropic materials, such as concrete. The solid element was used to create a mesh of the concrete column, dividing the volume of the concrete material into many small elements, each of which is represented by a single node (Zienkiewicz et al., 2003). In these simulations, the size of the seeds, which is the distance between nodes in the mesh, was set to 30 mm. This means that each element in the mesh was approximately 30 mm in size, allowing us to capture the behavior of the concrete at a very fine level of detail. Using a solid element for the concrete material in these numerical simulations allowed us to accurately simulate the behavior of the concrete column under eccentric loading, including the formation and propagation of cracks in the concrete. The mesh size and element type were chosen to balance the computational cost of the simulation with the desired level of accuracy and detail in the results. Overall, using a solid element in the meshing module of the Abaqus software was an important aspect of this modeling approach. Figure 4 depicts the column geometry with the structure mesh applied.

5. Results and Discussion

In this study, we conducted numerical simulations of three concrete columns with different eccentricities, namely 0.1, 0.25, and 0.9, relative to the effective section height (h_0). The finite element analysis was carried out using Abaqus. The objective was to investigate the effects of eccentric loading on the behavior of concrete columns and compare FEA results with a laboratory model. In the first part of the discussion, we presented figures 5 to 7, which compared concrete damages and cracks between these simulations and the laboratory patterns. The comparison revealed that numerical simulations were in good agreement with the laboratory model, indicating the accuracy of the numerical approach. The results of simulations showed that the lower the h_0 coefficient, the more accurate the compressive behavior of the column, which is consistent with the behavior observed in the laboratory model (figure 5 and 6). When the eccentricity was increased (i.e., larger h_0 coefficient), the bending behavior of the column became more pronounced and overcame the tensile behavior. This behavior was observed in all three columns, and the columns with higher eccentricities exhibited more significant bending behavior (Figure 7). Simulations also allowed us to examine the formation and propagation of cracks in the concrete columns. We observed that the cracks developed primarily in the regions of high-stress concentration, such as the corners of the column. The number and size of the tensile cracks increased with increasing eccentricity, and the formation of larger cracks in the concrete column was observed when the eccentricity was high. Also, the results showed that the compressive behavior of the column was most accurate at lower eccentricities, whereas at higher eccentricities, the column tended to exhibit bending behavior.

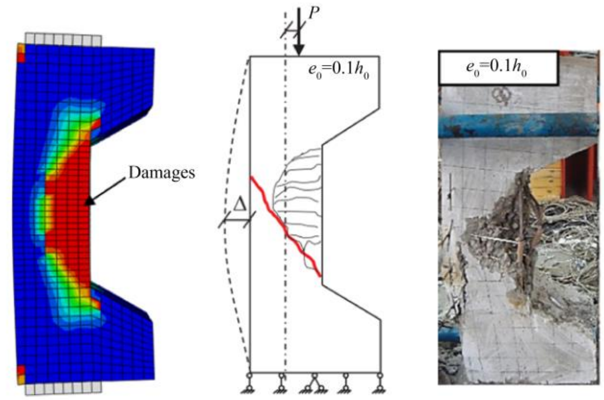

Fig. 5 Comparison of damages and crack distribution in modeled column with laboratory pattern (e_0= 0.1h_0)

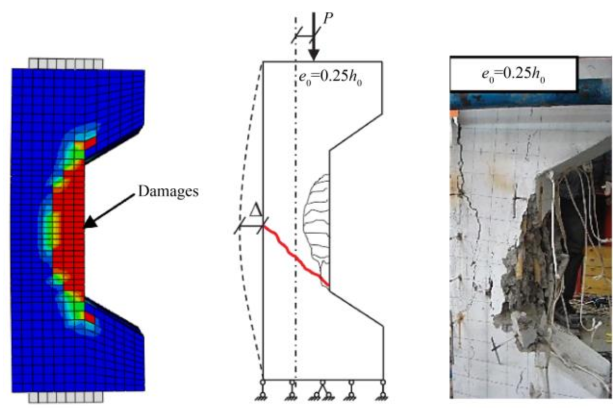

Fig. 6 Comparison of damages and crack distribution in modeled column with laboratory pattern (e_0= 0.25h_0)

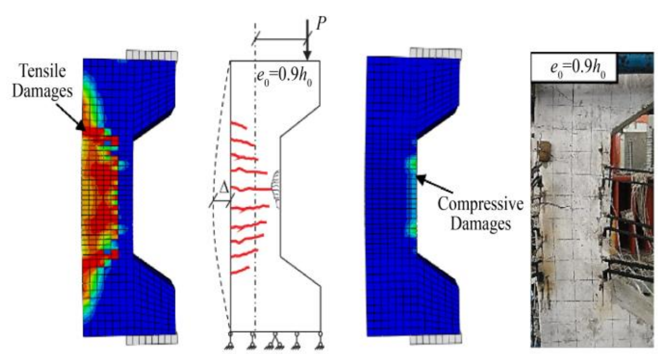

Fig. 7 Comparison of damages and crack distribution in modeled column with laboratory pattern (e_0= 0.9h_0)

In order to fully understand the behavior of concrete columns under eccentric loading, it is important to assess the column's bearing and ultimate capacities. A force-displacement curve is an important tool for this evaluation. Figures 8 to 10 present the bearing and ultimate capacities for eccentricities of 0.1, 0.25, and 0.9 relatives to the effective cross-sectional area of the columns (h_0).

