

Original Article

# Uplifting Effect on Structural Response to Elastic Foundation Caused by Earthquake Ground Motion

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**Abstract** - The effect of Uplift on structural response to the elastic foundation from earthquake ground motion was investigated. The uplifting of the foundation is an essential factor in structures' response during seismic vibration due to increased flexibility in the structure support. This structure is supported on the two-parameter elastic foundation, which accounts for continuity between the loaded and unloaded parts. Two conditions of foundation contact were analyzed: the foundation on complete contact with the supporting spring elements and the foundation during Uplift. For each of these cases, equations of motion were derived using Newton's second law of motion, applying D'Alembert's principle and considering the moment and lateral equilibrium of forces acting on the structure. Duhamel integral was applied in solving the system response, and the resulting response equations were solved using the numerical solution of Simpson's method. El-Centro maximum earthquake ground acceleration as input was applied for the structural responses. This study suggested that the Uplift of structures does not always reduce the structural response. An increased natural frequency during Uplift may be attributed to the surrounding soil spring element effects on the structure. It was seen that analysis of structural response to uplift occurrence showed an upward trend pointing that Uplift in a system might depend on the nature of excitation, structure parameter and foundation parameter as uplift effects can change the system dynamic properties.

**Keywords** - Dynamics, Earthquake, Elastic foundation, Flexibility, Uplift.

## 1. Introduction

The earth's surface is in continuous slow motion (lithospheric plates) in response to the flow of rock plates within the earth. The rock plate covers the entire surface of the earth, and since the plates are moving, they rub against each other sideways, edges to edges in some places [1, 2]. The strain at the edges of the plates builds up to when it cannot withstand any more bending, and with a lurch, the rock breaks. This sudden plate break releases enormous energy, propagating in various directions through the earth's crust as seismic waves at the earth's surface [1]. This phenomenon causes earthquakes, one of the earth's periodic adjustments in its evolution and may locate any weakness in the structure design. Despite earthquakes' destructive nature, there is evidence that structures can be designed to perform relatively safely. During solid ground motions, the foundation slowly rises due to increasing upward force applied from below due to earthquake forces causing the structure to uplift [3]. For buildings on elastic foundations that allow Uplift, its response to earthquake effects requires careful analysis to build a structure that can safely withstand earthquake forces due to its flexibility.

Foundation uplift is the physical separation of the structure base from the foundation or supporting soil. This separation is expected to be small [4] though it is often inaccessible for observation [5]. Uplift may be assumed to positively affect structures like towers, bridges, piers, etc. since the decrease in the contact area between the soil and foundation might increase the support's flexibility [4, 5]. There will also be a reduction in the soil stiffness, which may cause an increase in the natural Period of oscillation depending on the frequency content of the ground motion [4]. Foundation uplift is necessary for short-period structures as the natural Period of the soil is sensitive to the foundation flexibility, and the moment capacity of rocking mainly depends on the applied vertical load on the foundation [6]. Hence foundation uplift can modulate the distribution and damage level on a frame though the seismic response of a shear wall within a frame depends on the uplift response of its foundation [6]. Sometimes, foundation uplift effects on buildings tend to show an increase in the trend of the structural response [7, 8, 9, 27, 29] hence the need to understand uplift effects on buildings [10].



All superstructures lift off when the foundation moves due to ground motion. Heavy machinery and buildings sometimes lift off their foundations when subjected to extreme ground motions. Different researchers have filed evidence of significant Uplift. The first to report that several tall petroleum-cracking towers stretched their anchor bolts and rocked back and forth on their foundations during the Arvin-Tehachapi, California earthquake in 1952 was proposed [11, 31]. After the Alaska earthquake in 1964, ice was found under some oil tanks, which indicated the Uplift of these oil tanks. Analysis indicated that Veterans Hospital Building 41 had partial uplifting during the San Fernando earthquake [12, 27].

Some heavy power plant equipment was overturned by the Great Nicobar, India, earthquake [13]. Understanding the causes and effects of earthquakes on buildings and how to avoid damage to buildings and loss of lives during earthquakes is essential. The importance of earthquake analysis in structural dynamics is pertinent in light of the catastrophic consequences associated with major earthquakes. Examples include the 1923 earthquake in Tokyo, Japan, where a magnitude of 7.9 struck. The death toll was estimated to have exceeded 140,000, and hundreds of thousands of houses were either shaken down or burned in the ensuing fire by the quake.

Similarly, the San Francisco earthquake in 1906, with a magnitude of 7.8, left 3,000 people dead and destroyed more than 28,000 buildings (about 80% of the city). The 1985 earthquake in Mexico City, a magnitude 8.1, claimed about 10,000 people and 3,000 buildings destroyed [2]. Recently, the 2007 Southern Sumatra Indonesia earthquake, where a triple quake occurred, destroyed lives and properties. The 2010 earthquake in offshore Bio-Bio Chile left over 500 people dead and several buildings devastated. Also, Japan's 2011 earthquake left more than 15,000 people dead several missing, and hundreds of thousands of buildings collapsed. 2017 the two high-magnitude earthquakes in Mexico City left many catastrophic consequences. The recent 2023 earthquake that struck Turkey and Syria also had many catastrophic effects on humans and non-humans. It is clear that there are areas where strong earthquakes frequently occur, requiring buildings to be designed and built accordingly to resist earthquake forces. Over the central part of the globe, significant earthquakes occur so rarely that their effects are generally ignored in building construction. Even so, damaging earthquakes can occur anywhere.

An example is at Agadir, in Morocco, a region generally considered aseismic, where a moderate earthquake in 1960 caused much havoc and casualties to people and buildings. This is because none of the buildings was designed to resist earthquakes [2]. Earthquakes have destructive effects on buildings and human lives hence the need to pay particular attention to the safety of buildings and the response of buildings during earthquakes. Foundation uplift has long been recognized as a significant factor for determining the seismic

vulnerability of structures founded on elastic foundations. Hence, more research is needed on the Uplift of building foundations as it has made modern high-rise buildings the safest places to be during an earthquake [1]. Most of the works on foundation uplift were based on the one-parameter elastic foundation model (Winkler model). The use of the classical Winkler foundation model in most foundation uplift analyses has its shortcoming and deficiency, as there is no continuity between the loaded and unloaded part of the foundation surface [14]. The two-parameter elastic foundation model improves the Winkler model, incorporating shortcomings.

## 2. Literature Review

Uplift of multi-storey building foundations has rarely been observed because the Uplift is expected to be minor, and the foundation-soil interface is often inaccessible for observation [4]. In conventional buildings, the structure is meant to be fixed at the base, and the strength of the structural members resists the earthquake forces in the structure. This is achieved using unnecessary dead weight, large base mat projections and even artificial anchoring schemes [3, 15].

These sometimes make the cost of construction extremely high. However, uplift effects could be significant for strong seismic motion as it is helpful for the future design of the building foundation and superstructure [16, 17, 18, 19, 28] in understanding the influence which Uplift gives to building response for minimal damage. In recent years, there has been an increase in awareness of the effects of Uplift. During the Chilean earthquake of 1960, several golf-ball-on-a-tee types of elevated water tanks survived the ground shaking. In contrast, much more stable reinforced concrete elevated water tanks were severely damaged [4]. These behaviours lead many researchers to investigate uplift effects on structures.

Qin and Chou [15], in their work on the uplift behaviour of a shear frame in an earthquake, to show the effect of different dominant frequencies of ground motions on the response of structures with allowable Uplift, considered two support conditions: a fixed base and a footing with allowable Uplift. The result showed that when Uplift is permitted for the different dominant frequencies applied, a reduction of the forces activated in the structure is observed. The authors showed that foundation uplift can reduce the plastic hinge development of the structure [20, 21].

Iwashita and Taniguchi [16] considered the uplift effect on the earthquake response of buildings with a model consisting of one span and eleven stories of pure frames and another with a shear wall. Their analysis found that the frames with the shear wall model cause more Uplift than the pure frames model. The effect is to decrease seismic force on the building in Uplift since the maximum shear force decreases to about half allowing Uplift in frames with the shear wall. Malhotra [22] used the moment-rotation relationship to carry

out a simplified analysis for the performance-based seismic design of tanks. The analysis showed that unanchored tanks' overturning moment and base shear were reduced by more than 70% from the equivalent fully anchored tank. This reduction is because, in the unanchored tank, Uplift is possible. [7, 8, 12, 23, 30] their works showed that foundation uplift increases structural responses and may not be allowed.

Apostolou et al. [24] analyzed the rocking of rigid structures uplifting from their support under strong earthquake shaking. They investigated the structure resting on a flexible foundation and condition under which uplifting leads to large rotation angles and eventually to overturning. Ormeno et al. [25] worked on the influence of Uplift on liquid storage tanks during earthquakes. In their work, it was deduced that there is an upward increase in response trend.

A comparison of the seismic behaviour of a tank with and without anchorage was considered. Their findings showed that preventing Uplift in storage tanks does not always decrease the seismic forces acting on the structure, as some design documents affirm. It also showed that New Zealand Society for Earthquake Engineering (NZSEE) recommendation on Uplift overestimates the maximum Uplift needed. These studies were performed on a one-parameter elastic foundation model with little analysis of a two-parameter elastic foundation model with shear effects and continuity of the loaded part of the structure.

The elastic foundation model is a model known for its flexibility, elastic and deformable nature, and it includes the one-parameter foundation model (Winkler Model) and the multi-parameter foundation models consisting of the two-parameter models (Filonenko-Borodich model, Pasternak model, Hetenyi model, Vlasov and Loentiev model) and the three-parameter foundation model (Kerr model). Emil Winkler 1867 was the first to introduce the simple representation of the elastic foundation model called the Winkler foundation model [26, 27]. According to the Winkler model, the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point [26, 14].

The shortcomings of the Winkler foundation model, which include discontinuity of the loaded part of the foundation surface and the unloaded part and failure to include the shear effect among the soil springs, led some researchers to propose other models (multi-parameter foundation models), thereby improving on the classical Winkler model. This study investigated the effect of Uplift on the response of buildings resting on a two-parameter elastic foundation model (Filonenko-Borodich model, F-B model) from earthquake ground motion as a means to preventing structural failure.

### 3. Methodology

#### 3.1. System Description

The Filonenko – Borodich (F – B) model is a two-parameter elastic foundation model. It is idealized as a rigid body standing on distributed linear springs. The top ends of the springs are connected to an elastic membrane stretched to a constant 'T' for continuity. This system is presented in Figure 1 without dampers and Figure 2 with dampers.

The system is a building with height 'H' and width '2B' with a concentrated mass 'M' at the centre, and the direction of gravity is assumed to point vertically downwards. The building is assumed to have homogeneous and linear properties connected to an elastic foundation Figure 1. This model system has a foundation that rests on the spring elements by gravity and is not bonded to supporting soil elements. Thus, the supporting soil elements can provide an upward force to the foundation, not a downward pull.

During ground motion causing vibration, this upward reaction force will vary with time. At any instant, when one edge of the foundation gets to the natural unstressed level of the spring elements, that edge undergoes Uplift from the supporting elements. As upward displacement continues, some foundation uplifts from supporting spring elements. For uplift occurrence, two different contacts conditions are distinguished, which include:

- Full contact: here, the structure's base is in complete contact with supporting soil. In this condition, the equations of motion are linear for small displacements. This motion is governed by the standard classical theory of soil structure interaction and differential equations for a single degree of freedom system [4].
- When there is Uplift: Here, there is partial separation (Uplift) of the structure's base from supporting soil elements. The equations of motion are highly nonlinear because of the different degrees of contact between structure and foundation as the system continuously changes from one linear state to another. The governing equations can be derived by considering the lateral equilibrium of forces acting on the structure and the moment equilibrium of forces on the system.

It is important to perform dynamic analysis for structures subjected to dynamic loadings, which involves response spectrum analysis. The amount of Uplift that depends on the excitation from dynamic loads and parameters of the structure and foundation affects the system response. Previous research showed that horizontal translation of the base effects is usually negligible for the dynamic behaviour of structures on an elastic foundation. In this analysis, the horizontal translation is restricted, and there is no slippage between the structure's base and the supporting soil.