One important observation from the results is the decrease in bearing capacity with increasing eccentricity, which both the laboratory and numerical models correctly predicted. As eccentricity increases from 0.1h0 and $0.25h_0$ to $0.9h_0$, the maximum capacity decreases from 1377 KN and 920 KN to 161 KN, as shown in figures 8 to 10. This decrease in capacity is a result of the increased bending moment caused by eccentric loading. Another significant observation is that the Todeschini behavioral model accurately predicts the behavior of the column when the eccentricity is low. However, when the eccentricity increases to $0.9h_0$, the model does not accurately predict the forces corresponding to the primary displacements, even though it correctly predicts the occurrence of cracks and their location. This discrepancy may be attributed to the increased complexity of the behavior of the column as it approaches failure. Despite this discrepancy, the model predicts the column's final capacity accurately. In summary, the results indicate that the eccentricity of the load strongly influences the bearing and ultimate capacities of concrete columns under eccentric loading. Additionally, while the Todeschini behavioral model accurately predicts the behavior of the column for low eccentricities, it may not be sufficient for predicting the forces corresponding to primary displacements at high eccentricities.

6. Conclusion

Based on the results of this numerical modeling using the Abaqus software, it can be concluded that the behavior of concrete columns under eccentric loading can be accurately predicted by using the Concrete Damage Plasticity (CDP) criterion and the Todeschini method for nonlinear compressive strength. This study showed that as the load's eccentricity increases, the column's bearing capacity decreases, and its behavior becomes more similar to that of a beam column. Additionally, a comparison with a laboratory study conducted by Jin et al. (2016) showed that numerical modeling results were in good agreement with the experimental data. The results of this study have practical implications for the design of concrete columns under eccentric loading, as they highlight the importance of considering the eccentricity of the load in the design process to ensure the safety and stability of the structure. Moreover, this modeling approach and the results can be useful for engineers and researchers who seek to investigate further the behavior of concrete columns under various loading conditions. Overall, this research provides valuable insights into the behavior of concrete columns under eccentric loading and highlights the effectiveness of numerical modeling in predicting the behavior of concrete structures.

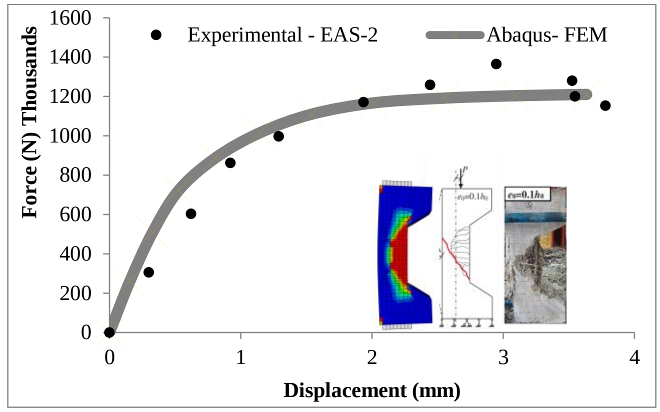

Fig. 8 Load-displacement curve (e_0= $0.1h_0$).

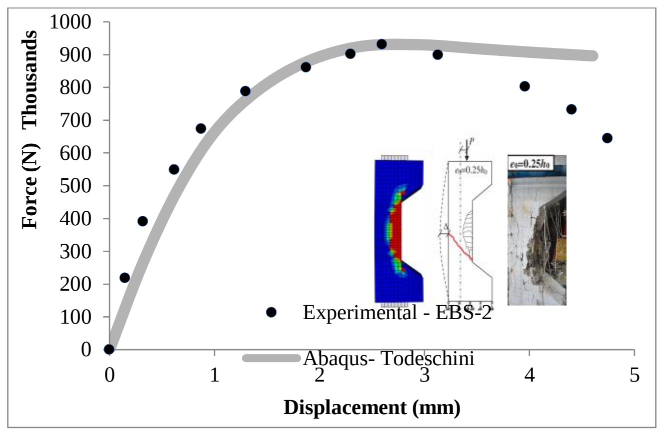

Fig. 9 Load-displacement curve (e_0= $0.25h_0$)

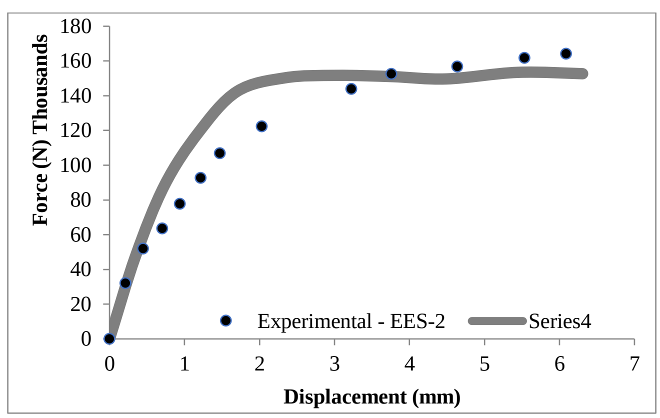

Fig. 10 Load-displacement curve (e_0= $0.9h_0$)

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