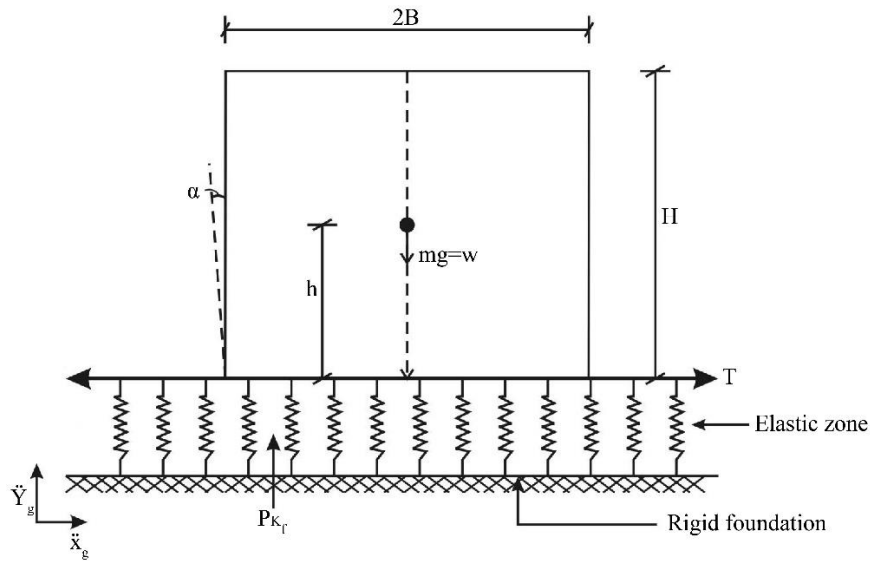


Fig. 1 Sketch of structure on F-B foundation on full-contact without dampers

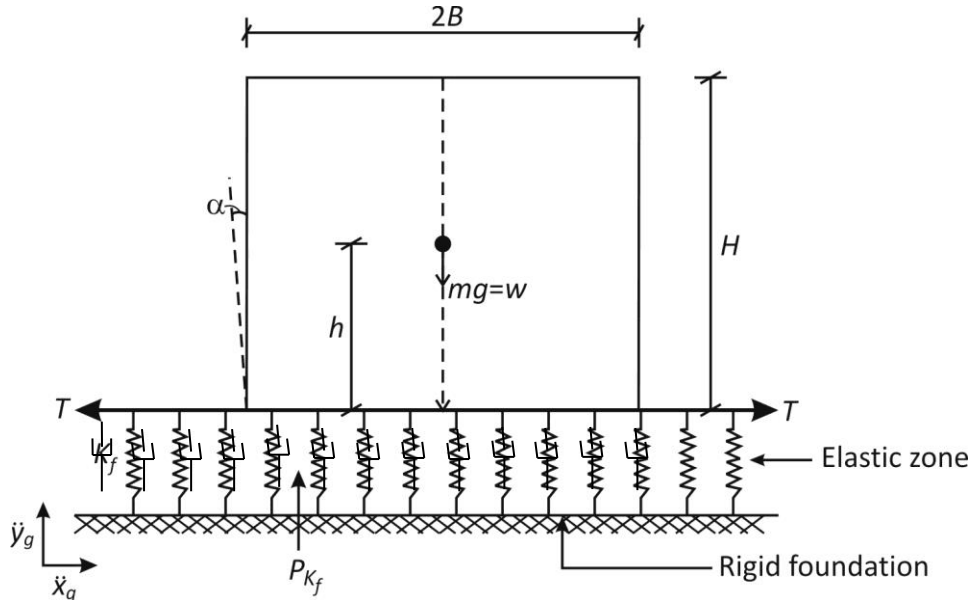


Fig. 2 Sketch structure on F-B foundation model with dashpots on full contact

Equations of motion for the two cases of each model are derived using Newton's second law of motion and by applying D'Alembert's principle. Figure 1 shows a two-dimensional structure on an elastic F-B foundation in the  $x$  and  $y$  directions with its two foundation parameters  $K_f$  and  $T$ . The elastic zone is connected to a base assumed to be rigid. The F-B system has the following properties: Width  $2B$ ; Spring stiffness  $P_{k_f}$  force; Height  $H$ ; Weight acting at the centre  $W$ ; Angle of structure tilting  $\alpha$ ;  $k_f = F - B$  spring stiffness. This system is affected by ground motion with vertical and horizontal components.

The system is assumed to rest on the  $F - B$  spring elements only through gravity force as it is not bonded to soil

elements. Because soil is poor in carrying tensile stresses, foundation uplift tends to occur. This foundation model behaves the same way as Winkler's but for the presence of two foundation parameters,  $K_f$  and  $T$ .

The body is presumed to not slip between the foundation and supporting elements. From this assumption, the body has two degrees of freedom; vertical motion is measured from the position of rest by vertical displacement and rotation by the angle of tilting  $\alpha$  from the vertical. Also, there is the assumption that the body is not bonded to soil elements and rests on F-B spring elements only through gravity. This is because of the poor performance of soil in carrying tensile stresses for foundation uplift.

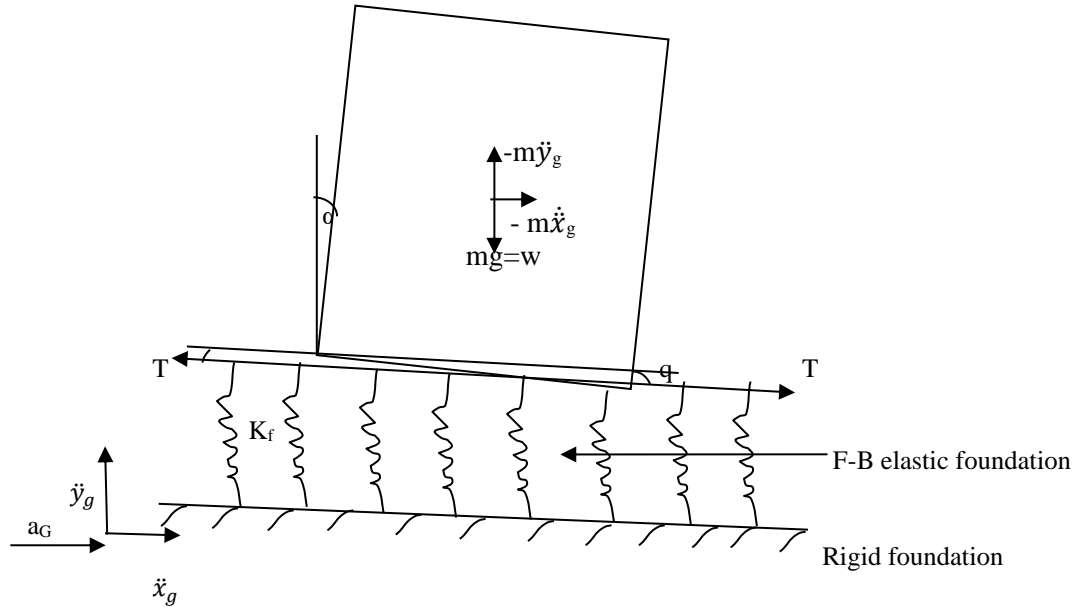


Fig. 3 Structure on F-B uplifting foundation with inertia force from ground motion

### 3.2. Equations of Motion

The equations of motion are derived for the two cases of before and during Uplift. From Figure 3, let;  $\alpha$  = angle of tilting measured from vertical;  $-ma_{Gx}$  = inertia force from ground acceleration acting horizontal;  $-ma_{Gy}$  = inertia force from ground acceleration acting vertical;  $T$  = tension force applied on elastic membrane;  $P_{kf} = (F - B)$  Spring stiffness force;  $W = mg =$  weight of structure at the centre. The structure weight is assumed to be acting at the centre. These are also forces acting in this F-B model.

Using these forces, the equations of motion are derived by considering the equilibrium of forces in the vertical direction and the equilibrium of moment about the centre of the structure for the rocking direction before and after the Uplift. Thus the soil-supporting elements can provide an upward force to the foundation but not a downward pull. During system vibration, this upward reaction force will vary with time and the ground motion acting horizontally. At any instant, when one edge of the foundation reaches the natural unstressed level of the spring elements, that edge is about to develop an Uplift from supporting elements. As upward displacements at that edge continue, a greater part of the foundation uplifts from soil-supporting elements, sometimes leading to overturning.

#### 3.2.1. Before Uplift

##### Vertical Direction:

Using Newton's second law of motion and D'Alembert's principles that  $-ma_{Gx}$  and  $ma_{Gy}$  are the ground acceleration applied as force,  $a_G$  is the input ground motion from El-Centro earthquake. From Figure 1 and Figure 3 and considering the equilibrium of forces in the vertical direction:

$$\sum fy = 0$$

Newton's second law of motion gives that  $F = ma$ , then summing forces in the vertical direction;

$$P_{kf} - W - ma_{Gy} = m\ddot{y} \quad (1)$$

But from Equation 1;

$$P_{kf} = W - 2k_fVy \quad (2)$$

$2V$  is the total distance of spring elements from both edges of the building, as in Figure 4. Substitute Equation 2 into Equation 1 to get:

$$m\ddot{y} + 2k_fVy = -ma_{Gy} \quad (3)$$

##### Rocking Direction:

Considering the equilibrium of moment about the centre of the structure, a horizontal force  $R_A$  was introduced to act between the structure and foundation since the horizontal displacement of the structure was prevented at the base. This is because frictional forces along the structure surface in contact with the foundation were assumed high enough to prevent sliding

$$wh\alpha - 2k_fVy h\alpha + R_A(h + B\alpha) - \frac{2k_fV^3\alpha}{3} - T(h + v\alpha) = 0 \quad (4)$$

But,  $R_A = -ma_G - mh\ddot{\alpha}$ , then,

$$T = -R_A = -(-ma_G - mh\ddot{\alpha}) = (ma_G + mh\ddot{\alpha}) \quad (5)$$

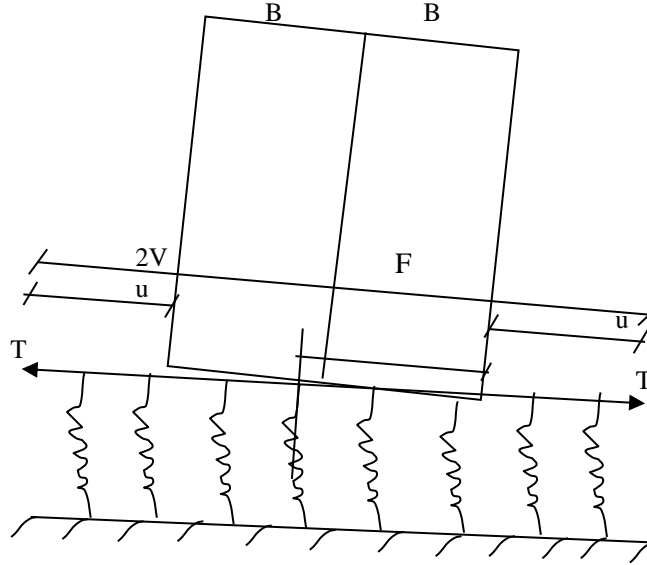


Fig. 4 Structure on F-B foundation with descriptions

Also, let;  $V=B+u$  (6)

Where,  $u$  = distance of elastic membrane from the edge of the building as in Figure 4.

$V$  = the distance from the centre of the structure to the surrounding spring elements where the elastic membrane ended. From Equation 4,

$$R_A(h + B\alpha) = -mha_G - mh^2\alpha - ma_G B\alpha - mhB\alpha\ddot{\alpha} \quad (7)$$

$$T(h + V\alpha) = mha_G + mh^2\alpha + ma_G V\alpha + mhV\alpha\ddot{\alpha} \quad (8)$$

Substitute Equation 7 and 8 into 4;

$$wh\alpha - 2k_f V y h \alpha - mha_G - mh^2\alpha - ma_G B\alpha - mhB\alpha\ddot{\alpha} - \frac{2k_f V^3 \alpha}{3} - mha_G - mh^2\alpha - ma_G V\alpha - mhV\alpha\ddot{\alpha} = 0 \quad (9)$$

Bringing like terms together, using  $I_c = mh^2$ , Equation 9, after rearranging, gives;

$$2I_c\alpha - wh\alpha + 2k_f V y h \alpha + ma_G B\alpha + mhB\alpha\ddot{\alpha} + \frac{2k_f V^3 \alpha}{3} + ma_G V\alpha + mhV\alpha\ddot{\alpha} = -2mha_G \quad (10)$$

Because the displacement is expected to be small and for simplicity, some nonlinear terms in Equation 10 are to be eliminated in linearizing the equation, the nonlinear terms are in the form;  $h\alpha, y\alpha, \alpha^2 V\alpha, B\alpha, \alpha\ddot{\alpha}$ , Then multiply Equation 10 by 'h' and divide with '2I<sub>c</sub>' gives;

$$h\ddot{\alpha} + \frac{k_f V^3 h \alpha}{3I_c} = \frac{-mh^2 a_G}{I_c} \quad (11)$$

Equation 3 and 11 are equations of motions for the foundation model when it is on full contact regime for the vertical and rocking directions.

### 3.2.2. During Uplift

#### Vertical Direction

Calculating forces in the vertical direction, the length of contact during vibration is no longer '2V' but 'V<sub>1</sub>' so that;

$$V_1 = F + U \quad (12)$$

Then using Newton's second law of motion and summing forces in the vertical direction;

$$\sum f_y = 0; F = ma;$$

$$P_{kf} = W - \frac{1}{2} k_f V_1 q \quad (13)$$

Using Equation 1 and substituting Equation 13 into it;

$$m\ddot{y} + \frac{1}{2} k_f V_1 q = -ma_{Gy} \quad (14)$$

Let,

$$V_1 = V + \frac{d}{\alpha} - \frac{y}{\alpha} \quad (15)$$

$$q = d - y + V\alpha \quad (16)$$

Then multiply Equation 15 and 16 and getting like terms together gives;

$$V_1 q = 2dV - 2Vy + V^2\alpha + \frac{d^2}{\alpha} - 2\frac{dy}{\alpha} + \frac{y^2}{\alpha} \quad (17)$$

The Static deflection being the deflection of the structure due to its own mass weight, is,

$$d_f = \frac{W}{2k_f V} \quad (18)$$

Substitute Equation 18 into 17 gives that

$$V_1 q = \frac{W}{k_f} - 2Vy + V^2 \alpha + \frac{W^2}{4k_f^2 V^2 \alpha} - \frac{Wy}{k_f V \alpha} + \frac{y^2}{\alpha} \quad (19)$$

Relating Equations 13 and 14, then eliminating nonlinear terms and dividing with 'm' gives;

$$\ddot{y} + \frac{k_f V y}{m} - \frac{k_f}{2m} V^2 \alpha = -a_{Gy} - \frac{W}{2m} \quad (20)$$

This can also be written as;

$$\ddot{y} + \frac{k_f V}{m} \left[ y - \frac{V \alpha}{2} \right] = -a_{Gy} - \frac{W}{2m} \quad (21)$$

Equation 21 is the equation of motion for the vertical direction during Uplift.

#### Rocking Direction

Considering the equilibrium of moments for the rocking direction about the centre of the structure;

$$wh\alpha - \frac{2k_f V^3 \alpha}{3} - 2k_f V y h \alpha + R_A(h + B\alpha) - T(h + v\alpha) - \frac{k_f}{2} \left[ V_1 q \times \frac{V_1}{3} \right] h \alpha = 0 \quad (22)$$

From Equation 22;

$$V_1 q \times \frac{V_1}{3} = \left( 2dV - 2Vy + V^2 \alpha + \frac{d^2}{\alpha} - 2 \frac{dy}{\alpha} + \frac{y^2}{\alpha} \right) \times \left( \frac{V}{3} + \frac{d}{3\alpha} - \frac{y}{3\alpha} \right) \quad (23)$$

Solving Equations 23 and 22 and applying Equation 18 gives that;

$$\frac{k_f}{2} \left[ V_1 q \times \frac{V_1}{3} \right] h \alpha = \frac{wVh\alpha}{4} + \frac{w^2 h}{8k_f} - \frac{why}{2} - \frac{k_f V^2 y h \alpha}{2} + \frac{k_f h V y^2}{2} + \frac{k_f V^3 h \alpha^2}{6} + \frac{w^3 h}{48k_f^2 V^3 \alpha} - \frac{w^2 h y}{8k_f V^2 \alpha} + \frac{why^2}{4V\alpha} - \frac{k_f h y^3}{6\alpha} \quad (24)$$

Using Equations 24 and 8 into 22, eliminating nonlinear terms, multiplying with 'h' and divide by 2I<sub>c</sub> gives that;

$$h\ddot{\alpha} + \frac{k_f V^3 h \alpha}{3I_c} - \frac{wh^2 y}{4I_c} = \frac{-mh^2 a_G}{I_c} - \frac{w^2 h^2}{16I_c k_f V} \quad (25)$$

Equation 25 can also be written as;

$$h\ddot{\alpha} + \frac{k_f V^2 h}{I_c} \left[ \frac{V\alpha}{3} - \frac{why}{4k_f V^2} \right] = \frac{-mh^2 a_G}{I_c} - \frac{w^2 h^2}{16I_c k_f V} \quad (26)$$

Equations 21 and 26 are equations of motion of the foundation model during Uplift.

### 3.3. Solution of Equations of Motion

The equations of motion before Uplift, Equation 3 and 11 and during uplift Equation 21 and 26 are solved using the classical solution, summing the complementary and particular solution and applying the Duhamel integral, then using the system's initial conditions before and during Uplift.

#### 3.3.1. Before Uplift

For the case of vertical and rocking directions, the initial condition here is the at rest condition.

$$\dot{y}(t) = -\frac{\sin\omega_7 t}{\omega_7} \int_0^t a_{Gy}(\tau) \cos\omega_7 \tau d\tau + \frac{\cos\omega_7 t}{\omega_7} \int_0^\tau a_{Gy}(\tau) \sin\omega_7 \tau d\tau \quad (27)$$

$$\dot{x}(t) = -\frac{\sin\omega_8 t}{\omega_8} \int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau) \cos\omega_8 \tau d\tau + \frac{\cos\omega_8 t}{\omega_8} \int_0^\tau \frac{mh^2 a_{Gx}}{I_c}(\tau) \sin\omega_8 \tau d\tau \quad (28)$$

Say that from Equation 27, the simple harmonic motion is represented as;

$$y(t) = -A(t)\sin\omega_7 t + B(t)\cos\omega_7 t \quad (29)$$

From Equation 29:

$$A(t) = \frac{1}{\omega_7} \int_0^t a_{Gy}(\tau) \cos\omega_7 \tau d\tau \quad (30)$$

$$B(t) = \frac{1}{\omega_7} \int_0^t a_{Gy}(\tau) \sin\omega_7 \tau d\tau \quad (31)$$

For Equation 28,

$$x(t) = -A(t)\sin\omega_8 t + B(t)\cos\omega_8 t \quad (32)$$

From Equation 32 where:

$$A(t) = \frac{1}{\omega_8} \int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau) \cos\omega_8 \tau d\tau \quad (33)$$

$$B(t) = \frac{1}{\omega_8} \int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau) \sin\omega_8 \tau d\tau \quad (34)$$

Simpson's rule is now used to evaluate for the structure response values in the vertical and rocking directions numerically.

### 3.3.2. During Uplift

The system's initial condition is the initial time of the start of Uplift. This is the time the static deflection of the system is equal to the vertical displacement of the system. But then assuming these conditions as the time of system uplift and these conditions were gotten from the point of before uplift analysis, Then,

$$y(t) = 4.5 \times 10^{-6} \cos \omega_{7A} t + \frac{6.5 \times 10^{-4}}{\omega_{7A}} \sin \omega_{7A} t - \frac{1}{\omega_{7A}} \int_0^t [a_{Gy} + \frac{W}{2m}] (\tau) \sin \omega_{7A} (t - \tau) d\tau \quad (35)$$

$$x(t) = 2.82 \times 10^{-6} \cos \omega_8 t + \frac{7.11 \times 10^{-4}}{\omega_8} \sin \omega_8 t - \frac{1}{I_c \omega_8} \int_0^t [mh^2 a_{Gx} + \frac{W^2 h^2}{16k_f V}] (\tau) \sin \omega_8 (t - \tau) d\tau \quad (36)$$

From which,

$$y(t) = 4.5 \times 10^{-6} \cos \omega_{7A} t + \frac{6.5 \times 10^{-4}}{\omega_{7A}} \sin \omega_{7A} t - (A(t) \sin \omega_{7A} t + B(t) \cos \omega_{7A} t) \quad (37)$$

$$x(t) = 2.82 \times 10^{-6} \cos \omega_8 t + \frac{7.11 \times 10^{-4}}{\omega_8} \sin \omega_8 t - [A(t) \sin \omega_8 t + B(t) \cos \omega_8 t] \quad (38)$$

Where for Equation 37

$$A(t) = -\frac{1}{\omega_{7A}} \int_0^t [a_{Gy} + \frac{W}{2m}] (\tau) \cos \omega_{7A} \tau d\tau \quad (39)$$

$$B(t) = \frac{1}{\omega_{7A}} \int_0^t [a_{Gy} + \frac{W}{2m}] (\tau) \sin \omega_{7A} \tau d\tau \quad (40)$$

Similar procedure applies to Equation 38 to have;

$$A(t) = -\frac{1}{I_c \omega_8} \int_0^t [mh^2 a_{Gx} + \frac{W^2 h^2}{16k_f V}] (\tau) \cos \omega_8 \tau d\tau \quad (41)$$

$$B(t) = \frac{1}{I_c \omega_8} \int_0^t [mh^2 a_{Gx} + \frac{W^2 h^2}{16k_f V}] (\tau) \sin \omega_8 \tau d\tau \quad (42)$$

The numerical integration solved by Simpson's method

### 3.3.3. Damping Effects

Most engineering systems possess damping as damping forces dissipate energy. Damping is a property of functional structures that causes vibrations to die away quite rapidly. The effect of damping is to increase the Period of natural frequency and then make the resonant frequency somehow less than the value obtained when there is no damping. In most practical structures, damping is considered small and very light, so the damped natural frequency is hardly distinguishable from the

un-damped natural frequency [2]. Friction at connections, micro-cracks in concrete and friction between the parts are some of the known damping types in constructions that influence the oscillatory and vibratory systems. The dashpots are introduced to the idealized F-B foundation model, as shown in Figure 2, to dissipate energy going into the structure. With the introduction of dashpots to the system and going with the derivation and solution of previous equations of motion when there is no damping, the damped equations of motion include;

Before Uplift:

$$y(t) = e^{-\omega \xi t} [y(0) \cos \omega_{d7} t + \frac{\dot{y}(0) + \omega \xi y(0)}{\omega_{d7}} \sin \omega_{d7} t] - \frac{1}{\omega_{d7}} \int_0^t a_{Gy} (\tau) e^{-\omega \xi t} \sin \omega_{d7} (t - \tau) d\tau \quad (43)$$

$$x(t) = e^{-\omega \xi t} [x(0) \cos \omega_{d8} t + \frac{\dot{x}(0) + \omega \xi x(0)}{\omega_{d8}} \sin \omega_{d8} t] - \frac{1}{\omega_{d8}} \int_0^t \frac{mh^2 a_{Gx}}{I_c} (\tau) e^{-\omega \xi t} \sin \omega_{d8} (t - \tau) d\tau \quad (44)$$

During Uplift:

$$y(t) = e^{-\omega \xi t} [y(0) \cos \omega_{d7A} t + \frac{\dot{y}(0) + \omega \xi y(0)}{\omega_{d7A}} \sin \omega_{d7A} t] - \frac{1}{\omega_{d7A}} \int_0^t [a_{Gy} + \frac{W}{2m}] (\tau) e^{-\omega \xi t} \sin \omega_{d7A} (t - \tau) d\tau \quad (45)$$

$$x(t) = e^{-\omega \xi t} [x(0) \cos \omega_{d8} t + \frac{\dot{x}(0) + \omega \xi x(0)}{\omega_{d8}} \sin \omega_{d8} t] - \frac{1}{I_c \omega_{d8}} \int_0^t [mh^2 a_{Gx} + \frac{W^2 h^2}{16k_f V}] (\tau) e^{-\omega \xi t} \sin \omega_{d8} (t - \tau) d\tau \quad (46)$$

For the case of during Uplift, the system's condition is that the initial time taken as the time of incipient Uplift, which is the static time deflection of the system, is equal to the vertical displacement of the system. The Simpson's ordinate multiplier is now affected by damping with a damping ratio of 0.05, and also, because the structure is assumed to be lightly damped [2], the damped frequency and un-damped frequency are equal. Thus, the damped and un-damped frequencies are the same for each vibration regime. In the following solution, initial conditions of the system are applied for the F-B models, and the equations are evaluated numerically using Simpson's rule.

### 3.4. Method Procedure

From this analysis procedure, the structure is assumed to undergo small displacements, and the vibratory system consists of a sequence of linear problems from the foundation model Filonenko-Borodich (F-B). It has two degrees of freedom. First, vertical motion is measured from the position of rest by vertical displacement 'y' at the centre of mass.



Second, the rotation is measured by the angle of tilting ' $\alpha$ ' from the vertical, as seen in Figure 1 and Figure 2. The horizontal direction is usually negligible for the dynamic behaviour of structures supported by an elastic foundation. Therefore only the vertical and rocking or rotation effects were considered. Two foundation uplift regimes were analyzed, including foundation on full contact with the supporting spring elements and Uplift.

For each of these regimes or cases, equations of motion were derived for the system's two degrees of freedom at vertical and rocking directions. The equations of motion were derived using Newton's second law of motion and D'Alembert's principle. The Duhamel integral was applied in solving the system response. The forcing function resulting from the support motion problem was taken as the seismographic record of the El-Centro earthquake ground

acceleration data of 1940, which is 0.32G, shown in Figure 5. This is the maximum ground acceleration during this earthquake, and 'G' is the acceleration due to gravity in  $m/s^2$ .

The seismographic record of the El-Centro earthquake ground acceleration data of 1940 can be seen in Figure 5. The maximum ground acceleration of  $3.2m/s^2$  was obtained from the seismographic record of 0.32g and used as the input motion. The natural frequency ' $\omega_n$ ' and period ' $T$ ' of system vibration from Table 1 were calculated and used to get the time intervals for the analysis for each regime before and during Uplift. Simpson's method was applied for the numerical analysis of the structural responses for the different regimes. The system's static deflection was 0.000186m due to its self-weight or weight due to gravity, as given in Equation 18.

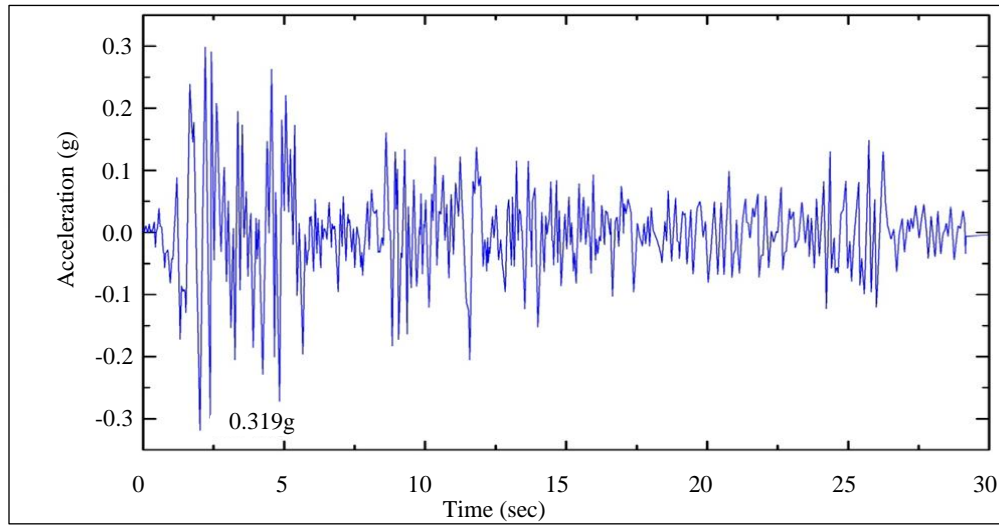


Fig. 5 Time history of El-Centro, 1940 earthquake ground motion (The time history of acceleration of North-South component of the El-Centro, 1940 earthquake ground motion), Chopra, 2011

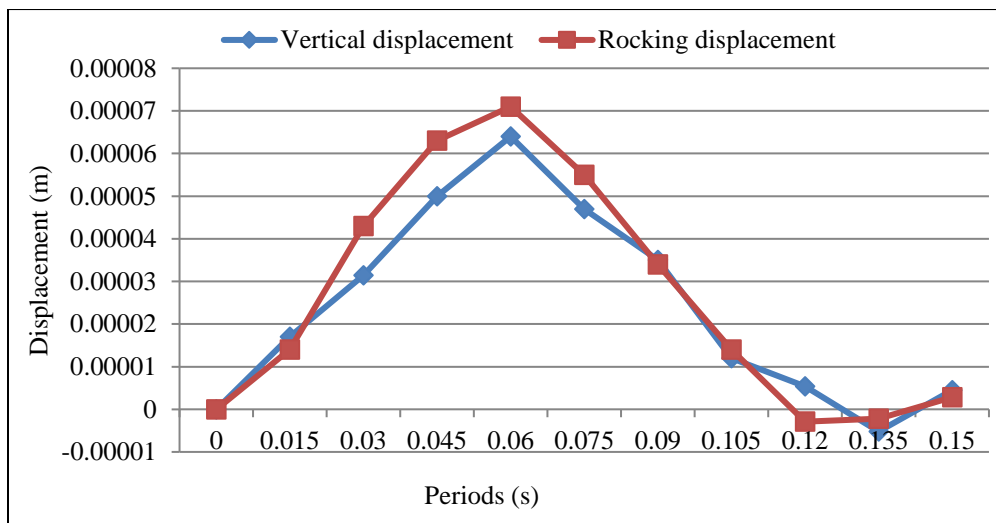
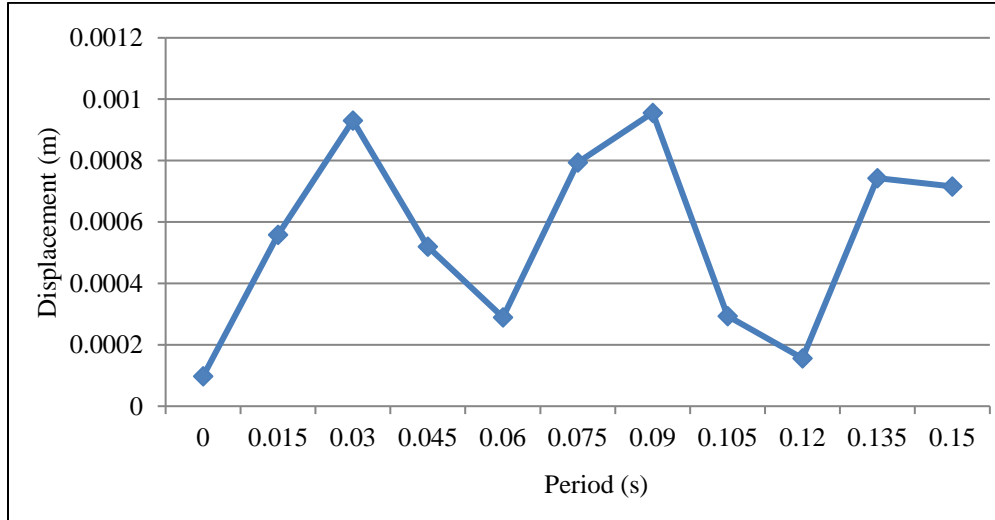


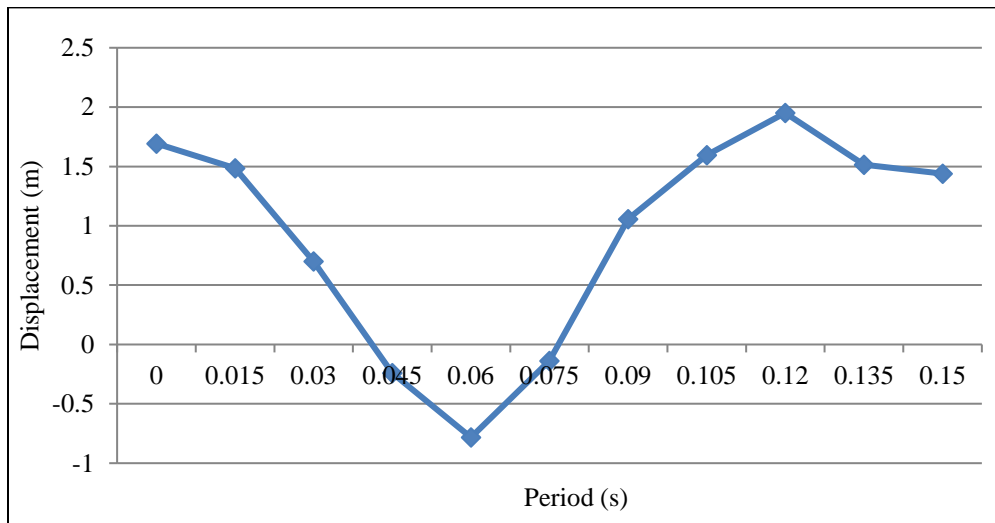
Fig. 6 Displacement effects before uplift

**Table 1. Natural frequency and period**

S. No	Model Type	Before Uplift		During Uplift	
		Vertical	Rocking	Vertical	Rocking
1	Natural frequency	36.47	50.13	18.24	53.27
2	Natural Period	0.027	0.02	0.0548	0.0187



**Fig. 7 Vertical displacement effects during uplift**



**Fig. 8 Rocking displacement effect during uplift**

Uplift will occur when the vertical displacement of the system before Uplift is equal to the static displacement. The vertical displacement from this analysis is far less than the static displacement; hence there is less likelihood of uplift occurrence in this analysis, as in Figure 6, compared to the static deflection. Including vertical components is significant in dynamic analysis, especially from earthquake motions, as vertical motions predominantly cause structural damage. The IS 1893 (part 1) 2016 code specifies that earthquake-generated vertical force effects should be given special attention and

considered in designs unless checked and proven by calculation to be insignificant for adequate earthquake effects.

For this analysis, the vertical displacement was less than the static displacements; hence there was less likelihood of uplift occurrence here. Vertical displacements are important when foundation uplift becomes pronounced, as Uplift can induce significant vertical displacement on structural response. This is because, at this stage, the static deflection of the structure can be affected, thereby being exceeded. The

result obtained in the analysis showed that the relative decrease in vertical displacement with respect to static deflection increased the rocking aspect. Hence accurate calculation in the vertical direction is important for analysing uplift problems since uplift occurrence depends on its vertical displacement before Uplift for seismic loading.

Table 1 shows that the system's fundamental natural frequency before Uplift is more remarkable than when Uplift occurs for the vertical and rocking direction. This reduction of natural frequency during Uplift results from the increased flexibility of the system. The flexibility of the system helps to reduce energy going into the structure. The F-B model tends to have an increased natural frequency during Uplift in the rocking direction from Table 1, which can result from the effects of surrounding soil spring elements on the structure. The natural periods of oscillation of the structure from Table 1 increased with the occurrence of Uplift as there might be a

reduction in the soil stiffness, which might have caused an increase in the natural Period of oscillation when Uplift occurs depending on the frequency content of the ground motion.

This effect is also shown in Figure 7 and Figure 8 below, hence the results obtained showed that the elongation of the natural Period is very significant. Foundation uplift is important when analyzing short-period structures, as the natural Period of the soil is sensitive to the foundation flexibility. However, considering the uplift effect, reducing the contact area between the soil and foundation increases support flexibility, thereby reducing soil-structure stiffness. This might increase in the natural Period of oscillation. So that during Uplift, there is the elongation of the Period of oscillation, or the Period increases as the foundation becomes softer.

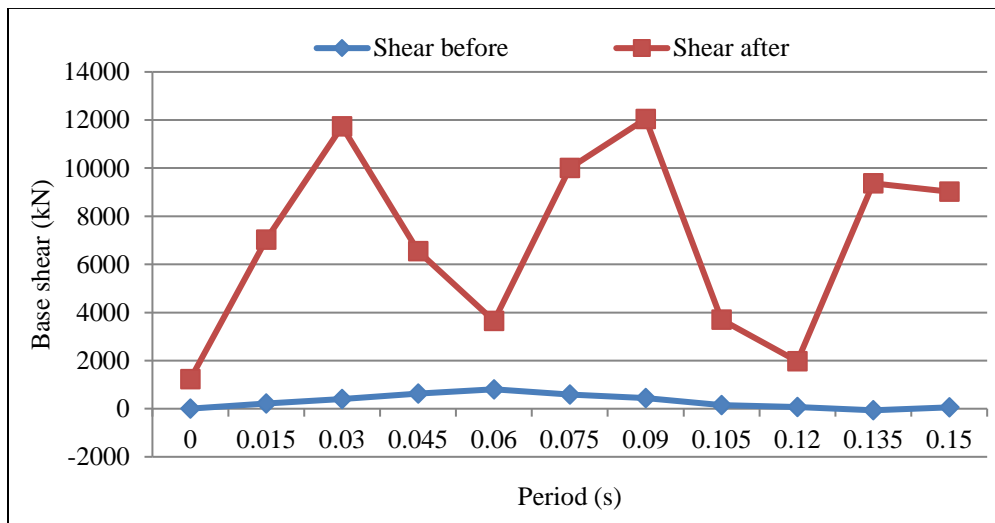


Fig. 9 Base shear effects of before and after uplift

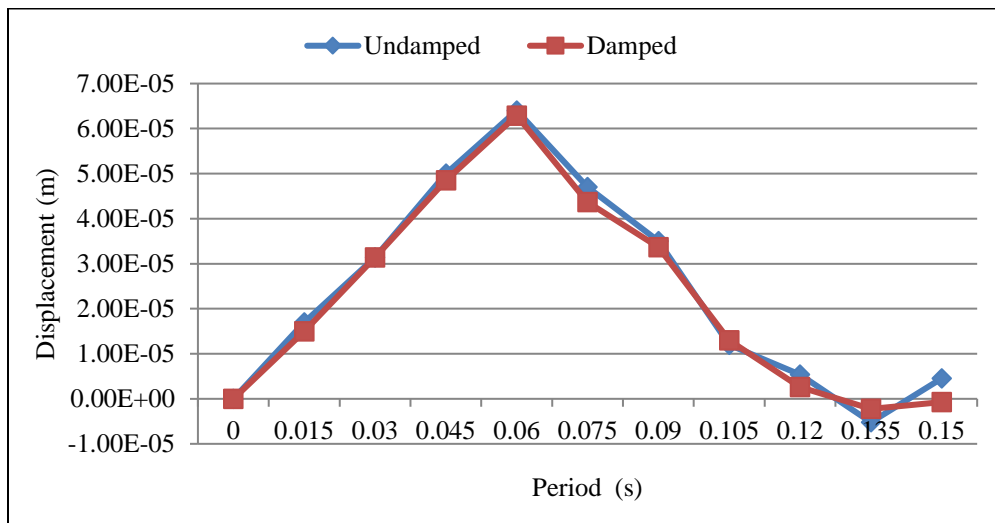


Fig. 10 Effect of damping on displacement vertical component before uplift

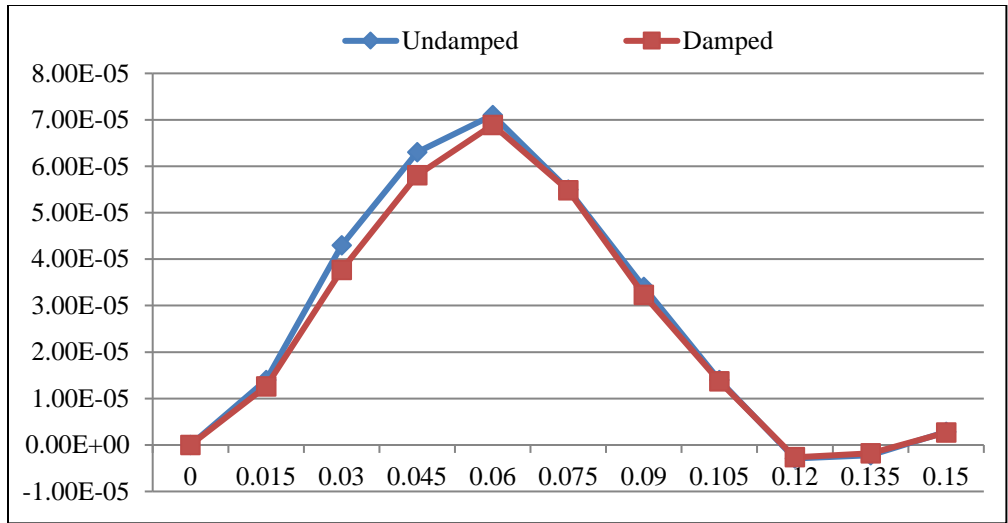


Fig. 11 Effect of damping on displacement rocking component before uplift

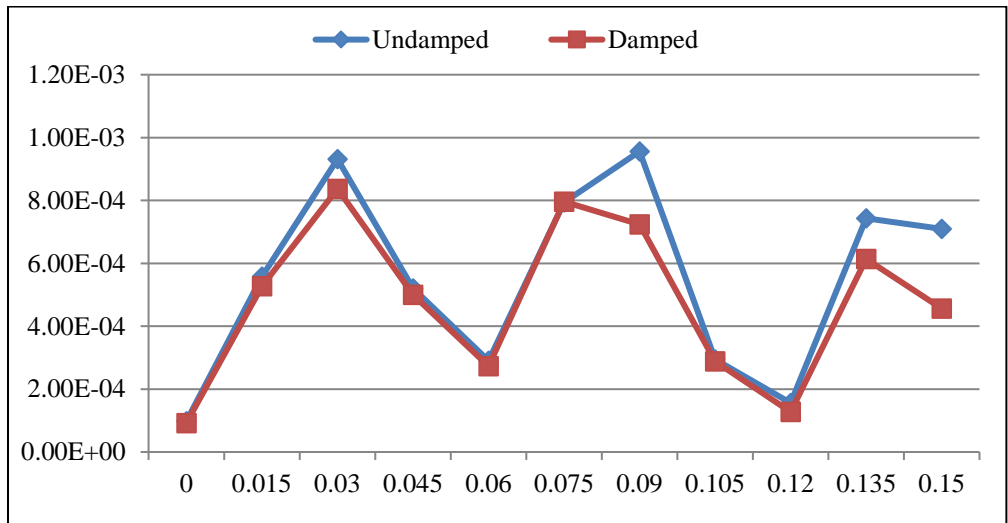


Fig. 12 Effect of damping on displacement vertical component after uplift

The initiation of uplifting can be quite beneficial for the superstructure under certain conditions relating to the fundamental Period of the structure and characteristics of ground shaking. Observing the vertical and rocking frequencies above showed increased frequencies in the rocking direction, and as a result, the rocking direction may be stiffer. The structure tends to have an increased natural frequency during Uplift from Table 1, which may be attributed to the effects of surrounding soil spring elements on the structure and the natural Period reduction.

The result of base shear representing the expected lateral force coming from the ground motion at the base of the structure also showed that during Uplift, there is an upward trend in the forces generated. This can be attributed to the increased flexibility in the structure support. The amplitude of the base shear during Uplift showed an upward trend than the case before Uplift as in Figure 9. The damped and un-damped

displacement effect shows that damping reduced the system response to Uplift. Figure 10 to Figure 12 show damping effects on displacement as it reduced uplifting on the system.

#### 4. Conclusion

The ground motion associated with earthquakes is very complex, with different components. This study examined the non-linearity behaviour of the system response in the neighbourhood of a linear state to determine its equilibrium during Uplift. The study suggested that the uplifting of structures during an earthquake does not always reduce the structure response, as some studies have shown. An increased natural frequency during Uplift may be attributed to the surrounding soil spring element effects on the structure. Also, analysis of structural response to uplift occurrence showed an upward trend pointing that allowing Uplift in a system can affect its safety as there might be a structural failure. In the design of conventional buildings, structures are meant to be

bonded to the base, and the weight and strength of the structural members resist the earthquake forces in the structure by gravity. This can be achieved by increasing the dead weight of the structure, using large base mat projections and even artificial anchoring schemes. Though these sometimes made the cost of construction extremely high but can be encouraged to minimize structural damages during ground motion. Also, damping reduced the structure response before and during Uplift showing the efficiency of dampers in energy dissipation as seen in Figure 10 to Figure 12, thereby reducing the chances of structure overturning. Uplift effects can change the system's dynamic properties; thus, earthquake responses then depend on the nature of excitation, structure parameters and

foundation parameters; hence, it cannot be deduced whether Uplift is beneficial. Sometimes, soil spring elements surrounding the structure can influence its structural responses.

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